Solving Linear Programming Problems By Using Excel's Solver

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Abstract:

This paper describes advanced methods for finding a verified global optimum and finding all solutions of a system of linear programming, as implemented in the Premium Solver Platform, an extension of the Solver bundled with Microsoft Excel. It also describes the underlying tools that allow Excel spreadsheets to be used over linear data, with fast computation of optimization.

Also it provides: a brief overview of Excel's **Add-in** Solver; basic theory of optimization as implemented within the Solver; advantages of the Excel Solver in linear programming, and three numerical examples outlining the steps involved in carrying out adjustment of Solver to solve the linear programming problems. The reasons to use of Excel for optimization can be considered a viable option are: (a) Excel is readily available in any Windows platform without any additional cost. (b) Excel is easy to use. (c) The data transfer to and from Excel is very flexible.

Keywords; Operations Research, Linear Programming, Excel Solver, Optimization.

Introduction:

Since its introduction in February 1991, the Microsoft Excel Solver has become the most widely distributed and almost surely the most widely used general purpose optimization modeling system.

Bundled with every copy of Microsoft Excel and Microsoft Office shipped during the last eight years, the Excel Solver is in the hands of 80 to 90 percent of users of office-productivity software worldwide. The remaining 10 to 20 percent of this audience use either Lotus 1-2-3 or Quattro Pro, both of which now include very similar spreadsheet solvers, based on the same technology used in the Excel Solver. This widespread availability has spawned many applications in industry and government [1].

In review of the background and design philosophy of the Excel Solver. It was clear up some common misunderstandings and pitfalls, and to suggest ideas for good modeling practice when using spreadsheet optimization. It was found many applications of the Excel Solver in industry and education and describe how practitioners who are not affiliated with the OR/MS (Operations Research/ Management Science) community use it [2].

The Microsoft Excel Solver combines the functions of a graphical user interface (GUI), an algebraic modeling language like GAMS [3] or AMPL [4], and optimizers for linear, nonlinear, and integer programs. Each of these functions is integrated into the host spreadsheet program as closely as possible.

On the basis of the relevant literature and given that this can be easily formulated as Linear Programming (LP) techniques would have been widely used in every business or management school worldwide, they have been, so far, hardly used in real world conditions by management. This is because the LP formulation of, even quite simple, business situations involves an exceptionally big number of variables and constraints, and hence, expensive dedicated software requiring specialized personnel needed to be used for handling the resulting models. Thus LP, for a number of years, has been used only by very big business, government agencies and organizations or in the frames of academic research [5].

The extensive use of personal computers, the dramatic reduction of their cost and the tremendous increase of their computing ability have influenced the management culture worldwide. Senior, medium and front-line management have now access to personal computers and spreadsheet software such as Microsoft Excel [Microsoft Corporation, (1985-2007)] is extensively used. The package contains Solver, an exceptionally evolved and impressively powerful tool that is very effective for handling linear and non linear optimization problems [6]. Each problem of LP situations are not only easily handled by Solver but additional decision support information can also be obtained [5]. The advantages of spreadsheets include the power and breadth of their functions for quantitative analysis, and their intuitive grid-like user interface with which many users are familiar and comfortable.

Spreadsheets are omnipresent in many organizations, so there is already a large knowledge base upon which to draw. Specifically for OR, spreadsheets offer a multitude of resources such as dynamic recalculation and chart updating, statistical analysis, built-in optimization algorithms (such as Excel Solver), programming languages (such as Excel's VBA), database connectivity, rapid application development with visual components, and the widespread availability of specialist "Add-Ins" [7].

In the following pages the paper introduces and describes the method of using the Microsoft Excel's Solver to find the optimal solution of some Linear Programming problems.

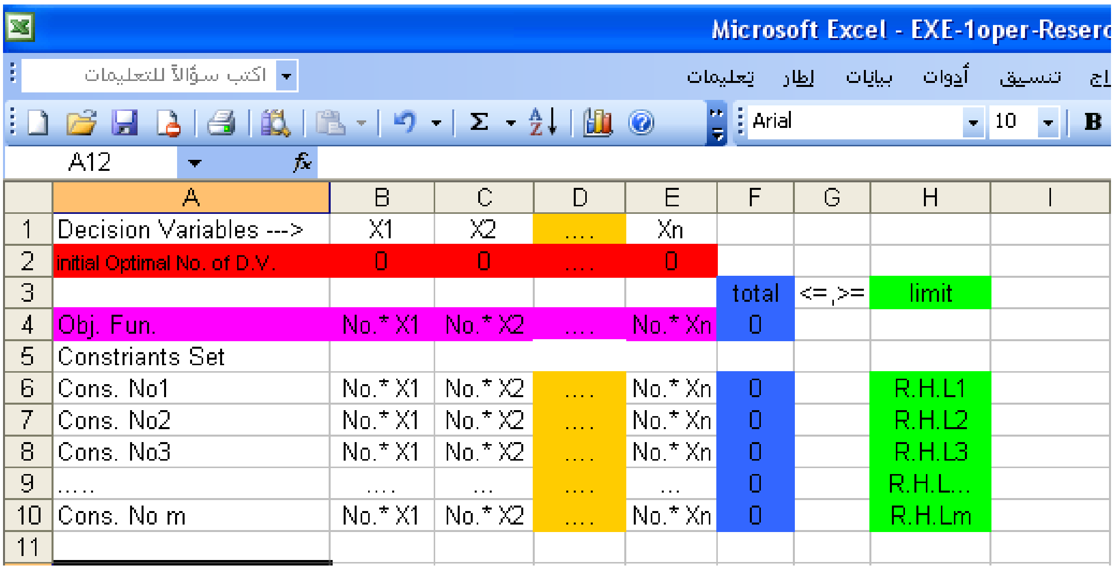
Steps of LP solution by Excel Solver

Step 1: User has to familiarize his/herself with the LP data set.

Step 2: Set up the optimization model (Model Construction)**.**

Step 3: Setting up Excel Solver to solve LPs by the following sub steps**:**

1. Open a new Excel spreadsheet and name it to "*Name of the Problem*".
2. Lay out the problem data in Excel spreadsheet as follows:



3. Type the formula ;
F4 = SUMPRODUCT ($B$2:$C$3: …:$E$2;B4:C4: … :E4) and pull-down it to cells F6:F10.
4. Next, invoke the Excel Solver. To do this select Solver from the *Tools* pull-down menu. (If Solver is not on the *Tools* menu, it will be necessary to install this add-in using the *Add-Ins* option on the *Tools* pull-down menu). In the Solver Dialog box, specify the following ;
   Target Cell: F4
   Constraints: F6:F10 <=, =, or >= H6:H10
   Changing Cells: $B$2:$C$3: …:$E$2
   Equal To: Max or Min
5. While still in the Solver dialog box, click *Options* and set the following options ;
   Assume Linear Model: On
   Assume Nonnegative: On
   This tells Solver that your model is linear in variables and the choice variables are all nonnegative, and then click *OK*.
6. While still in Solver, click *Solve*. This should return a dialog box with the notice: "*Solver found solution*". If not, you have an error somewhere, so go back and re-check all of the steps.
7. While still in the solver, click "*Answer, Sensitivity, Limits*" or " Each you *need*" under "*Reports*" and click *OK*. This should produce the following output under your worksheet entitled " *Answer Report*", "*Sensitivity Report* "," *Limits Report*".

Linear Programming Problems

The following linear programming problems will introduces you to the exciting world of linear programming and describes the method of using the *Excel Solver* in optimization of such problems.

Problem1: Maximizing Profit

Step 1: **Familiarizing with the data set**:

Stratton Co. problem states that it

- Produces two basic types of plastic pipes;
- Three resources have been identified as critical to pipe output "Pipe extrusion hours, Packaging hours, and Special additive mix ".

Stratton Company Data was summarized in the following Table:

	Product		Resource
Resource	Type 1	Type 2	Availability
Extrusion	4 hrs.	6 hrs.	48 hrs.
Packaging	2 hrs.	2 hrs.	18 hrs.
Additive Mix	2 lbs.	1 lbs.	16 lbs.
Profit	$34	$40	

All data given is for a package of pipe – 100 feet

The problem requirement to formulate an LP model to determine how much of each type of pipe should be produced to maximize profit.

Step 2: Set up the optimization model (Model Construction):

Decision Variables:

Figure 1: Excel Add-Ins Dialog Box

]

P1= No. of pipe 1 to be produced
P2= No. of pipe 2 to be produced

Objective Function:

MAX $Z = 34\ P1 + 40\ P2$

Subject to Model Constraints:

$4\ P1 + 6\ P2 \leq 4$ *Extrusion hours*
$2\ P1 + 2\ P2 \leq 18$ *Packaging hours*
$2\ P1 + 1\ P2 \leq 16$ *Additive supply*
$P1\ ,\ \ P2 \geq 0$ *Non-negativity*

Step 3: Setting up Excel Solver to solve LPs

- Solver is an add-in to Excel
- Not automatically ready,
- To get solver ready

(In Excel points to → *Tools* → *Add Ins* then Scroll down to *Solver Add In* → Check the box → Click on *OK* as shown figure 1)

- Only need to do this one time

To solve an LP using Excel Solver;

- Setup the spreadsheet;
  - ■ TYPE data in one place (as shown in figure 2);
  - ■ CREATE in D4 Cell the function (as shown in figure 3);
  - ■ Pull- Down D4 Function to The cells D5 to D7 (as shown in figure 4);

■ OPEN Solver box with *Tools* → Solver (as shown in figures 5, 6 and 7);

■ CREATE Cells for decisions variables and formulas to calculate LHS of constraints (figure 8);

■ ENTER formulas to calculate Objective Function (Figs. 9 and 10);

■ CLICK *OK* then *Solve* to get the optimal solution (Figs.11 and 12);

■ Now solver found the optimal solution, click on all reports to keep the solution (as in figure 13);

■ Click on *OK* to get the optimal solution (as in figure 14);

■ All reports can be found in figures (15, 16, and 17);

■ Compare SOLVER solution with Graphical and SIMPLEX solutions in figures (18 and 19);

Figure 2 The spreadsheet

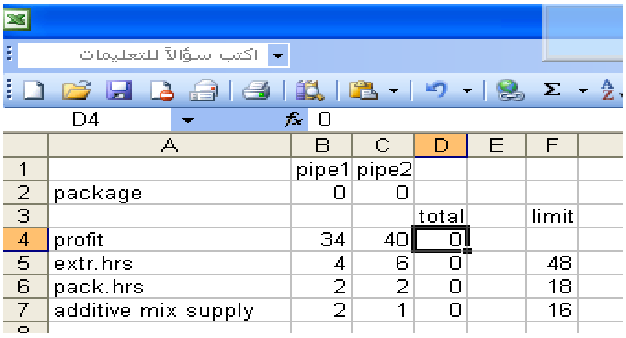


Figure 3 Create the function cell

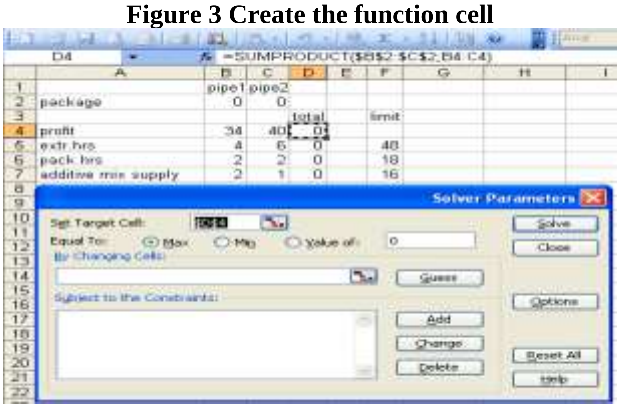


Figure 4 Pull- Down fun. cell

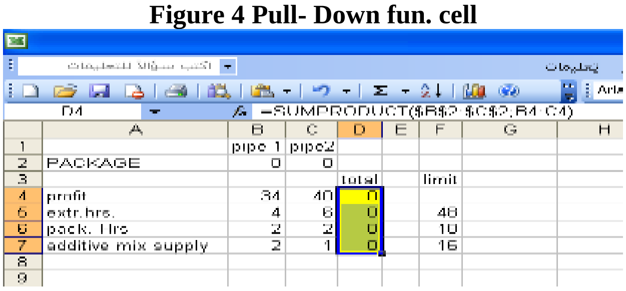


Figure 5 Open Solver box

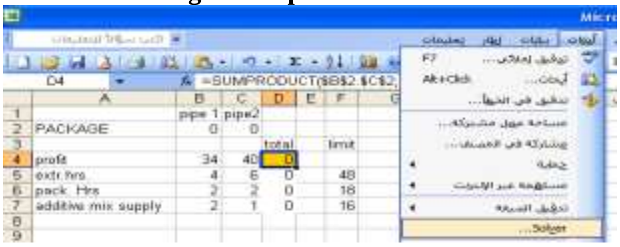


Figure 6
Excel Solver Dialog Box

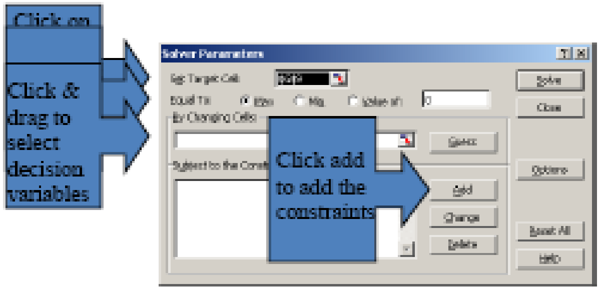


Figure 7
Excel solver – constraints dialog box

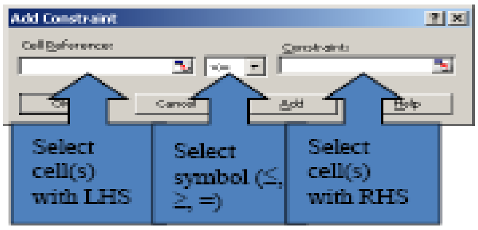


Figure 8 Create D.V.& Constraints

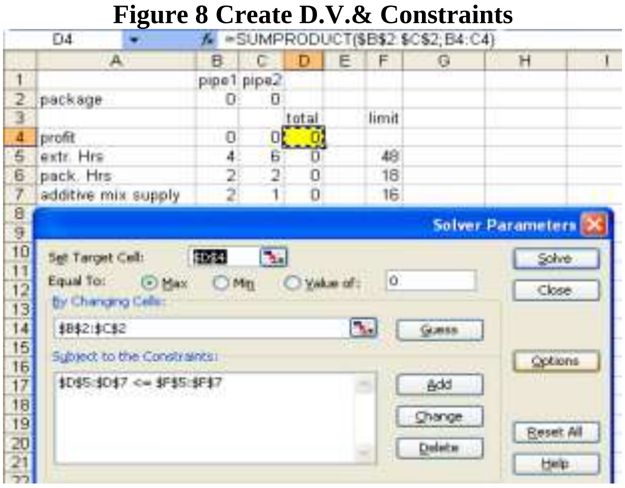


Figure 9

Go to the Options Dialog box

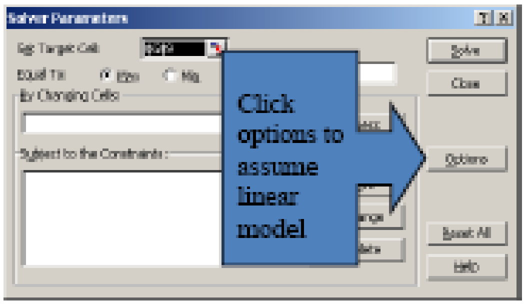


Figure 10 Formula of Obj. Fun.

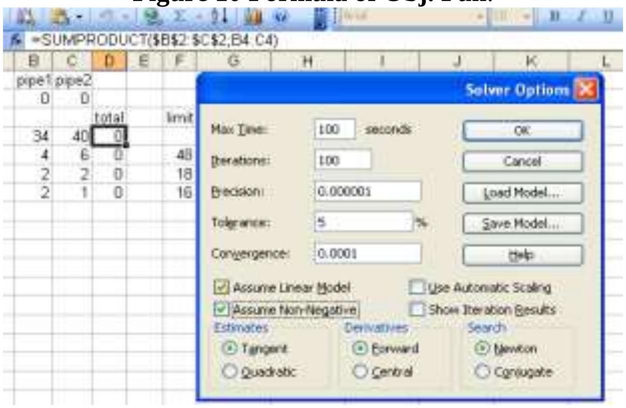


Figure 11

Last dialog box - options

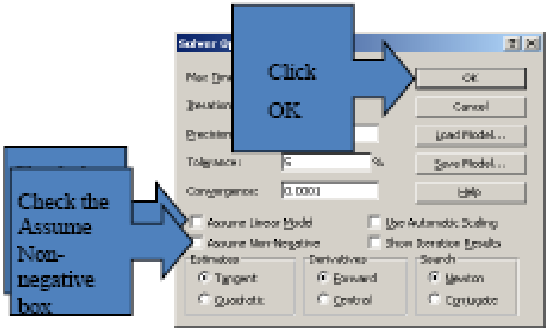


Figure 12

Now SOLVE

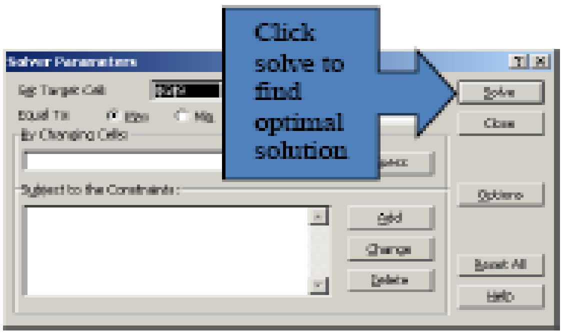


Figure 13

Solver found a solution

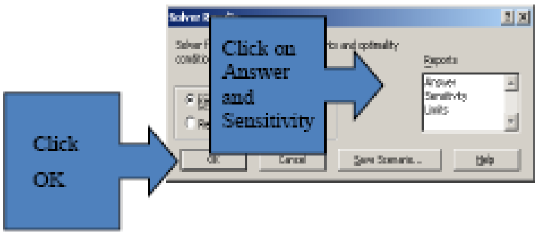


Figure 14 Keep all Reports

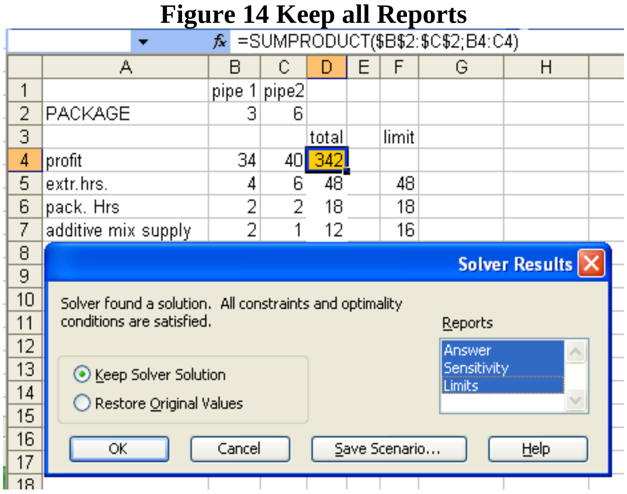
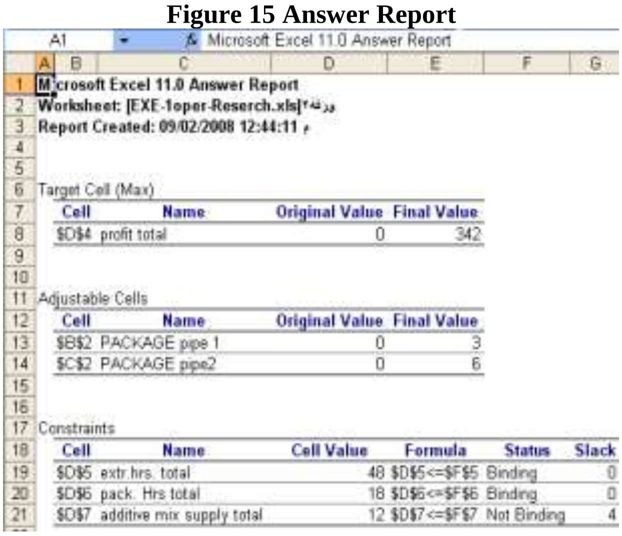


Figure 15 Answer Report



Microsoft Excel 11.0 Answer Report
Worksheet: [EXE-1oper-Reserch.xls]ورقة2
Report Created: 09/02/2008 12:44:11 م

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	profit total	0	342

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	PACKAGE pipe 1	0	3
$C$2	PACKAGE pipe2	0	6

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	extr.hrs. total	48	$D$5<=$F$5	Binding	0
$D$6	pack. Hrs total	18	$D$6<=$F$6	Binding	0
$D$7	additive mix supply total	12	$D$7<=$F$7	Not Binding	4

Figure 16 Sensitivity Report

A1 — Microsoft Excel 11.0 Sensitivity Report

Microsoft Excel 11.0 Sensitivity Report
Worksheet: [EXE-1oper-Reserch.xls]ورقة1
Report Created: 09/02/2008 12:44:12 م

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	PACKAGE pipe 1	3	0	34	6	7.333333333
$C$2	PACKAGE pipe2	6	0	40	11	6

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	extr.hrs. total	48	3	48	6	8
$D$6	pack. Hrs total	18	11	18	2	2
$D$7	additive mix supply total	12	0	16	1E+30	4

Figure 17 Limits Report

A1 — Microsoft Excel 11.0 Limits Report

Microsoft Excel 11.0 Limits Report
Worksheet: [EXE-1oper-Reserch.xls]Limits Report 2
Report Created: 09/02/2008 12:44:12 م

Cell	Target Name	Value
$D$4	profit total	342

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
$B$2	PACKAGE pipe 1	3	0	240	3	342
$C$2	PACKAGE pipe2	6	0	102	6	342

Figure 18 Graphical Solution

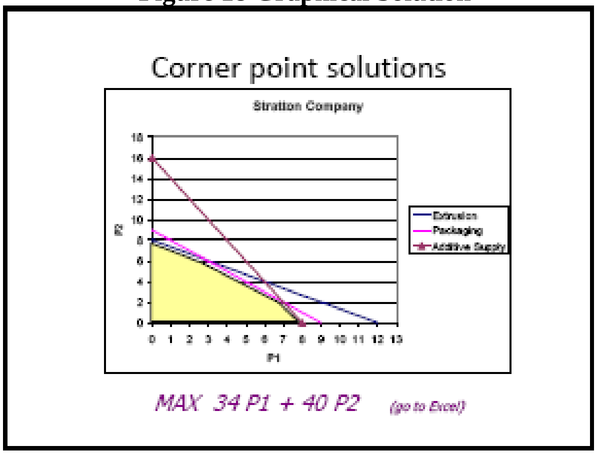
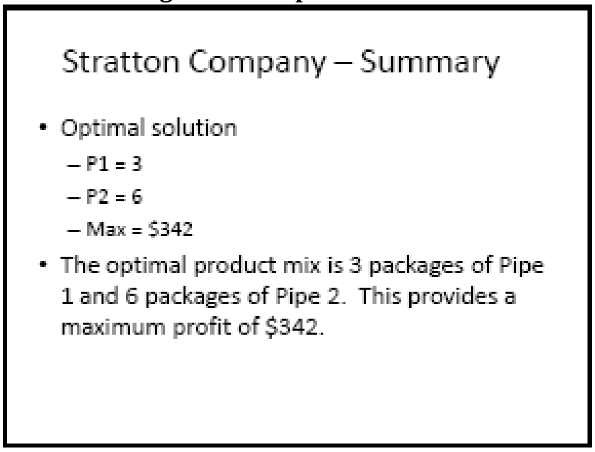


Figure 19 Simplex Solution



Problem2: Minimizing Problem

Consider the following problem:

Minimize $Z = 0.6\ X1 + 0.5\ X2$

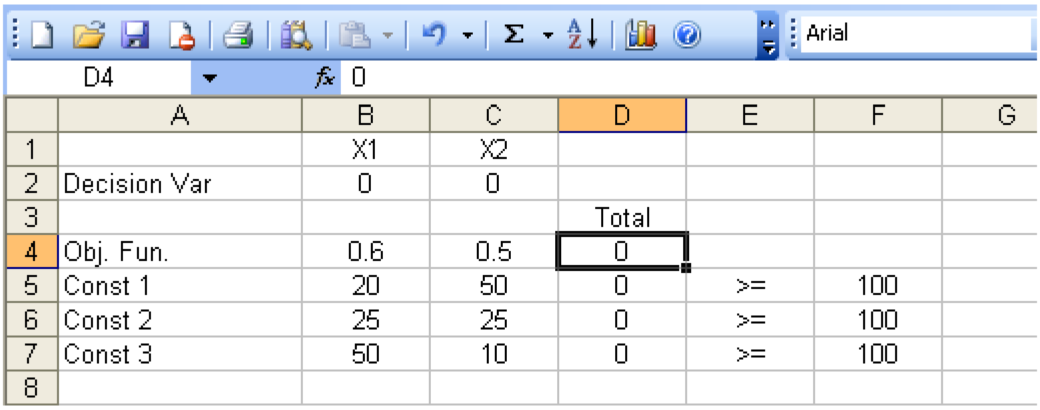
S.T. $20\ X1 + 50\ X2 \geq 100$

$25\ X1 + 25\ X2 \geq 100$

$50\ X1 + 10\ X2 \geq 100$

$X1\ ,\quad X2 \geq 0$

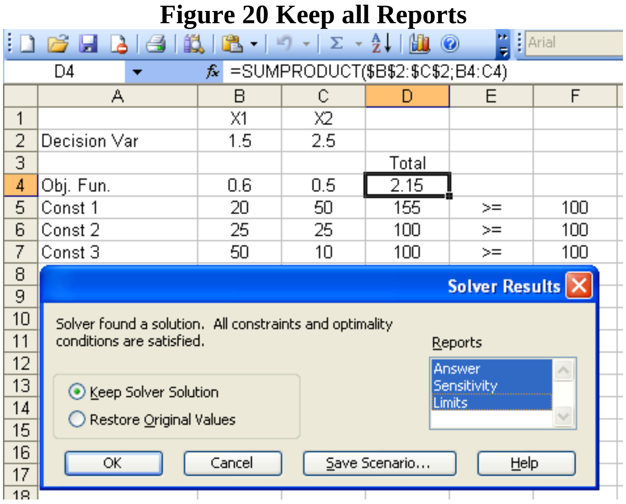
By applying the solution steps by the Excel Solver that were applied in the first problem, with changing the inequality symbols to (>=), it has to be begun with the following spreadsheet of the problem:



	A	B	C	D	E	F	G
1		X1	X2				
2	Decision Var	0	0				
3				Total			
4	Obj. Fun.	0.6	0.5	0			
5	Const 1	20	50	0	>=	100	
6	Const 2	25	25	0	>=	100	
7	Const 3	50	10	0	>=	100	
8							

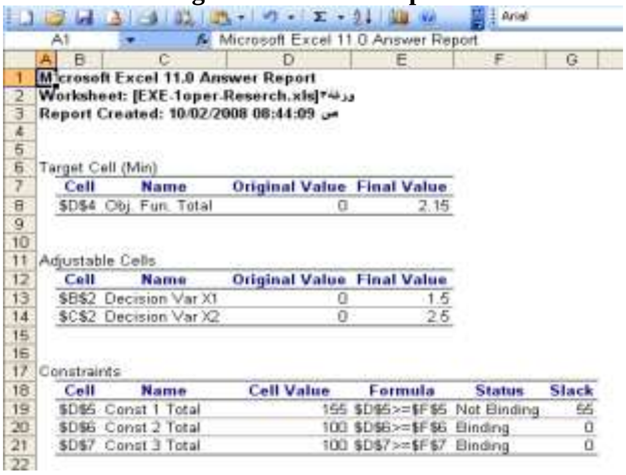
The optimal solution, through Step 3 with changing sub step3-4 "*Equal to* by Min ", can be found as in figures (20, 21, 22, and 23).

Figure 20 Keep all Reports



	A	B	C	D	E	F
1		X1	X2			
2	Decision Var	1.5	2.5			
3				Total		
4	Obj. Fun.	0.6	0.5	2.15		
5	Const 1	20	50	155	>=	100
6	Const 2	25	25	100	>=	100
7	Const 3	50	10	100	>=	100

Figure 21 Answer Report



Microsoft Excel 11.0 Answer Report
Worksheet: [EXE-1oper-Reserch.xls]ورقة7
Report Created: 10/02/2008 08:44:09 ص

Target Cell (Min)

Cell	Name	Original Value	Final Value
$D$4	Obj. Fun. Total	0	2.15

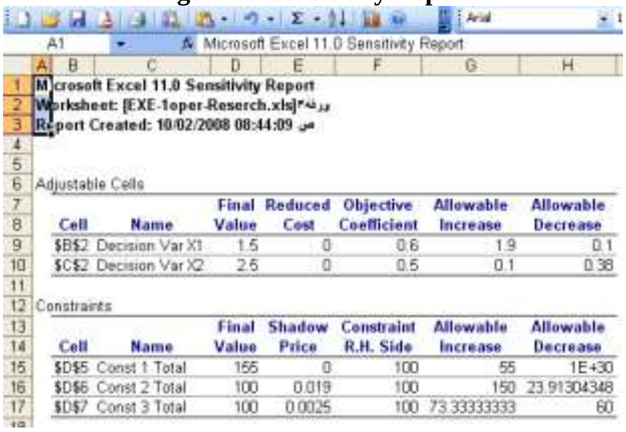
Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	Decision Var X1	0	1.5
$C$2	Decision Var X2	0	2.5

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	Const 1 Total	155	$D$5>=$F$5	Not Binding	55
$D$6	Const 2 Total	100	$D$6>=$F$6	Binding	0
$D$7	Const 3 Total	100	$D$7>=$F$7	Binding	0

Figure 22 Sensitivity Report



Microsoft Excel 11.0 Sensitivity Report
Worksheet: [EXE-1oper-Reserch.xls]ورقة7
Report Created: 10/02/2008 08:44:09 ص

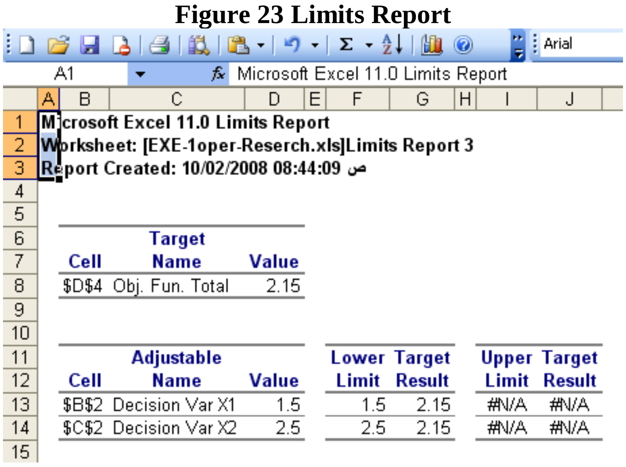
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Decision Var X1	1.5	0	0.6	1.9	0.1
$C$2	Decision Var X2	2.5	0	0.5	0.1	0.38

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Const 1 Total	155	0	100	55	1E+30
$D$6	Const 2 Total	100	0.019	100	150	23.91304348
$D$7	Const 3 Total	100	0.0025	100	73.33333333	60

Figure 23 Limits Report



Microsoft Excel 11.0 Limits Report
Worksheet: [EXE-1oper-Reserch.xls]Limits Report 3
Report Created: 10/02/2008 08:44:09 ص

Cell	Target Name	Value
$D$4	Obj. Fun. Total	2.15

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
$B$2	Decision Var X1	1.5	1.5	2.15	#N/A	#N/A
$C$2	Decision Var X2	2.5	2.5	2.15	#N/A	#N/A

Problem3: Artificial Starting Solution Problem

Consider the following problem [8]:

Minimize $Z = 4X1 + X2$

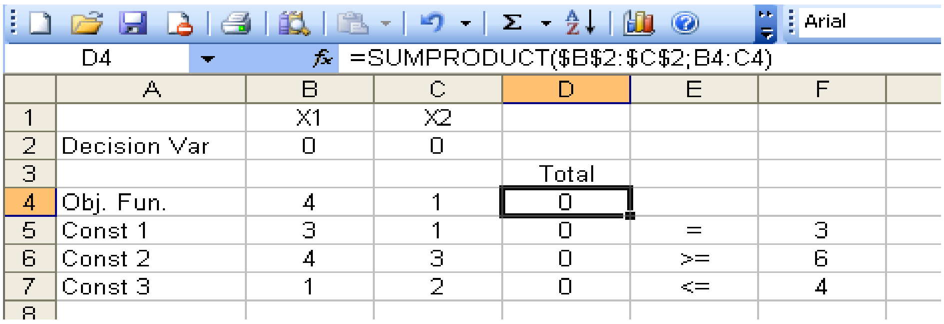
S.T. $3X1 + X2 = 3$

$4X1 + 3X2 \geq 6$

$X1 + 2X2 \leq 4$

$X1, X2 \geq 0$

By applying the solution steps by the Excel Solver that were applied in the first problem, with adding each constraint and its inequality symbol of (=,>=,<=) at a time individually by using *Add* in figure 7, it has to be begun with the following spreadsheet of the problem:

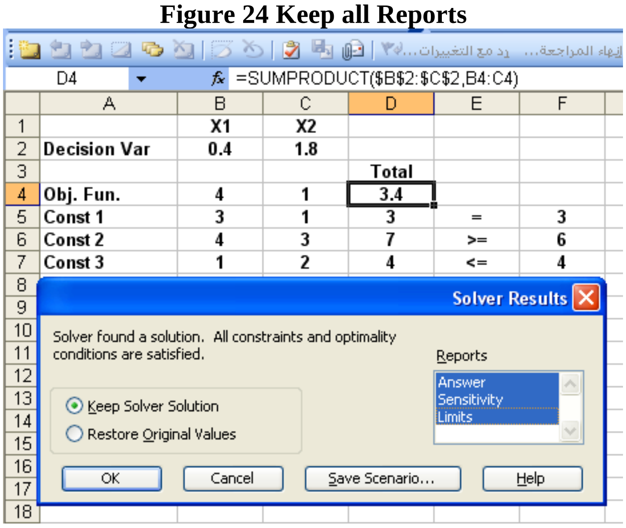


D4 =SUMPRODUCT($B$2:$C$2;B4:C4)

	A	B	C	D	E	F
1		X1	X2			
2	Decision Var	0	0			
3				Total		
4	Obj. Fun.	4	1	0		
5	Const 1	3	1	0	=	3
6	Const 2	4	3	0	>=	6
7	Const 3	1	2	0	<=	4

The optimal solution, through Step 3 with changing sub step3-4" *Equal to* by Min ", can be found as in figures (24, 25, 26, and 27).

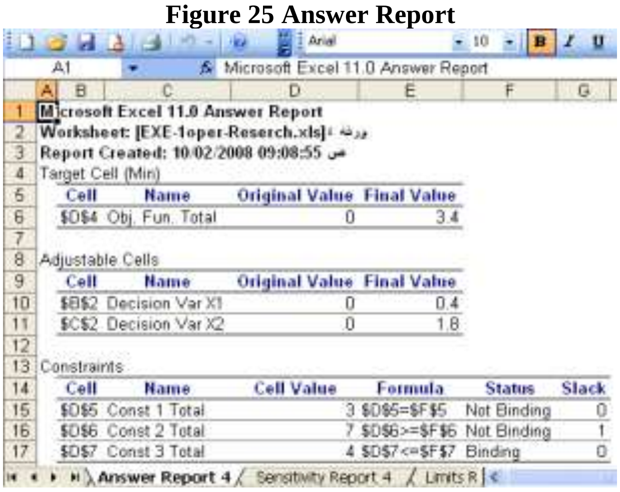
Figure 24 Keep all Reports



D4 =SUMPRODUCT($B$2:$C$2,B4:C4)

	A	B	C	D	E	F
1		X1	X2			
2	Decision Var	0.4	1.8			
3				Total		
4	Obj. Fun.	4	1	3.4		
5	Const 1	3	1	3	=	3
6	Const 2	4	3	7	>=	6
7	Const 3	1	2	4	<=	4

Figure 25 Answer Report



Microsoft Excel 11.0 Answer Report
Worksheet: [EXE-1oper-Reserch.xls]
Report Created: 10/02/2008 09:08:55

Target Cell (Min)

Cell	Name	Original Value	Final Value
$D$4	Obj. Fun. Total	0	3.4

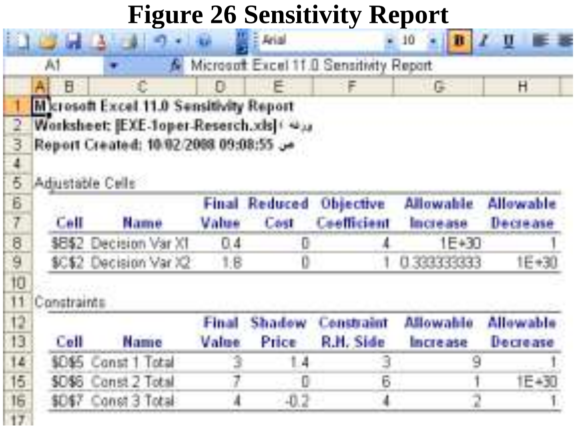
Adjustable Cells

Cell	Name	Original Value	Final Value
$B$2	Decision Var X1	0	0.4
$C$2	Decision Var X2	0	1.8

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	Const 1 Total	3	$D$5=$F$5	Not Binding	0
$D$6	Const 2 Total	7	$D$6>=$F$6	Not Binding	1
$D$7	Const 3 Total	4	$D$7<=$F$7	Binding	0

Figure 26 Sensitivity Report



Microsoft Excel 11.0 Sensitivity Report
Worksheet: [EXE-1oper-Reserch.xls]
Report Created: 10/02/2008 09:08:55

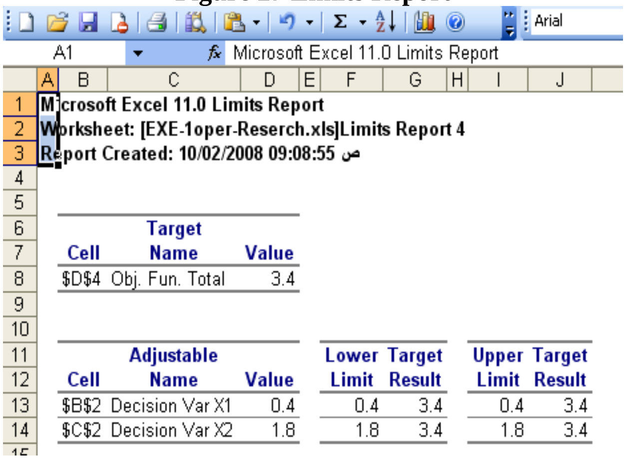
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$2	Decision Var X1	0.4	0	4	1E+30	1
$C$2	Decision Var X2	1.8	0	1	0.333333333	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Const 1 Total	3	1.4	3	9	1
$D$6	Const 2 Total	7	0	6	1	1E+30
$D$7	Const 3 Total	4	-0.2	4	2	1

Figure 27 Limits Report



Microsoft Excel 11.0 Limits Report
Worksheet: [EXE-1oper-Reserch.xls]Limits Report 4
Report Created: 10/02/2008 09:08:55 ص

Cell	Target Name	Value
$D$4	Obj. Fun. Total	3.4

Cell	Adjustable Name	Value		Lower Limit	Target Result		Upper Limit	Target Result
$B$2	Decision Var X1	0.4		0.4	3.4		0.4	3.4
$C$2	Decision Var X2	1.8		1.8	3.4		1.8	3.4

Conclusions

Excel Solver provides a simple, yet effective, medium for allowing users to explore linear programming problems. It can be used for large problems containing hundreds of variables and constraints, and does these relatively quickly, but as a teaching tool using small illustrative problems it is very potent, particularly as the user must appreciate the structure of a LP when entering it into the spreadsheet.

On the downside, one can't view the Tableau as it is generated at each iteration and so those users who want to be proficient in the manual methods of LP would find Solver less superior to allow this. It does, however, produce a superior set of results and sensitivity reports when compared to Simplex method, and, due to the spreadsheet nature, does allow the student very quickly to observe the effects of any changes made to constraints or the objective function.

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حَلّ مشاكلِ البرمجة الخطية باستعمال حلاّل نظام الإكسل

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الملخّص:

تَصِفُ هذه الورقةِ طرقَ متقدّمةَ قد حقّقتْ شهرةِ عالميَة لإيجاد كُلّ الحلول لنظام المعادلاتِ الخطيّةِ، كما نفذ في البناء الأساسي للحَلاّل (Solver)، حيث حزمت إمتداداته ببرنامج Microsoft Excel. هذا الحَلاّل يَصِفُ أيضاً الأدواتَ التحتيةَ التي تَسْمحُ باستعمال صفحات النشر في نظام الإكسل(Excel spreadsheets) لكي تتعامل مع الدوال الخطية من خلال الحساباِت السريعِة لتحقيقِ الأمثلية.

تقدمُ أيضاً نظرة عامّة قصيرة عن Excel's Add-in Solver؛ النظرية الأساسية لتحقيقِ الأمثلية كما هي مطبقة ضمن هذا الحَلاّل؛ فوائده في البرمجة الخطيةِ، وثلاثة أمثلةِ عدديةِ تُلخّصُ الخطواتَ التي تشترك في ترتيبات هذا الحَلاّل من اجل حَلّ مسائل البرمجة الخطيةَ. يُمْكِنُ أَنْ يعتبر إستعمالِ نظام الإكسل لتحقيقِ الأمثلية خيارا فعّالا للأسباب التالية : (أ) جاهزية توفره وبسهولةُ في البناء الأساسي لأيّ نظام نوافذِ (any Windows) بدون أيّ كلفة إضافية، (ب) نظام الإكسل سهل الاستعمال، (ج) طريقة تُحوّيلُ البياناتَ إليه ومِنْه تكون مرنةُ جداً.

الكلمات الدالة: بحوث العمليات، برمجة خطية، حَلاّل الإكسل، الأمثلية.