On t^*-Generalized Compact and t^* -Generalized C-Compact Mappings

Shymaa Hussain Salih
Applied Sciences Department, University of Technology/ Baghdad
Email:hussain.shaymaa@yahoo.com

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ABSTRACT

This paper, will introduce some types of compact mapping called $t^* -$generalized compact mapping by using $t^* -$generalized compact sets and giving some properties of their mappings, Moreover, showing the relationship between these types.

Keywords: (($t^* - g$ -compact set, g- compact set, g- closed set , compact mapping , C-compact mapping, strong $t^* - G -$ compact mapping,))

حول تطبيقات t^* المولدة المتراصة و t^* المولدة المتراصة في الفضاءات التبولوجية

الخلاصة

يقدم هذا البحث تم تقديم أنواع من التطبيقات المتراصة وتسمى التطبيقات المتراصة-g $t^* -$ باستخدام المجموعات المتراصة $t^* -$g- وإعطاء بعض خواص هذه الدول بالإضافة إلى إثبات العلاقة بين هذه الأنواع.

INTRODUCTION

The new types of mapping introduced by Halfar in 1957 [3], which namely compact mapping, and he gave some properties of these mappings, also many researchers have studied the generalized types of these mapping also introducing new class of compact space is called C-compact space upon some types of sets.

The aim of this work, is to study some class of compact mapping which are $t^* -$generalized compact mapping by using $t^* -$generalized compact sets and $t^* -$generalized C-compact mapping, giving some properties and characterization of these types of mappings, moreover, showing the relationship between these mappings.

FUNDAMENTAL DEFINITIONS AND REMARKS

In this section, let us give some basic definitions and notations of compact mappings. **2.1 Definition:** [1]

A subset A of a topological space (X,T) is called generalized closed set (g-closed in short) if $cl(A)\subseteq W$ whenever $A\subseteq W$, where W is open set in X. The complement of A is called generalized open set (g-open in short).

Remarks: [1]

(1) Every closed (open) set is g-closed (g-open) set, but the converse is not necessary to be true.

(2) *If* X *is* $T_{\frac{1}{2}}$ *- space then the converse of (1) is true.*

In [6], using definition (2.1), to give the following definition

Definition

A subset A of a topological space (X,T) is called generalized compact set (g-compact in short) if every g-open cover has finite g-open subcover.

It is clear that, every g-compact set is compact set but the converse is not true in general, but the converse is true when X is $T_{\frac{1}{2}}$ - space.

Now, we give the definitions appeared in [5].

Definitions

Let (X,T) be a topological space and $A\subseteq X$, then

(1) the intersection of all g-closed sets containing A is said to be the generalized operator and denoted by $cl^*(A)$.

(2) the topology t^* is defined by $t^*=\{G: cl^*(G^c)=G^c\}$, where (X,t^*) is topological space on X.

Next, we recall the definition of t^*-generalized closed set appear in [5].

Definition

Let (X,T) be a topological space, a subset A of X is said to be t^*-generalized closed set (t^*-g-closed in short) if $cl^*(A)\subseteq W$ whenever $A\subseteq W$, where W is g-open set in X. and the complement of A is called generalized open set (t^*-g-open in short).

Remarks [5]

(1) It is easy to check that, every g-closed (g-open) set is t^*-g-closed (t^*-g-open) set but the converse is not true in general.

(2) A topological space (X,t^*) is called t^*-T_g- space if t^*-g-closed (t^*-g-open) set is g-closed (g-open) set.

By using definition (2.5), S. Eswaran and A. Pushalatha introduce new topological space is called t^*-generalized compact space (t^*-g-compact in short) and defined as follow:

Definition

Let (X,t^{*}) be a topological space, a subset A of X is said to be t^{*}-generalized compact set (t^{*}-g-compact in short) if every t^{*}-g-open cover has finite subcover.

Proposition

Let (X,t^{*}) be a topological space. If A is t^{*}-g- compact set then A is g-compact set.

Proof

Let A is t^{*}-g- compact set and $\{U_i : i \in I\}$ is g-open cover of A and by using remark () , can have $\{U_i : i \in I\}$ is t^{*}-g-open cover of A but A is t^{*}-g-compact , so, has finite subcover. Therefore A is g-compact set.

But, the converse of above proposition is not necessary to be true and became true when (X,t^{*}) is $t^{*} - T_g -$ space.

t^{*}-GENERALIZED COMPACT MAPPINGS

in this section, we give certain types of t^{*}-generalized compact mapping by using t^{*}-g-compact set and shows the relation between these types. We start by the following definition is modified of definition appeared in [5].

Definition

A mapping $f:(X,t^{*}) \longrightarrow (Y,s^{*})$ is said to be t^{*}-generalized compact mapping (t^{*}-G-compact in short) if for every t^{*}-g-compact subset K of Y then $f^{-1}(K)$ is g-compact subset of X.

Definition

A mapping $f:(X,t^{*}) \longrightarrow (Y,s^{*})$ is said to be strongly t^{*}-generalized compact mapping (strongly t^{*}-G-compact in short) if for every g-compact subset K of Y then $f^{-1}(K)$ is t^{*}-g-compact subset of X.

Now, the following proposition shows the relation between t^{*}-G-compact and strongly t^{*}-G-compact mappings.

Proposition

If $f:(X,t^{*}) \longrightarrow (Y,s^{*})$ is strongly t^{*}-G-compact mapping f is t^{*}-G-compact mapping.

proof

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be strongly t^*-G-compact mapping and K be t^*-g-compact subset K of Y, so by using proposition (2.8) we get K is g-compact set and from definition (3.2), one can have $f^{-1}(K)$ is t^*-g-compact subset of X. thus $f^{-1}(K)$ is g-compact set, therefore f is t^*-G-compact mapping.

It is clear that, the converse of above proposition is not true in general and the following corollary give the condition in order to the converse of above proposition is true.

Proposition

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is t^*-G-compact mapping and X,Y are t^*-T_g-space then f is strongly t^*-G-compact mapping.

Next, the following definition give another type of t^*-G-compact mapping.

Definition

A mapping $f:(X,t^*)\longrightarrow(Y,s^*)$ is said to be irresolute t^*-generalized compact mapping (irresolute t^*-G-compact in short) if for every t^*- g-compact subset K of Y then $f^{-1}(K)$ is t^*-g-compact subset of X.

Now, the following proposition shows the relation between irresolute t^*-G-compact and strongly t^*-G-compact mappings.

Proposition

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is strongly t^*-G-compact mapping then f is irresolute t^*-G-compact mapping.

proof

let $f:(X,t^*)\longrightarrow(Y,s^*)$ be strongly t^*-G-compact mapping and K be t^*-g-compact subset K of Y, so by using proposition (2.8) we get K is g-compact set and from definition (3.2), one can have $f^{-1}(K)$ is t^*-g-compact subset of X. therefore f is irresolute t^*-G-compact mapping.

It is easy to cheek that, the converse of above proposition is not necessary to be true and the following corollary give the condition to make the converse is true.

Corollary

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is irresolute t^*-G-compact mapping and Y is t^*-T_g-space then f is strongly t^*-G-compact mapping.

Also, the relation between t^*-G-compact and strongly t^*-G-compact mappings can be shows by the following proposition.

Proposition

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is irresolute t^*-G-compact mapping.Then f is t^*-G-compact mapping.

proof

let $f:(X,t^*)\longrightarrow(Y,s^*)$ be irresolute t^*-G-compact mapping and K be t^*-g-compact subset K of Y, so by using definition (3.5), one can have $f^{-1}(K)$ is t^*-g-compact subset of X.so by using proposition (2.8) we get $f^{-1}(K)$ is g-compact set , therefore f is t^*-G-compact mapping.

Now, the following corollary addition the condition in order to make the converse of proposition (3.8) is true.

Corollary

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is t^*-G-compact mapping and X is t^*-T_g-space then f is irresolute t^*-G-compact mapping.

The following diagram shows the relationship between the types of t^*-generalized compact mapping have been studied.

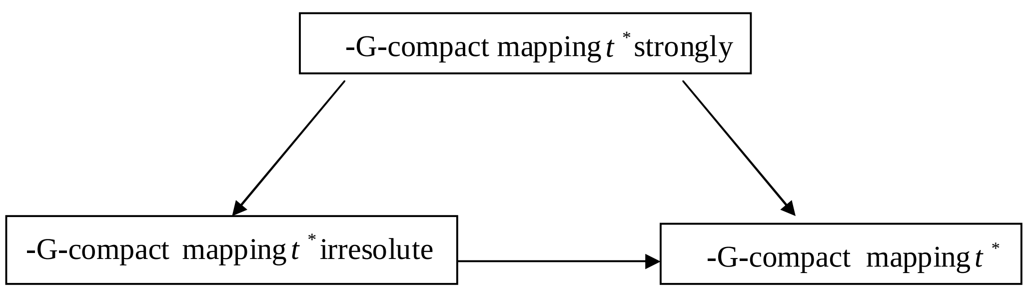

Diagram I

CERTAIN PROPERTIES OF t^*-GENERALIZED COMPACT MAPPINGS

In this section, let us introduce some properties of t^*-generalized compact mappings such as the composition and restriction and getting start with the following theorem.

Theorem

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be t^*-G-compact mapping

1) If $g:(Y,s)\longrightarrow(Z,g)$ is t^*-G-compact mapping and Y is t^*-T_g-space then gof is t^*-G-compact mapping.

2) If $g:(Y,s)\longrightarrow(Z,g)$ is strongly t^*-G-compact mapping and *X is t^*-T_g-space then gof is strongly t^*-G-compact mapping.*

3) If $g:(Y,s)\longrightarrow(Z,g)$ is irresolute t^*-G-compact mapping then gof is t^*-G-compact mapping.

Proof

1) Let K be a t^*-g-compact subset in Z, then $g^{-1}(K)$ is g-compact set in Y and since Y is t^*-T_g-space, so by using definition(3.1), we have $g^{-1}(K)$ is t^*-g-compact also, since f is t^*-G-compact mapping then $f^{-1}(g^{-1}(K))$ is g-compact set in X, but $(gof)^{-1}(K)=f^{-1}(g^{-1}(K))$, therefore; gof is t^*-G-compact mapping.

2) Let K be a g-compact subset in Z, thus by using definition (3.5), one can have, $(g)^{-1}(K)$ is t^*-g-compact set in Y, and since f is t^*-G-compact mapping , we get, $(gof)^{-1}(K)$ is g-compact set in X, but X is t^*-T_g-space, so $(gof)^{-1}(K)$ is t^*-g-compact set, therefore; gof is strongly t^*-G-compact mapping.

3) Let K be a t^*-g-compact subset in Z, thus by using definition (3.5), one can have, $(g)^{-1}(K)$ is t^*-g-compact set in Y, and since f is t^*-G-compact mapping , we get, $(gof)^{-1}(K)$ is g-compact set in X, therefore; gof is t^*-G-compact mapping.

Now, the following theorem gives, the composition when f is strongly t^*-G-compact mapping.

Theorem

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be strongly t^*-G-compact mapping

1) If $g:(Y,s)\longrightarrow(Z,g)$ is t^*-G-compact mapping. Then gof is irresolute t^*-G-compact mapping.

2) If $g:(Y,s)\longrightarrow(Z,g)$ is strongly t^*-G-compact mapping. Then gof is strongly t^*-G-compact mapping.

3) If $g:(Y,s)\longrightarrow(Z,g)$ is irresolute t^*-G-compact mapping. Then gof is t^*-G-compact mapping.

Proof

1) Let K be a t^*-g-compact subset in Z, then $g^{-1}(K)$ is g-compact set in Y and , since f is strongly t^*-G-compact mapping then $f^{-1}(g^{-1}(K))$ is t^*-g-compact set in X, but $(gof)^{-1}(K)=f^{-1}(g^{-1}(K))$, therefore; gof is irresolute t^*-G-compact mapping.

2) Let K be a g-compact subset in Z, thus by using definition (3.2), one can have, $(g)^{-1}(K)$ is t^*-g-compact set in Y, and by using proposition (2.8), we get, $(g)^{-1}(K)$ is g-compact and since f is strongly t^*-G-compact mapping , we get, $(gof)^{-1}(K)$ is t^*g-compact set in X, therefore; gof is strongly t^*-G-compact mapping.

3) Let K be a t^*-g-compact subset in Z, thus by using definition (3.5),we have, $(g)^{-1}(K)$ is t^*-g-compact set in Y, and by using proposition (2.8), one can have, $(g)^{-1}(K)$ is g-compact and since f is strongly t^*-G-compact mapping , we get, $(gof)^{-1}(K)$ is t^*g-compact set in X, therefore; gof is irresolute t^*-G-compact mapping.

Next, the following theorem without proof give the composition when f is irresolute t^*-G-compact mapping.

Theorem

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be irresolute t^*-G-compact mapping

1) If $g:(Y,s)\longrightarrow(Z,g)$ is t^*-G-compact mapping. Then gof is t^*-G-compact mapping.

2) If $g:(Y,s)\longrightarrow(Z,g)$ is strongly t^*-G-compact mapping. Then gof is strongly t^*-G-compact mapping.

3) If $g:(Y,s)\longrightarrow(Z,g)$ is irresolute t^*-G-compact mapping then gof is t^*-G-compact mapping.

Now, the following proposition a shows the restriction of t^*-G-compact mapping under the condition.

Proposition

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be a t^*-G-compact mapping. If $A\subseteq X$ is closed then $f|A:A\longrightarrow Y$ is a t^*-G-compact mapping..

Proof

Let K is a t^*-g-compact set in Y, thus $f^{-1}(K)$ is a g-compact in X and $(f|A)^{-1}(K)=A\cap f^{-1}(K)$ but, $A\cap f^{-1}(K)$ is a g-compact set. and from definition () one can get, $f|A$ is a t^*-G-compact mapping.

Now, from above proposition we can get the following corollary

Corollary

Let $f:X\longrightarrow Y$ be a strongly t^*-G-compact (irresolute t^*-G-compact mapping.) mapping. If $A\subseteq X$ is closed then $f|A:A\longrightarrow Y$ is a strongly t^*-G-compact (irresolute t^*-G-compact) mapping.

t^*-GENERALIZED C-COMPACT MAPPINGS

in this section, we give new type of t^*-generalized compact mapping namely, t^*-generalized C-compact mappings with some characterizations of these mapping.

Definition

A mapping $f:(X,t^*)\longrightarrow(Y,s^*)$ is said to be t^* generalized C-compact mapping (t^*-G-C-compact in short) if $f\times I_W:X\times W\longrightarrow Y\times W$ is t^*-G-compact mapping, for any topological space W.

Definition

A mapping $f:(X,t^*)\longrightarrow(Y,s^*)$ is said to be strongly t^* generalized C-compact mapping (strongly t^*-G-C-compact in short) if $f\times I_W:X\times W\longrightarrow Y\times W$ is strongly t^*-G-compact mapping, for any topological space W.

Now, the following lemma shows the relation between t^*-G-C-compact and strongly t^*-G-C-compact mappings.

Lemma

If $f:(X,t^*)\longrightarrow(Y,s^*)$ is strongly t^*-G-C-compact mapping then f is t^*-G-C-compact mapping.

Proof

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be strongly t^*-G-C-compact mapping, thus $f\times I_W:X\times W\longrightarrow Y\times W$ is strongly t^*-G-compact mapping, for any topological space W and by using proposition (3.3), we get $f \times I_W : X \times W \longrightarrow Y \times W$ is t^*-G-compact mapping thus, from definition (5.1), we have f is t^*-G-C-compact mapping

Now, we can give anther type of t^*-G-C-compact mapping namely irresolute t^*generalized C-compact mapping.

Definition

A mapping $f:(X,t^*) \longrightarrow (Y,s^*)$ is said to be irresolute t^*generalized C-compact mapping (irresolute t^*-G-C-compact in short) if $f \times I_W : X \times W \longrightarrow Y \times W$ is irresolute t^*-G-compact mapping, for any topological space W.

Next, the following proposition give the relation between strongly t^*-G-C-compact and irresolute t^*-G-C-compact mappings.

Proposition

If $f:(X,t^*) \longrightarrow (Y,s^*)$ is strongly t^*-G-C-compact mapping then f irresolute t^*-G-C-compact mappings.

Proof

Let $f:(X,t^*) \longrightarrow (Y,s^*)$ be strongly t^*-G-C-compact mapping, thus $f \times I_W : X \times W \longrightarrow Y \times W$ is strongly t^*-G-compact mapping, for any topological space W and by using proposition (3.6), we get $f \times I_W : X \times W \longrightarrow Y \times W$ is irresolute t^*-G-compact mapping thus, from definition(5.4), we have f is irresolute t^*-G-C-compact mapping

Now, the following proposition shows the relation between irresolute t^*-G-C-compact and t^*-G-C-compact mapping

Proposition

If $f:(X,t^*) \longrightarrow (Y,s^*)$ is irresolute t^*-G-C-compact mapping then f is t^*-G-C-compact mappings.

Proof

Let $f:(X,t^*) \longrightarrow (Y,s^*)$ be irresolute t^*-G-C-compact mapping, thus $f \times I_W : X \times W \longrightarrow Y \times W$ is irresolute t^*-G-compact mapping, for any topological space W and by using proposition (), we get

$f \times I_W : X \times W \longrightarrow Y \times W$ is t^*-G-compact mapping thus, from definition(5.4), we have f is irresolute t^*-G-C-compact mapping

Now, to illustrate the relationship between all types of generalized compact mapping have been studied in this section, see the following diagram.

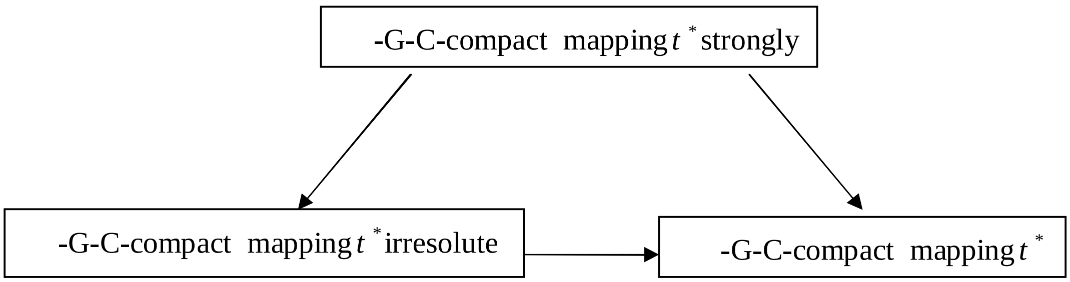

Diagram II

Now, Let us study certain properties of t^*-G-C-compact mapping and the following theorem give the composition of some types of t^*-G-C-compact mapping.

Theorem

Let $f : (X, t^*) \longrightarrow (Y, s^*)$ be t^*-G-C-compact mapping

1) If $g : (Y, s) \longrightarrow (Z, g)$ is t^*-G-C-compact mapping and $X \times W$ is t^*-T_g-space. Then gof is t^*-G-compact mapping.

2) If $g : (Y, s) \longrightarrow (Z, g)$ is strongly t^*-G-C-compact mapping. Then gof is irresolute t^*-G-C-compact mapping.

3) If $g : (Y, s) \longrightarrow (Z, g)$ is irresolute t^*-G-C-compact mapping. Then gof is irresolute t^*-G-C-compact mapping.

Proof

1) since f is t^*-G-C-compact mapping so by using definition (5.1), we get $f \times I_W$ is t^*-G-compact mapping for any topological space W, also since g is t^*-G-C-compact mapping , thus $g \times I_W$ is t^*-G-compact mapping , for any topological space W, then from theorem (4.1(1)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is t^*-G-compact mapping mapping, therefore; from definition (5.1), we have ; gof is t^*-G-C-compact mapping

2) since f is strongly t^*-G-C-compact mapping so ,we get $f \times I_W$ is strongly t^*-G-compact mapping for any topological space W, also since g is strongly t^*-G-C-compact mapping , thus we get $g \times I_W$ is strongly t^*-G- compact mapping, for any topological space W, and from theorem (4.1(2)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is irresolute t^*-G- compact mapping, therefore; from definition (), we have ; gof is irresolute t^*-G-C-compact mapping.

3) since f is irresolute t^*-G-C-compact mapping so ,we get $f \times I_W$ is irresolute t^*-G-compact mapping for any topological space W, also since g is irresolute t^*-G-C-compact mapping , thus we get $g \times I_W$ is irresolute t^*-G- compact mapping, for any topological space W, and from theorem (4.1(3)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is irresolute t^*-G- compact mapping, therefore; from definition (5.4), we have ; gof is irresolute t^*-G-C-compact mapping

Moreover, the following theorem give the composition of strongly t^*-G-C-compact mapping.

Theorem

Let $f:(X,t^*) \longrightarrow (Y,s^*)$ be strongly t^*-G-C-compact mapping

1) If $g:(Y,s) \longrightarrow (Z,g)$ is t^*-G-C-compact mapping then gof is irresolute t^*-G-C-compact mapping.

2) If $g:(Y,s) \longrightarrow (Z,g)$ is strongly t^*-G-C-compact mapping and X is t^*-T_g-space then gof is strongly t^*-G-C-compact mapping.

3) If $g:(Y,s) \longrightarrow (Z,g)$ is irresolute t^*-G-C-compact mapping then gof is t^*-G-compact mapping.

Proof

1) since f is strongly t^*-G-C-compact mapping so by using definition (5.2), we get $f \times I_W$ is strongly t^*-G-compact mapping for any topological space W, also since g is t^*-G-C-compact mapping , thus $g \times I_W$ is t^*-G-compact mapping , for any topological space W, then from theorem (4.2(1)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is irresolute t^*-G-compact mapping, therefore; from definition (5.4), we have ; gof is irresolute t^*-G-C-compact mapping

2) since f is strongly t^*-G-C-compact mapping so ,we get $f \times I_W$ is strongly t^*-G-compact mapping for any topological space W, also since g is strongly t^*-

G-C-compact mapping , thus we get $g \times I_W$ is strongly t^*-G- compact mapping, for any topological space W, and from theorem (4.2(2)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is strongly t^*-G- compact mapping, therefore; from definition (5.2), we have ; gof is strongly t^*-G-C-compact mapping.

3) since f is strongly t^*-G-C-compact mapping so ,we get $f \times I_W$ is strongly t^*-G-compact mapping for any topological space W, also since g is irresolute t^*-G-C-compact mapping , thus we get $g \times I_W$ is irresolute t^*-G- compact mapping, for any topological space W, and from theorem (4.2(3)), one can have $(f \times I_W)o(g \times I_W) = (f \times g)oI_W$ is irresolute t^*-G- compact mapping, therefore; from definition (5.4), we have ; gof is irresolute t^*-G-C-compact mapping

Next, the following theorem without proof give the composition when f is irresolute t^*-G-compact mapping.

Theorem

Let $f:(X,t^*)\longrightarrow(Y,s^*)$ be irresolute t^*-G-compact mapping

1) If $g:(Y,s)\longrightarrow(Z,g)$ is t^*-G-compact mapping and $X \times W$ is t^*-T_g-space then gof is irresolute t^*-G-compact mapping.
2) If $g:(Y,s)\longrightarrow(Z,g)$ is strongly t^*-G-compact mapping then gof is strongly t^*-G-compact mapping.
3) If $g:(Y,s)\longrightarrow(Z,g)$ is irresolute t^*-G-compact mapping then gof is irresolute t^*-G-compact mapping.

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