Bandwidth Efficiency Enhancement for OFDM System Using Hybrid Equalization Techniques

Thamer M. Jamel [*]   **&**   **Ali N. Abdulla**[*]

Abstract

The Cyclic prefixed OFDM systems are very robust to multipath, provided that the delay spread of the transmission channel is less than the length of the CP inserted between transmitted symbols. Moreover, appending CP to each OFDM symbol decreases the bandwidth efficiency of the OFDM system, since the CP does not carry useful information. To maintain bandwidth efficiency, a time-domain equalizer (TEQ) and frequency-domain equalizer (FEQ) is used in the OFDM system to mitigate ISI and ICI when the CIR interval is larger than the CP interval. However, the performances of these two equalizers are degraded under severe channel condition. So a proposed hybrid equalizer structure is suggested in this paper. The hybrid equalizer gives about 10dB, 5dB and 3dB advantages at 10-4 BER compared with OT-FEQ, TEQ and PTEQ respectively. This comparison at fd=50Hz with sufficient length of CP. Also, for this case but with an insufficient length of CP, the hybrid equalizer gives about 8dB and 7dB advantage compared with both TEQ and PTEQ respectively. Moreover, at fd=100Hz with sufficient CP length, the hybrid equalizer gives about 6dB, 3.5dB and 3dB advantage at 10-4 BER compared with OT-FEQ, TEQ and PTEQ respectively. Also, for this case but with an insufficient length of CP, the hybrid equalizer gives about 6dB and 4dB advantage compared with both TEQ and PTEQ respectively.

Keywords: TEQ, FEQ, OFDM system

تحسين كفاءة الموجة لنظام مزج تقسيم التردد المتعامد باستخدام تقنيات المعادل الهجين

ألخلاصه

أنظمة (OFDM) ذات البادئة الدورية (CP) تكون قوية جداً ضد ظاهرة المسارات المتعددة، بشرط أن تأخير إنتشارَ قناةِ الإرسالَ أقل مِنْ طولِ البادئة الدورية ألمحشورة بين الرموزِ المُرسَلةِ. علاوة على ذلك، إضافة البادئة الدورية إلى كُلّ رمزِ OFDM يَنْقصُ كفاءة الموجة لنظامِ OFDM ، حيث أن البادئة الدورية لاتَحْملُ معلوماتَ مفيدةَ. لإبْقاء كفاءةِ الموجةِ , المعادل ذو المجالِ الزمني (TEQ) والمعادل ذو المجالِ الترددي (FEQ) يستعملان في نظامِ OFDM لتقليل تأثير تداخل الرموز و تداخل الحوامل عندما طول البادئة الدورية اقل من تأخير إنتشارِ القناة. على أية حال، أداءات مثل هذه المعادلات يتدهور تحت ظروف القناةِ الحادّةِ. لذا تم إقتراحَ تركيبَ معادلٍ هجين في هذا العملِ. يعطي المعادلُ الهجينُ حوالي 10dB , 3dB, 5dB ربح عند معدل خطأ 10⁻⁴ بالمقارنه مع OT-FEQ, TEQ و PTEQ على التوالي. هذه المقارنةِ عندما تردد دوبلر fd =50Hz مع طول بادئة دورية كافي. أيضاً، لهذه الحالةِ لكن بطولٍ غير كافي للبادئة الدورية ، يَعطي المعادلَ الهجينَ حوالي 8dB و7dB ربح بالمقارنه مع TEQ و PTEQ على

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التوالي. علاوة على ذلك، عندما f_d=100Hz مع طول بادئة دورية كافي ، يَعطي المعادلَ الهجينَ حوالي 6dB , 3.5dB، 3dB ربح عند معدل خطأ 10^{-4} بالمقارنه مع OT-FEQ, TEQ و PTEQ على التوالي. أيضاً، لهذه الحالةِ لكن بطولٍ غير كافي للبادئة الدورية ، يَعطي المعادلَ الهجينَ حوالي 6dB و4dB ربح بالمقارنه مع TEQ و PTEQ على التوالي.

Introduction

OFDM is a scheme used in the area of high-data-rate mobile wireless communications such as cellular phones and satellite communications. OFDM is a multi-carrier modulation scheme in which the whole system bandwidth is divided into many narrow band subcarriers whose waveforms are orthogonal to each other. Therefore, the originally high rate serial data stream will be split into several low rate data stream and transmitted in parallel fashion over different subcarriers.

A cyclic prefix (CP) is inserted between the OFDM symbols in order to mitigate intersymbol interference (ISI) and intercarrier interference (ICI) effects for signal transmission over multipath channels. To avoid ISI and also ICI, the CP interval should not be shorter than the channel-impulse response (CIR) interval. However in some cases, such as satellite OFDM systems, the very long delay spreads pose the possibility that the duration of the ISI exceeds the length of the moderate CP length, resulting in performance degradation. On one hand, a long CP is required in order to mitigate ISI/ICI. On the other hand, a long CP wastes the valuable bandwidth since the CP does not carry useful information. Thus the CP interval should be a small fraction of the OFDM symbol interval in order not to lose the bandwidth efficiency [1].

In order to mitigate this problem, various equalization schemes have been proposed. These are mainly classified into two categories, one is a time domain equalizer and the other is a frequency domain equalizer

System Model

The time domain equalizer (TEQ) is based on the impulse response shortening approach to shorten the effective length of the overall CIR to be in the range of the guard interval (CP) was proposed in [2, 3]. The time-domain equalizers (TEQ) compensate the CIR by means of a time-domain finite impulse response (FIR) filter.

The frequency-domain equalizers (FEQ) [4, 5] achieve further improvement by shifting the FIR filter from the time domain to the frequency domain.

OFDM system uses symbols generated by a finite length FFT with size N. The orthogonality of the consecutive OFDM symbols is maintained by appending a length cyclic prefix (CP) at the start of each symbol [6]. The CP is obtained by taking the last v samples of each symbol and so the total length of the transmitted OFDM symbol is (N+v) samples. By this means the linear convolution of the transmitted signal with the CIR is converted into a circular one. For each OFDM symbol to be independent and to avoid any Inter Symbol Interference (ISI) or Inter Carrier Interference (ICI), the length of the CIR should be less than v+1 samples. Hence the distortion caused by the CIR only affects the samples within the CP. The receiver takes only the last N samples for decoding at the receiver FFT,

disregarding the CP. Consequently, the effects of the CIR can then be easily equalized by an array of one-tap Frequency Domain Equalizers (FEQ) following demodulation by the FFT. Figure 1 shows the block diagram of the system, where P/S and S/P mean parallel to serial and serial to parallel conversion respectively [6].

Time Domain Equalizer (Teq)

To maintain the bandwidth efficiency and to avoid using a long CP, a short FIR time-domain equalizer (TEQ) is used in the OFDM systems receiver to eliminate the ISI and ICI effects when the CIR interval is longer than the CP interval. Two major approaches for TEQ design have been proposed.

A number of algorithms for calculating the optimum coefficient of the TEQ digital filter have been proposed. The first group of algorithms is represented by methods that shorten the impulse response. They are mainly the Minimum Mean Square Error (MMSE) method, the Maximum Shortening Signal to Noise Ratio (MSSNR) method.

The second group of algorithms is created by methods that maximize the transmission rate. The signal-to-noise ratio (SNR) is maximized in the computation. They are mainly the Maximum Geometric Signal to Noise Ratio (MGSNR) method, the Minimum Intersymbol Interference (MIN-ISI) method and the Maximum Channel Capacity (MCC) method.

The underlying idea of this algorithm is the minimization of the mean square error between the impulse response of channel h and the target impulse response (TIR) h_{eff} using the TEQ equalizer, which is realized as an FIR digital filter with impulse response w.

The impulse response of transmission channel is thus shortened via the TEQ equalizer to the optimum target impulse response h_{eff}, which is also being sought [7].

The structure of the algorithm is shown in Fig. (2). If the error in Fig. (2) could be forced to be zero, then the OIR (upper path) would be equal to the TIR (lower path) with a time delay difference. Given the length of the TIR, it is desired to find the coefficients vector w of TEQ to minimize the mean square error (MSE) signal, given by [7]: Given the length of the TIR, it is desired to find the coefficients vector w of TEQ to minimize the mean square error (MSE)

signal, given by:

$$MSE = E[e^2] = h_{eff}^T R_{xx} h_{eff} - h_{eff}^T R_{xy} w - w^T R_{yx} h_{eff} + w^T R_{yx} w$$

$$......(1)$$

Where $R_{xx} = E[xx^T]$,
$R_{xy} = E[xy^T]$
, $R_{yx} = E[yx^T]$, $R_{yy} = E[yy^T]$

In order to minimize the mean square error of MSE expressed in equation (1), the well-known orthogonality principle of the linear estimation theory can be exploited, to require:

$$E[ey^T] = 0 \qquad ..... (2)$$

which gives the solution as

$$h_{eff}^* R_{xx} = w^* R_{yy} \qquad .... (3)$$

For the unit energy constraint (UEC) on TEQ coefficients w the optimum solution can be calculated by the following equation:

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$$w_{opt} = \left(\sqrt{R_{yy}} \right)^{-1} g_{opt} \qquad \text{…… (4)}$$

where g_{opt} is the eigenvector corresponding to the minimum eigenvalue which satisfies the equation:

$$w^T R_{yy} w = w^T \sqrt{R_{yy}}^T \sqrt{R_{yy}} w = g^T g \qquad \text{……(5)}$$

with $g = \sqrt{R_{yy}} w$. The above calculations involve solving the minimum eigenvalue and eigenvector corresponding to the channel and noise independent matrices.

One-Tap Frequency Domain Equalizer (Ot-Feq)

For an OFDM system with sufficient CP or with TEQ properly shortened the CIR to be in the range of CP length, the orthogonality among subcarriers is maintained and thus ISI and ICI do not occur. Only the amplitude and phase distortions occur. To compensate this distortion, the simple complex-valued one-tap FEQ is sufficient with value at each subchannel is set as the inverse of the product of the channel gain and TEQ gain at the subchannel. This is certainly one of the key advantages of OFDM [8].

The operation of the FEQ is very simple. At the output of FFT on the receiver side, the sample at each subcarrier is multiplied by the coefficient of the corresponding channel equalizer. The coefficient of an equalizer can be calculated based on the ZF criterion or the MMSE criterion The ZF criterion forces ISI to be zero at the sampling instant of each subcarrier. The coefficient (weight) of the one-tap ZF equalizer is calculated as follows:

utilizing CP and OT-FEQ

$$W_k = \frac{1}{H_k}$$

where H_k is the channel frequency response within the bandwidth of the nth subcarrier. The disadvantage of ZF criterion is that it enhances noise at the nth subcarrier if H_k is small, which corresponds to spectral nulls, so MMSE criterion can be used to provide better performance at a cost of additional complexity.

Zero Forcing Equalizer (Zf)

Complete elimination of the ISI requires the use of an inverse filter to the linear filter model $Q(Z)$. Such an equalizer called a zero-forcing (ZF) equalizer. Fig. (4) illustrates in block diagram the equivalent discrete-time channel and ZF equalizer [9].

Decision Feedback Equalizer (Dfe)

The decision-feedback equalizer (DFE) is made up of two parts, the feedforward and the feedback section. The feedforward section is the same as the linear transversal equalizer The feedback section is also a linear Transversal Equalizer (LTE), but operates on previously detected symbols instead of the received signal. The output of each filter is added to form the equalized signal [9].

Functionally, the decision-feedback equalizer works as follows. The feedforward filter removes the ISI caused by future symbols on the current symbol through linear filtering. The feedback filter is used to remove that part of the ISI from the present estimate caused by previously detected symbols. A diagram of a DFE is shown in Fig.(5).

The Proposed Hybrid Equalizer

The proposed equalizer structures based on the following observations:
1. A one-tap TEQ is applied at the receiver to convert the doubly-selective channel into a purely frequency-selective

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channel. Then this channel is equalized in the frequency domain using the proposed FEQ.

2. The proposed FEQ is a per-tone frequency domain equalizer which consists of two cascaded parts:

A. A one-tap ZF equalizer is used to compensate for amplitude and phase distortion caused by the channel.

B. A multitap DFE equalizer which performs two functions. Firstly the feedforward portion of the DFE used to suppress ICI for each OFDM symbols. Secondly the feedback portion of the DFE responsible for removing ISI.

3. Due to the fact that in practical applications, a user utilizes only a subset of channel for data transmission, hence only those used channels need to be equalized, and eventually a reasonable complexity is required. The block diagram of the proposed equalizer is shown in Fig. (6)

The operation of the equalizer is asfollow:

1. The transmitted signal is propagated through a doubly-selective channel so it is contaminated by noise and corrupted by ISI and ICI.

2. The OT-TEQ is used for shortening the CIR so that the doubly-selective channel can be reduced into purely frequency-selective so the equalization can be performed in the frequency-domain.

3. After FFT, the frequency domain received corrupted signal first equalizer by OTE-ZF

Here two cases will be considered as follow:

A. if the channel was used for data transmission then the value of the OTE-ZF will be the inverse of the channel frequency response, as explained below

$$W_n = \frac{1}{H_n} \qquad \text{……..(6)}$$

So the W_n need to be the inverse of the channel frequency response if the channel contain a given user data.

B. From equation (6) one can observe that for a given channel W_n does not have to be the inverse of H_n if this channel is not used practically for data transmission.

Therefore, only used channels need to be equalized i.e. the value of OTE-ZF has to be the inverse of the channel, while the value of OTE-ZF is not constrained for the unused channels.

4. As discussed earlier, the structure of the DFE is composed of two parts. In the feedforward part, ISI replica is generated by using the previous detected symbols passing through the feedback path. The ISI replica is subtracted from the received signal in the feedforward part to yield the ISI free signal. Then the feedforward part suppresses the ICI in the ISI free signal by keeping the orthogonality among subcarriers.

The proposed equalizer is a hybrid equalizer. This is true because of the following two justifications:

1. This equalizer is composite of time-domain and frequency-domain equalizers for two different purposes, thus the equalizer may consider as a hybrid equalizer.

2. The proposed frequency domain equalizer employs a hybrid criterion, i.e., the ZF criterion for compensating desired channel in OTE-ZF, and the MMSE criterion for ISI and ICI cancellation in the DFE equalizer. Again the equalizer may consider as a hybrid equalizer.

Simulation And Results

The flowchart of OFDM system under noisy Rayleigh fading channel that utilizing CE technique for compensating the fading effect is shown in Fig. (7).

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OFDM signal was generated and plotted in the time. Figure (8) shows the time wave form for the transmitted part of 6 symbols (after IFFT-128). Each symbol 128 samples long ($6 \times 128 = 768$ samples).

In Figure (9), the OFDM signal spectrum can be seen corresponding to a random binary input sequence. The frequency is normalized to the subcarrier spacing with 128 subcarrier used for this plot. The DC subcarrier has not been used, making the signal symmetrical around DC.

The parameters and system configuration used in the simulation can be summarized as shown in tables1, 2 , and 3.

It is worthy to mention here that in this work only uncoded OFDM system has been considered, but it is clear that the performance can be further improved by incorporating error correction codes.

In the simulation results presented below, four equalizer structures have been considered: OT-FEQ, TEQ, Per Tone Equalizer (PTEQ), and the proposed hybrid equalizer. In addition, the performance of data detection without any compensation or equalization technique (no compensation) is also presented for comparison purpose. In these simulations, the duration of cyclic prefix is assumed to be $3T_s/20$ or equivalently 15 percent of the symbol duration. This case corresponding to the insufficient CP case, because the duration of CP is shorter than the maximum delay spread of the channel (equals to $T_s/4$ or 25 percent of the symbol duration). Alternatively, when the duration of cyclic prefix is assumed to be $7T_s/20$ or equivalently 35 percent of the symbol duration. This case corresponding to the sufficient CP case, because the duration of CP is longer than the maximum delay spread of the channel.

Case A:

The doubly-selective fading channel with fd=50 Hz has been considered. Also, two sub cases have been considered, including sufficient and insufficient length of CP.

(A-1): fd =50 Hz, sufficient CP.

From Figure (10), the top curve shows the performance of OFDM system without any technique to recover the noisy faded signal. Obviously said, increasing Eb/No does not reduce the BER due to the severe ISI/ICI resulting in irreducible BER. In this case, the simulation results show that the performance of OT-FEQ is good over CP sufficiently larger than the delay spread of the channel. Further, both TEQ and PTEQ offer a great improvement in the BER. Also, the results show that the proposed equalizer outperforms the TEQ and PTEQ. The proposed equalizer shows 1dB and 2dB gain at 10^{-6} BER performance compared to the PTEQ and TEQ respectively. Consequently, the proposed equalizer has the best equalization performance among the all considered equalizers.

(A-2): fd =50 Hz, insufficient CP.

The result of this case is shown in Fig. (11). This figure indicates that at low Eb/No values, since channel noise is dominant, all equalization methods cannot improve the BER performance. When the Eb/No is higher than 20dB, the effects of ISI and ICI become obvious. The OT-FEQ without sufficient CP reaches an error floor soon caused by ISI and ICI, while both TEQ and PTEQ have evident BER improvement. On the contrary, the proposed equalizer can also offer a BER high enough for successful demodulation. Even in this case, the proposed equalizer can achieve performance offset by 4dB and 5dB at 10^{-4} BER performance compared to the PTEQ and TEQ

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respectively. This result confirms the effectiveness of the proposed equalizer with insufficiency of CP.

Case B:

The doubly-selective fading channel with fd=100 Hz has been considered. Also, two sub cases have been considered, including sufficient and insufficient length of CP.

(B-1): fd =100 Hz, sufficient CP.

The performance of equalizers under this case is shown in Fig. (12). In this case, the performance of the OT-FEQ is not degraded severely. This result indicates that the influences of the delayed signals are mitigated over the sufficient CP interval with OT-FEQ. This shows the OT-FEQ has acceptable performance with long CP. Also, TEQ and PTEQ have a comparable and good performance. On the other hand, the proposed equalizer has the best performance.

In contrast, the proposed equalizer succeeds in combating ISI and ICI and thus outperforms OT-FEQ by orders of magnitude. Furthermore, for the case considered, the proposed equalizer achieves better performance than TEQ and PTEQ equalization. To achieve 10^{-5} BER, TEQ and PTEQ require an Eb/No of 37 dB whereas the proposed equalizer achieves the same performance at an Eb/No of only 34 dB. This performance advantage can be explained by the observation that the proposed equalizer performs hybrid equalization.

(B-2): fd =100 Hz, insufficient CP.

In this case, the BER performance of the OT-FEQ is severely degraded as expected. This is due to the insufficiency of CP and noise enhancement introduced by this equalizer. Moreover, the BER of TEQ and PTEQ is also degraded, where TEQ does not demonstrate good

performance because of enhanced noise. This case depicted in Fig. (13).

Conclusions

In this paper the equalization techniques for OFDM system have been considered. A hybrid equalizer structure is proposed for OFDM system with insufficient CP and thus bandwidth efficiency has to be improved due to the reduced CP length. Also, the performance of the proposed equalizer is much better than the conventional equalizer and is not degraded under severe channel conditions

References

[1] Hossein Jafarian, Hossein Khoshbin, and S. Pasupathy, "Time-Domain Equalizer for OFDM Systems Based on SINR Maximization" IEEE Transactions on Communications, Vol. 53, No. 6, pp. 924–929, June 2005.

[2] P.J.W. Melsa, R.C. Younce, and C.E. Rohrs, "Impulse Response Shortening for Discrete Multitone Transceivers", IEEE Trans. on Commun., vol. 44, pp. 1662–1672, Dec. 1996.

[3] N. Al-Dhahir and J. M. Cioffi, "Optimum Finite-Length Equalization for Multicarrier Transceivers," IEEE Trans. on Commun., vol. 44, pp. 56–64, Jan. 1996.

[4] K. V. Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, "Per Tone Equalization for DMT-Based Systems," IEEE Trans. Commun., vol.49, no.1, pp.109–119, Jan. 2001.

[5] I. Barhumi, G. Leus, and M. Moonen, "Per-tone Equalization for OFDM over Doubly-Selective Channels," IEEE Trans. Proc. IEEE Int. Conf. Communications, ICC, vol.5, pp. 2642–2647, June 2004.

[6] V.S. Abhayawardhana, I.J. Wassell, "Frequency Scaled Time Domain Equalization for OFDM in Broadband Fixed Wireless Access Channels", University of Cambridge, UK, 2002.

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http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.16.680

[7] Pavel Šilhavý, "Time Domain Equalization in Modern Communication Systems Based on Discrete Multitone Modulation" Proceedings of the Sixth International Conference on Networking (ICN'07), IEEE Computer Society, 2007.

[8] H. Kang, W. Hwang and K. Kim, "Performance Analysis of the OFDM System with One Tap Equalizer Against the Two-ray Multipath Channel" IEEE TENCON, pp. 45–48, 1999.

[9] S.U. Qureshi, "Adaptive Equalization", Proceeding of IEEE, vol. 37, no. 9, pp. 1340-1387, September 1985

Table (2) Channel parameters

Type	Doubly-selective
Fading distribution	Rayleigh
Phase	Random between (p to $-p$)
Frequency	2GHz
Required bandwidth	2 MHz
Number of paths	2 paths
2nd path gain	0.043-0.171i
2nd path delay	Ts/4
Maximum channel delay spread	Ts/4
Doppler frequency	10 Hz, up to 300 Hz

Table (1)OFDM system parameters

Modulation scheme	BPSK
Number of subcarriers	128
OFDM symbol duration Ts	10 μs
Guard interval–type	7Ts/20, Ts/4, 3Ts/20, Ts/10–CP
No. of generated symbols per subcarrier	768
No. of transmitted symbols	25 000
No. of bits per each symbol	6 bits
No. of transmitted bits =25k * 768	19200000 bits

Table (3)Equalizer parameters

OT-TEQ	
Type/algorithm	MMSE
OT-ZF	
Algorithm	ZF
DFE	
Feedforward part length	16
Feedback part length	15
algorithm	LMS/NLMS
Step size constant (μ) value	0.00028

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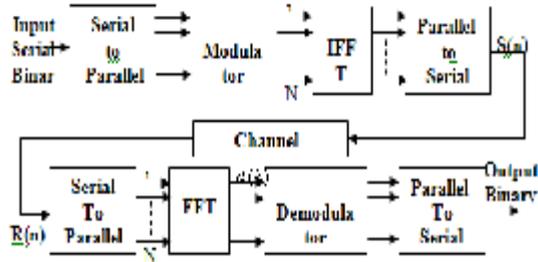
Figure (1) Block Diagram of the

OFDM System
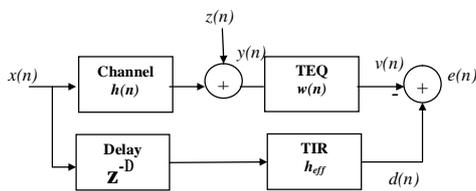
**Figure (2) Structure of MMSE
equalizer**
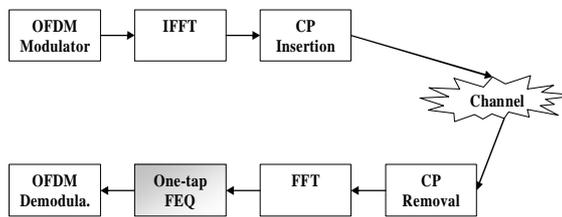
**Figure (3) A simplified OFDM system**
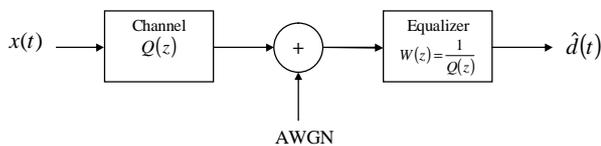
**Figure (4) Block diagram of channel
with zero-forcing equalizer**
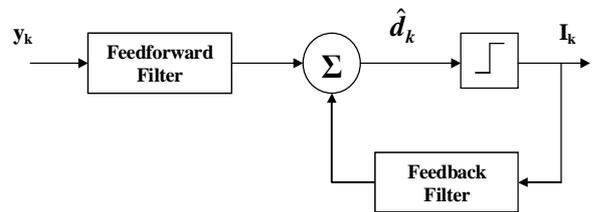
**Figure (5) A decision-feedback
equalizer**
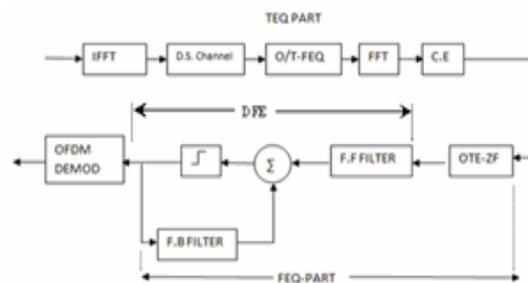
**Figure (6) The proposed hybrid
equalizer Structure**

    D.S.channel: doubly-selective
channel
    OT-TEQ: one-tap time-domain
equalizer
    CE: channel estimation
    OTE-ZF: one-tap equalizer-zero
forcing
    F.F: feedforward part of DFE
    F.B: feedback part of DFE

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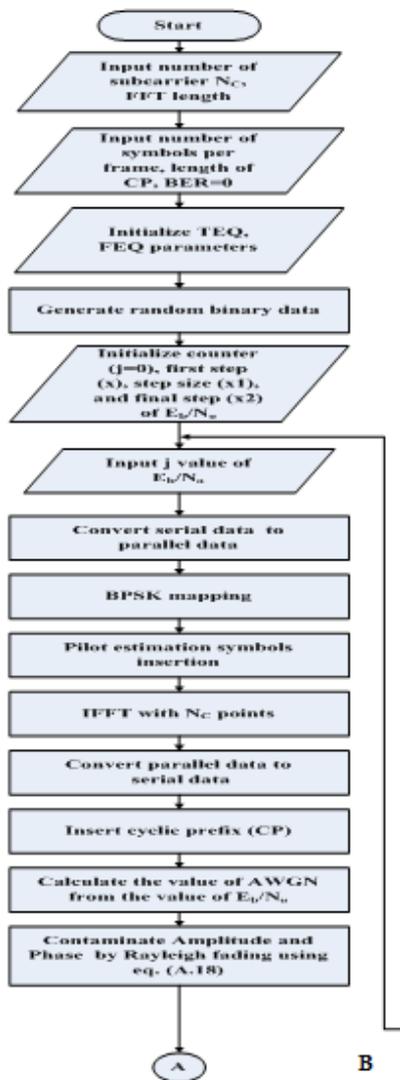
**Figure (7) A flowchart of the proposed
equalizer continued……**

**Figure (7)A flowchart of the proposed
equalizer**

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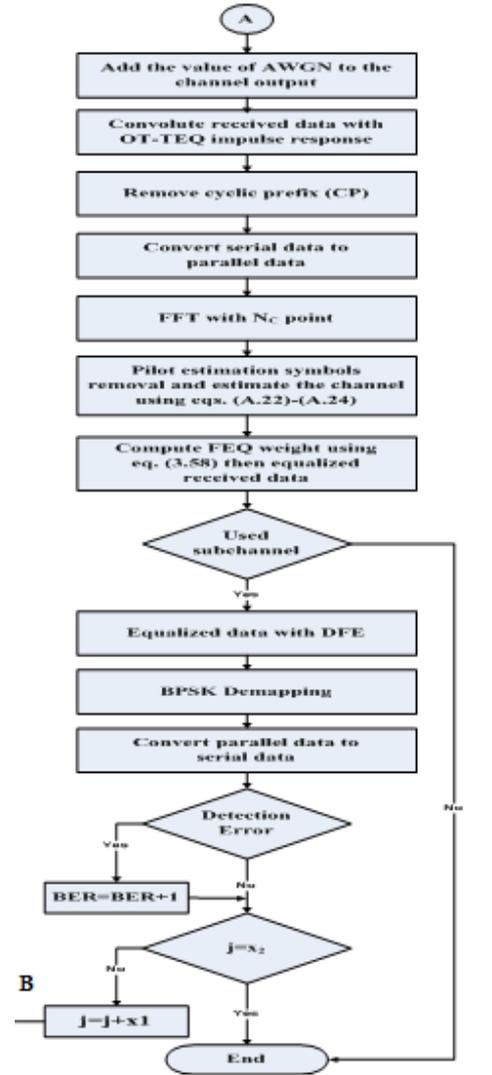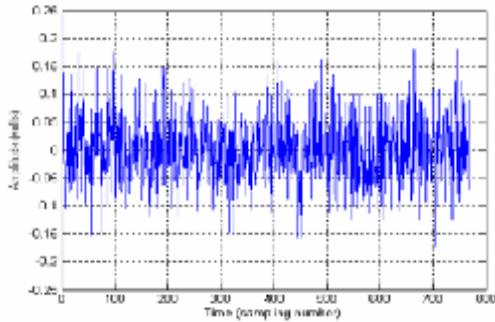
**Figure (8)Fig. (8) Wave form for
OFDM signal in time domain**
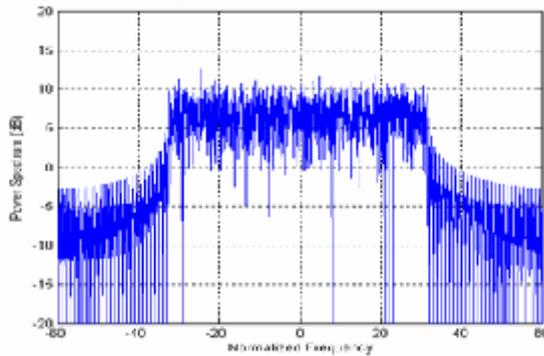
**Figure (9)Fig. (9) Spectrum of the
OFDM signal**
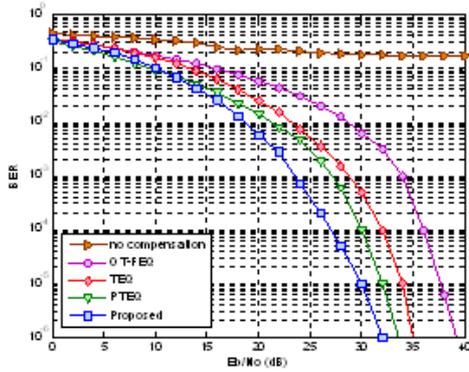
Fig. (10) Performance of each equalizer in the
OFDM system with sufficient cyclic prefix using case
(A-1).
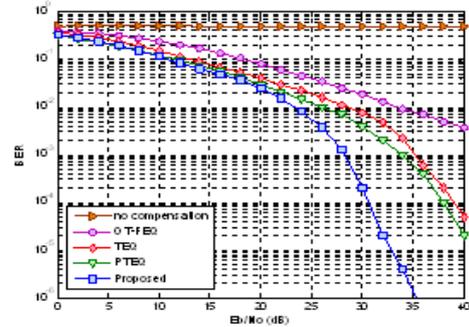
Fig. (11) Performance of each equalizer in the
OFDM system with insufficient cyclic prefix using case
(A-2).
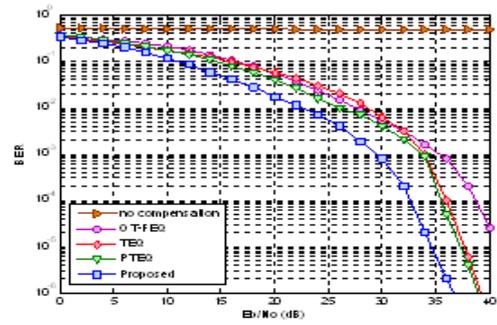
Fig. (12) Performance of each equalizer in
the OFDM system with sufficient cyclic
prefix using case (B-1).

Fig. (13) Performance of each equalizer in the
OFDM system with insufficient cyclic prefix
using case (B-2).