J. of university of anbar for pure science : Vol.4:NO.1 : 2010

ISSN: 1991-8941

ON GENERALIZED ALMOST CONTRA CONTINUOUS FUNCTIONS AND SOME RELATIONS WITH ANOTHER KINDS OF CONTINUITY ON INTUITIONISTIC TOPOLOGICAL SPACES

Yunis J. Yaseen Ali M. Jasim
College of Education - Tikrit University

Received:11/1/2009 Accepted:25/6/2009

Abstract: We study in this paper the concept of almost contra continuous functions and generalized them in intuitionistic topological spaces and we studied the relations of each kind of these function by properties, examples and a diagram to summarize these functions. Also we study some relation between almost contra continuous function and some continuous functions.

Key words: ALMOST CONTRA CONTINUOUS FUNCTIONS ,CONTINUITY ON INTUITIONISTIC ,TOPOLOGICAL SPACES

Introduction

Almost contra continuous functions were introduced by Joseph and Kwack [4], almost contra pre continuous fun- ction was introduced by Ekici [3]. So we are going generalized them on ITS's.

In this paper we investigate defin-itions of almost contra continuous, almost contra semi continuous,almost contra pre continuous, almost contra α continuous,almost contra θ continuous, almost contra β continuous, almost contra g continuous, almost contra gs continuous,almost contras$g$ continuous almost contra gp continuous, almost contra pg continuous, almost contra $g\alpha$ continuous, almost contra αg contin- uous, almost contra $g\beta$ continuous and almost contra θg continuous functions and we show the relations of each kind of these functions by properties and counter examples and we illustrate the result by a diagram and we introduced the definitions of almost semi-regular, almost regular closed,regular irresolute and regular set connected and study the relation among them and almost contra continuous functions.

2.Preliminaries

Let X be anon- where A_1 and A_2 are disjoint subset of X. the set A_1 is called the set of member of A, while A_2 is called the set of non member of A, an intuitionistic topology (IT, for short) on a non-empty set X, is a family T of IS in X containing $\tilde{\emptyset}, \tilde{X}$ and closed under arbitrary unions and finitely intersections. In this case the pair (X,T) is called an intuitionistic topological space (ITS, for short), any IS in T is known as an intuitionistic open set (IOS, for short) in X. The complement of IOS is called intuitionistic closed set (ICS, for short), so the interior and closure of A are denoted by $int(A)$ and $cl(A)$ respectively and defined by

$$int(A) = \cup \{G_i : G_i \in T \text{ and } G_i \subseteq A\} \text{ and } cl(A) = \cap \{F_i : F_i \text{ is ICS in X and } A \subseteq F_i\}$$

So $int(A)$ is the largest IOS contained in A,and $cl(A)$ is the smallest ICS contain A, a set A is called intuitionistic regular-closed set (IRCS, for short) if $A = clintA$ intuitionistic α-ciosed set (I$^{\alpha}$CS, for short) if $clintclA \subseteq A$, intuitionistic semi-closed set (ISCS, for short) if $intclA \subseteq A$, intuitionistic pre-closed set (IPCS, for short) if $clintA \subseteq A$,intuitionistic β-closed set (I$^{\beta}$CS, for short) if $\text{intclint}A \subseteq A$. The complement of IRCS (resp. I^{α}CS, ISCS, IPCS and I^{β}CS) is called intuitionistic regular-open set (resp. intuitionistic α-open set, intuitionistic semi-open set, intuitionistic pre-open set and intuitionistic β-open set) in X. (IROS, I^{α}OS, ISOS, IPOS and I^{β}OS, for short), A is said to be intuitionistic semi-regular set (ISRS, for short) [6] if A is ISOS and ISCS in X, so A is called intuitionistic B-set (IBS, for short) [6] if A is the intersection of an IOS and ISCS and A is said to be an intuitionistic θ-closed set (I^{θ}CS, for short) if $A = cl_{\theta}A$ where $cl_{\theta}A = \{x \in X: cl(U) \cap A \neq \emptyset, U \in T \text{ and } x \in U\}$.

A is called intuitionistic θ generalized-closed set ($I\theta g$-closed for short) if $cl_{\theta}A \subseteq U$, whenever $A \subseteq U$ and U is IOS.

3. Generalized almost contra continuous functions on ITS's.

The definitions of almost contra continuous functions which appears in general topology by [2],[5] and [6], so we generalized them on ITS's.

Definition 3.1. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then f is said to be:

An intuitionistic almost contra continuous (I almost contra cont., for short) function if the inverse image of each IROS in Y is ICS in X.

An intuitionistic almost contra semi-continuous (I almost contra semi-cont., for short) function if the inverse image of each IROS in Y is ISCS in X.

An intuitionistic almost contra α-continuous (I almost contra α-cont., for short) function if the inverse image of each IROS in Y is I^{α}CS in X.

An intuitionistic almost contra pre-continuous (I almost contra pre-cont., for short) function if the inverse image of each IROS in Y is IPCS in X.

An intuitionistic almost contra β-continuous (I almost contra β-cont., for short) function if the inverse image of each IROS in Y is I^{β}CS in X.

An intuitionistic almost contra θ-continuous (I almost contra θ-cont., for short) function if the inverse image of IROS in Y is I^{θ}CS in X.

Definition 3.2. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then f is said to be an intuitionistic almost contra g-cont. (resp. almost contra gs-cont., almost contra sg-cont., almost contra gp-cont., almost contra pg-cont., almost contra $g\alpha$-cont., almost contra αg-cont., almost contra θg-cont. and almost contra $g\beta$-cont. functions if the inverse image of each IROS in Y is Ig-closed (resp. Igs-closed, Isg-closed, Igp-closed, pg-closed, I$g\alpha$-closed, Iαg-closed, Iθg-closed and I$g\beta$-closed) set in X.

Proposition 3.3. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then:

1- If f is I almost contra cont. function then f is I almost contra g-cont. function.

2- If f is I almost contra θ-cont. function then f is I almost contra cont. function.

3- If f is I almost contra θg-cont. function then f is I almost contra g-cont. function.

4- If f is I almost contra cont. function then f is I almost contra α-cont. function.

5- If f is I almost contra θ-cont. function then f is I almost contra θg-cont. function.

6- If f is I almost contra α-cont. function then f is I almost contra semi-cont. function.

7- If f is I almost contra semi-cont. function then f is I almost contra β-cont. function.

8- If f is I almost contra α-cont. function then f is I almost contra pre-cont. function.

9- If f is I almost contra pre-cont. function then f is I almost contra β-cont. function.

10- If f is I almost contra α-cont. function then f is I almost contra gα-cont. function.

11- If f is I almost contra β-cont. function then f is I almost contra gβ-cont. function.

12- If f is I almost contra semi-cont. function then f is I almost contra sg-cont. function.

13- If f is I almost contra g-cont. function then f is I almost contra αg-cont. function.

14- If f is I almost contra g-cont. function then f is I almost contra gs-cont. function.

15- If f is I almost contra gα-cont. function then f is I almost contra αg-cont. function.
16- If f is I almost contra sg-cont. function then f is I almost contra gs-cont. function.
17- If f is I almost contra pg-cont. function then f is I almost contra gp-cont. function.
18- If f is I almost contra pre-cont. function then f is I almost contra gβ-cont. function.
19- If f is I almost contra gα-cont. function then f is I almost contra pre-cont. function.
20- If f is I almost contra αg-cont. function then f is I almost contra gp-cont. function.
21- If f is I almost contra αg-cont. function then f is I almost contra gs-cont. function.
22- If f is I almost contra gα-cont. function then f is I almost contra gs-cont. function.
23- If f is I almost contra gs-cont. function then f is I almost contra gβ-cont. function.
24- If f is I almost contra gp-cont. function then f is I almost contra gβ-cont. function.

Proof:

We are give the proof of (21) as example and others can be proved in a similar way.

Let V be IROS in Y then $f^{-1}(V)$ is Iαg-closed set in X (since f is I almost contra αg-cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl(f^{-1}(V)) \subseteq A$. Now since every I$\alpha$CS is ISCS then $scl(f^{-1}(V)) = \cap \{F_i : F_i$ is ISCS and $f^{-1}(V) \subseteq F_i\} \subseteq \alpha cl(f^{-1}(V))$. So we have that for each IOS A in X and $f^{-1}(V) \subseteq A$ then $scl(f^{-1}(V)) \subseteq A$. There fore, $f^{-1}(V)$ is Igs-closed set in X and hence f is I almost contra gs-cont. function.♦

We start with example to show that I almost contra g-cont. is not imply I almost contra cont.

Example 3.4. Let $X = \{a, b, c, d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a, b\}, \{c\}\rangle, B = \langle x, \{a\}, \{b\}\rangle$ and $C = \langle x, \{a, b\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E, F, H\}$ where $D = \langle y, \{1\}, \emptyset\rangle$, $E = \langle y, \{2\}, \{1,3\}\rangle$, $F = \langle y, \{1,2\}, \emptyset\rangle$ and $H = \langle y, \emptyset, \{1,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = f(c) = 1, f(b) = 2$ and $f(d) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, D\}$ Now let $G = f^{-1}(D) = \langle x, \{a, c\}, \emptyset\rangle$ then G is Ig-closed set in X since the only IOS containing G is X and $clG = X \subseteq X$ but G is not ICS in X since $G \neq clG = X$. So f is I almost contra g-cont. function but not I almost contra cont. function.

In this example we are going to show I almost contra α-cont. function is not imply I almost contra cont. function

Example 3.5. Let $X = \{a, b, c, d\}$ and let $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a, b\}, \{c\}\rangle, B = \langle x, \{b, d\}, \{a\}\rangle, C = \langle x, \{b\}, \{a, c\}\rangle$ and $D = \langle x, \{a, b, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E, F\}$ where $E = \langle y, \emptyset, \{1,2\}\rangle$ and $F = \langle y, \{1\}, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1$, $f(b) = f(c) = 2$ and $f(d) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, F\}$. So let $G = f^{-1}(E) = \langle x, \{a\}, \{b, c\}\rangle$ then G is IαCS set in X since $clintclG = \emptyset \subseteq G$ but G is not ICS in X since $clG = \bar{C} \neq G$. Then f is I almost contra α-cont. but not I almost contra cont.

The following example shows I almost contra θg-cont. is not imply I almost contra θ-cont.

Example 3.6. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a, c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, B, C\}$ where $B = \langle y, \{1\}, \{2\}\rangle$ and $C = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = f(b) = 1$ and $f(c) = 2$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, B\}$ Now let $G = f^{-1}(C) = \langle x, \{a, b\}, \{c\}\rangle$ then G is Iθg-closed set in X since the only IOS containing G is X and $cl_\theta G = X \subseteq X$. But G is not I$\theta$CS since $G \neq cl_\theta G = X$, then f is I almost contra θg-cont. function. But f is not I almost contra θ-cont. function.

The next example shows that:

1. I almost contra semi-cont. is not imply I almost contra α-cont.
2. I almost contra semi-cont. is not imply I almost contra pre-cont.
3. I almost contra semi-cont. is not imply I almost contra cont.

Example 3.7. Let $X = \{a, b, c, d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{b\}\rangle$ and $B = \langle x, \emptyset, \{a, b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \emptyset, \{1\}\rangle$ and $D = \langle y, \{2\}, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, C\}$ Now a set $G = f^{-1}(C) = \langle x, \emptyset, \{a\}\rangle$ is ISCS in X since $intclG = B \subseteq G$ but G is not IαCS (resp. IPCS and ICS) in X since $clintclG = clintG = clG = \bar{B} \nsubseteq G$. So the inverse image of each IROS in Y is ISCS in X. There fore, f is I almost contra semi-cont. function but f is not I almost contra α-cont. (resp. I almost contra pre-cont. and I almost contra cont.) function.

We are going to show that:

1- I almost contra cont. is not imply I almost contra θ-cont.

2- I almost contra cont. is not imply I almost contra θg-cont.

3- I almost contra g-cont. is not imply I almost contra θg-cont.

4- I almost contra g-cont. is not imply I almost contra θ-cont.

Example 3.8. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{a, b\}, \{c\}\rangle$ and $C = \langle x, \{a, c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y, \{1\}, \{2\}\rangle$ and $E = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = f(b) = 2$ and $f(c) = 1$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, D\}$ Now let $G = f^{-1}(D) = \langle x, \{c\}, \{a, b\}\rangle$, then G is ICS and I$g$-closed set in X but G is not IθCS in X since $cl_\theta G = X \nsubseteq G$ so G is not Iθg-closed set since the only IOS containing G in X is C and $cl_\theta G = X \nsubseteq C$. So f is I almost contra cont. function and I almost contra g-cont. function but f is not I almost contra θ-cont. function so f is not I almost contra θg-cont. function.

The next example shows that:

1. I almost contra pre-cont. is not imply I almost contra α-cont.
2. I almost contra pre-cont. is not imply I almost contra semi-cont.
3. I almost contra pre-cont. is not imply I almost contra cont.

Example 3.9. Let $x = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a, c\}, \emptyset\rangle$, $B = \langle x, \{c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \{1\}, \{3\}\rangle$ and $D = \langle y, \emptyset, \{1,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 3, f(b) = 2, f(c) = 1$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, C\}$. Now let $G = f^{-1}(C) = \langle x, \{c\}, \{a\}\rangle$, then G is IPCS in X since $clintG = \emptyset \subseteq G$ but G is not IαCS (resp. ISCS and ICS) in X since $clintclG = intclG = clG = X \nsubseteq G$. There fore, f is I almost contra pre-cont. function but f is not I almost contra α-cont. (resp. I almost contra semi-cont. and I almost contra cont.) function.

The following example shows that:

1. I almost contra β-cont. is not imply I almost contra cont.
2. I almost contra β-cont. is not imply I almost contra pre-cont.
3. I almost contra β-cont. is not imply I almost contra semi-cont.
4. I almost contra β-cont. is not imply I almost contra α-cont.

Example 3.10. Let $X = \{a, b, c, d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{c, d\}, \{a\}\rangle, C = \langle x, \{a, c, d\}, \emptyset\rangle$ and $D = \langle x, \emptyset, \{a, b, c\}\rangle$ and let

$Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E, F\}$ where $E = \langle y, \{2\}, \{1\}\rangle$ and $F = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = f(d) = 2, f(b) = 1$ and $f(c) = 3$.ROY $= \{\tilde{\emptyset}, \tilde{Y}, E\}$. Then a set $G = f^{-1}(E) = \langle x, \{a,d\}, \{b\}\rangle$ is IβCS in X since $intclintG = A \subseteq G$ but G is not ICS (resp. IαCS, IPCS and ISCS) in X since $clG = intclG = clintclG = X \not\subseteq G$ so $clintG = D \not\subseteq G$. Then f is I almost contra β-cont. function but f is not I almost contra cont. (resp. I almost contra semi-cont., I almost contra α-cont. and I almost contra pre-cont.) function.

We are going in the following example to show that:

1. I almost contra gs-cont. is not imply I almost contra αg-cont.
2. I almost contra gs-cont. is not imply I almost contra g-cont.
3. I almost contra sg-cont. is not imply I almost contra g-cont.

Example 3.11. Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a,b\}\rangle, B = \langle x, \{b\}, \{c\}\rangle$ and $C = \langle x, \{b,c\}, \emptyset\rangle$ and let $Y = \{\tilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y, \{2\}, \{3\}\rangle$ and $H = \langle y, \emptyset, \{2,3\}\rangle$ Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$and $f(c) = 3$. ROY $= \{\tilde{\emptyset}, \tilde{Y}, D\}$ and SOX$= \{\tilde{\emptyset}, \tilde{X}, A, B, C, E, F\}$ where $E = \langle x, \{c\}, \{b\}\rangle$ and $F = \langle x, \{a,b\}, \{c\}\rangle$. So αOX$=$T. We have $B = f^{-1}(D)$ is Igs-closed and Isg-closed in X since the only IOS and ISOS in X that containing B are B,C and F so $sclB = \bar{F} = B$. but B is not Ig-closed set and it's not Iαg-closed set in X since the only IαOS in X containing B is B and C and $clB = \alpha clB = \bar{A} \not\subseteq B$ or C. There fore, f is I almost contra gs-cont. (resp. I almost contra sg-cont.) function but not I almost contra αg-cont.(resp. I almost contra g-cont.) function.

The following example shows that:

1. I almost contra gs-cont. is not imply I almost contra gα-cont.
2. I almost contra g-cont. is not imply I almost contra gα-cont.
3. I almost contra αg-cont. is not imply I almost contra gα-cont.

Example 3.12. Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b,c\}\rangle, B = \langle x, \{b\}, \{a,c\}\rangle$ and $C = \langle x, \{a,b\}, \{c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y, \{3\}, \{1\}\rangle$ and $E = \langle y, \emptyset, \{1,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 3, f(b) = 1$ and $f(c) = 2$. ROY $= \{\tilde{\emptyset}, \tilde{Y}, D\}$ and SOX$= \{\tilde{\emptyset}, \tilde{X}, A, B, C, G, K, I, N, F\}$ where $G = \langle x, \{a\}, \{b\}\rangle, K = \langle x, \{a,b\}, \emptyset\rangle, I = \langle x, \{a,c\}, \{b\}\rangle, N = \langle x, \{b\}, \{a\}\rangle$ and $F = \langle x, \{b,c\}, \{a\}\rangle$. So αOX$= \{\tilde{\emptyset}, \tilde{X}, A, B, C, K\}$. We have $G = f^{-1}(D)$ is Ig-closed (resp. Igs-closed, Iαg-closed) set in X since the only IOS containing G is X and $clG = \alpha clG = \bar{B} \subseteq X$ and $sclG = N \subseteq X$ but G is not Igα-cosed since $G \subseteq F$ where F is IαOS in X but $\alpha clG = \bar{B} \not\subseteq F$. Then the inverse image of each IROS in Y is Ig-closed (resp. Igs-closed and Iαg-closed) set in X. So f is I almost contra g-cont. (resp. I almost contra gs-cont., I almost contra αg-cont.) function but not I almost contra gα-cont. function.

We are going to show I almost contra gα-cont. is not imply I almost contra α-cont.

Example 3.13. Let $X = \{a,b,c,d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a,c\}\rangle$ and $B = \langle x, \{a\}, \{b,c\}\rangle$ and $C = \langle x, \{a,b\}, \{c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E, F, H\}$

where $D = \langle y, \{2\}, \{1,3\} \rangle, E = \langle y, \{1,2\}, \emptyset \rangle, F = \langle y, \{1\}, \emptyset \rangle$ and $H = \langle y, \emptyset, \{1,3\} \rangle$.

Define a function $f: X \to Y$ by $f(a) = 2, f(b) = f(c) = 1$ and $f(d) = 3$.

$ROY = \{\tilde{\emptyset}, \tilde{Y}, F\}$ and $\alpha OX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, K\}$ where $K = \langle x, \{a, b\}, \emptyset \rangle$. So a set $G = f^{-1}(F) = \langle x, \{b, c\}, \emptyset \rangle$ is I$g\alpha$-closed set in X since the only IαOS containing G is X and $\alpha clG = X \subseteq X$ but G is not IαCS in X since $clintclG = X \nsubseteq G$ then f is I almost contra $g\alpha$-cont. function but not I almost contra α-cont. function.

The next example shows I almost contra gs-cont. is not imply I almost contra sg-cont.

Example 3.14. Let $X = \{a, b, c, d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a, b\} \rangle, B = \langle x, \{a\}, \{b, c\} \rangle$ and $C = \langle x, \{a, c\}, \{b\} \rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E, F, H\}$ where $D = \langle y, \{2\}, \{1,3\} \rangle, E = \langle y, \{1,2\}, \emptyset \rangle, F = \langle y, \emptyset, \{1,3\} \rangle$ and $H = \langle y, \{1\}, \emptyset \rangle$. Define a function $f: X \to Y$ by $f(a) = 2, f(b) = 3$ and $f(c) = 1$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, H\}$ and $SOX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, K, L, M, N, I\}$ where $K = \langle x, \{c\}, \{a\} \rangle, L = \langle x, \{a, c\}, \emptyset \rangle, M = \langle x, \{b, c\}, \{a\} \rangle, N = \langle x, \{a\}, \{c\} \rangle$ and $I = \langle x, \{a, b\}, \{c\} \rangle$.

Now let $G = f^{-1}(H) = \langle x, \{c\}, \emptyset \rangle$, then G is I$g$s-closed set in X since the only IOS containing G is X and $sclG = X \subseteq X$ but G is not Isg-closed set in X since $G \subseteq L$ where L is ISOS in X and $sclG = X \nsubseteq L$. Then the inverse image of each IROS in Y is Igs-closed set in X so f is I almost contra gs-cont. function but not I almost contra sg-cont. function.

The following example shows that I almost contra $g\beta$-cont. is not imply I almost contra gp-cont.

Example 3.15. Let $X = \{a, b, c\}$ and let $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\} \rangle, B = \langle x, \{b\}, \{a, c\} \rangle$ and $C = \langle x, \{a, b\}, \{c\} \rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y, \{1\}, \{2\} \rangle$ and $E = \langle y, \emptyset, \{1,2\} \rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = f(c) = 2$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, D\}$ and $\beta OX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, F, H, K, L, I, M, O, N, G, V, J\}$ where $F = \langle x, \{b\}, \{a\} \rangle, H = \langle x, \{b\}, \{c\} \rangle, K = \langle x, \{b\}, \emptyset \rangle, L = \langle x, \{a\}, \{b\} \rangle, I = \langle x, \{a\}, \{c\} \rangle, M = \langle x, \{a\}, \emptyset \rangle, O = \langle x, \{b, c\}, \{a\} \rangle, N = \langle x, \{a, b\}, \emptyset \rangle, G = \langle x, \{b, c\}, \emptyset \rangle, V = \langle x, \{a, c\}, \{b\} \rangle$ and $J = \langle x, \{a, c\}, \emptyset \rangle$.

$POX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, H, K, I, N, G, J\}$.

Now a set $A = f^{-1}(D)$ is I$g\beta$-closed set in X since A is IOS and $\beta clA = A$. But A is not Igp-closed set since $pclA = O \nsubseteq A$. Then f is I contra $g\beta$-cont. function since the inverse image of each IROS in Y is I$g\beta$-closed set in X. so f is not I contra gp-cont. function.

We are going to show that:

1. I almost contra pre-cont. is not imply I almost contra $g\alpha$-cont.
2. I almost contra β-cont. is not imply I almost contra sg-cont.
3. I almost contra β-cont. is not imply I almost contra gs-cont.
4. I almost contra gp-cont. is not imply I almost contra sg-cont.
5. I almost contra gp-cont. is not imply I almost contra αg-cont.
6. I almost contra $g\beta$-cont. is not imply I almost contra sg-cont.

7. I almost contra gβ-cont. is not imply I almost contra gs-cont.

Example 3.16. Let $X = \{a,b,c\}$ and let $T = \{\widetilde{\emptyset}, \widetilde{X}, A, B\}$ where $A = \langle x,\{c\},\{b\}\rangle$ and $B = \langle x,\{b,c\},\emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \widetilde{Y}, C, D\}$ where $C = \langle y,\{1\},\{2\}\rangle$ and $D = \langle y,\emptyset,\{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 2, f(b) = 3$ and $f(c) =$ 1. $ROY = \{\widetilde{\emptyset}, \widetilde{Y}, C\}$ and $\beta OX = POX = T \cup \{K_i\}_{i=1}^{17}$

where

$K_1 = \langle x,\{c\},\emptyset\rangle, K_2 = \langle x,\{c\},\{a\}\rangle, K_3 = \langle x,\{b,c\},\{a\}\rangle, K_4 = \langle x,\{b,c\},\emptyset\rangle, K_5 = \langle x,\{a,c\},\{b\}\rangle, K_6 = \langle x,\{c\},\{a,b\}\rangle, K_7 = \langle x,\{a,b\},\emptyset\rangle, K_8 = \langle x,\{a,c\},\{b\}\rangle, K_9 = \langle x,\{a\},\emptyset\rangle, K_{10} = \langle x,\{a\},\{c\}\rangle,$

$K_{11} = \langle x,\{a\},\{b\}\rangle, K_{12} = \langle x,\emptyset,\{a,b\}\rangle, K_{13} = \langle x,\emptyset,\{b\}\rangle, K_{14} = \langle x,\emptyset,\{a\}\rangle, K_{15} = \langle x,\{a\},\{b,c\}\rangle, K_{16} = \langle x,\{b\},\emptyset\rangle$ and $K_{17} = \langle x,\{b\},\{a\}\rangle$. so $\alpha OX = SOX = T \cup \{K_1, K_5, K_7\}$ Then a set $K_2 = f^{-1}(C)$ is IPCS (resp. IβCS, Igp-closed set and Igβ-closed set) in X since $clintK_2 = intclintK_2 = \emptyset \subseteq K_2$, so the only IOS containing K_2 is B and $pclK_2 = \beta clK_2 = K_2 \subseteq B$ but K_2 is not Igs-closed (resp. Isg-closed, Igα-closed, Iαg-closed) set in X since the only IOS, IαOS and ISOS containing K_2 is B and K_7 so $\alpha clK_2 = sclK_2 = X \not\subseteq B$ or K_7. There for, f is I almost contra pre-cont. (resp. I almost contra β-cont, I almost contra gβ-cont. and I almost contra gp-cont.). but f is not I almost contra gs-cont. (resp. I almost contra sg-cont., I almost contra gα-cont. and I almost contra αg-cont.) function.

The following example shows that:

1- I almost contra gp-cont. is not imply I almost contra pre-cont.
2- I almost contra g-cont. is not imply I almost contra sg-cont.
3- I almost contra gp-cont. is not imply I almost contra sg-cont.
4- I almost contra gp-cont. is not imply I almost contra pg-cont.
5- I almost contra gβ-cont. is not imply I almost contra pre-cont.
6- I almost contra gβ-cont. is not imply I almost contra β-cont.
7- I almost contra gβ-cont. is not imply I almost contra sg-cont.
8- I almost contra gβ-cont. is not imply I almost contra pg-cont.

Example 3.17. Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \widetilde{X}, A, B\}$ where $A = \langle x,\{b,c\},\emptyset\rangle$ and $B = \langle x,\{b\},\{c\}\rangle$. and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \widetilde{Y}, C, D, E, F\}$ where $C = \langle y,\{1\},\emptyset\rangle, D = \langle y,\{1,2\},\emptyset\rangle, E = \langle y,\{2\},\{1,3\}\rangle$ and $F = \langle y,\emptyset,\{1,3\}\rangle$.

Define a function $f: X \to Y$ by $f(a) = f(b) = 1$ and $f(c) = 2$. $ROY = \{\widetilde{\emptyset}, \widetilde{Y}, C\}, POX = \beta OX = T \cup \{K_i\}_{i=1}^{17}$

where

$K_1 = \langle x,\{b\},\emptyset\rangle, K_2 = \langle x,\{b\},\{a\}\rangle, K_3 = \langle x,\{a\},\{b\}\rangle, K_4 = \langle x,\{a,c\},\emptyset\rangle, K_5 = \langle x,\{b,c\},\{a\}\rangle, K_6 = \langle x,\{a,b\},\{c\}\rangle, K_7 = \langle x,\emptyset,\{a,c\}\rangle, K_8 = \langle x,\{a,b\},\emptyset\rangle, K_9 = \langle x,\{a\},\emptyset\rangle, K_{10} = \langle x,\{c\},\emptyset\rangle,$

$K_{11} = \langle x,\{a,c\},\{b\}\rangle, K_{12} = \langle x,\emptyset,\{c\}\rangle, K_{13} = \langle x,\{a\},\{b,c\}\rangle, K_{14} = \langle x,\{b\},\{a,c\}\rangle, K_{15} = \langle x,\{c\},\{a\}\rangle, K_{16} = \langle x,\{a\},\{c\}\rangle$ and $K_{17} = \langle x,\emptyset,\{a\}\rangle$.

SOX= $T \cup \{K_1, K_6, K_8\}$ Now a set $K_8 = f^{-1}(C)$ is Ig-closed (resp. Igp-closed and Igβ-closed) set in X since the only IOS containing K_8 is X and

$clK_8 = pclK_8 = \beta clK_8 = X \subseteq X$ but K_8 is not IPCS (resp, IβCS, Isg-closed set, Ipg-closed) set in X since $clintK_8 = intclintK_8 = X \nsubseteq K_8$ so $sclK_8 = pclK_8 = X \nsubseteq K_8$ Then f is I almost contra g-cont. (resp. I almost contra gp-cont. and I almost contra $g\beta$-cont.) function but it's not I almost contra pre-cont. (resp. I almost contra β-cont, I almost contra sg-cont. and I almost contra pg-cont.) function.

In the next example we show that I almost contra sg-cont. is not imply I almost contra semi-cont.

Example 3.18. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{c\}, \{a\}\rangle$ and $C = \langle x, \{a, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D, E, F, H\}$ where $D = \langle y, \{2\}, \{1,3\}\rangle$, $E = \langle y, \emptyset, \{1,3\}\rangle$, $F = \langle y, \{1,2\}, \emptyset\rangle$ and $H = \langle y, \{1\}, \emptyset\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$.

$ROY = \{\tilde{\emptyset}, \tilde{Y}, H\}$,

$SOX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, K, L\}$ where $K = \langle x, \{a\}, \{c\}\rangle, L = \langle x, \{b, c\}, \{a\}\rangle$.

Now let $G = f^{-1}(H) = \langle x, \{a, b\}, \emptyset\rangle$ then G is Isg-closed set in X since the only ISOS containing G is X and $sclG = X \subseteq X$ but G is not ISCS in X since $intclG = X \nsubseteq G$. So the inverse image of each IOS in Y is Isg-closed set in X and we have f is I almost contra sg-cont. function but not I almost contra semi-cont. function.

In the last example we show that:

1- I almost contra αg-cont. is not imply I almost contra g-cont.

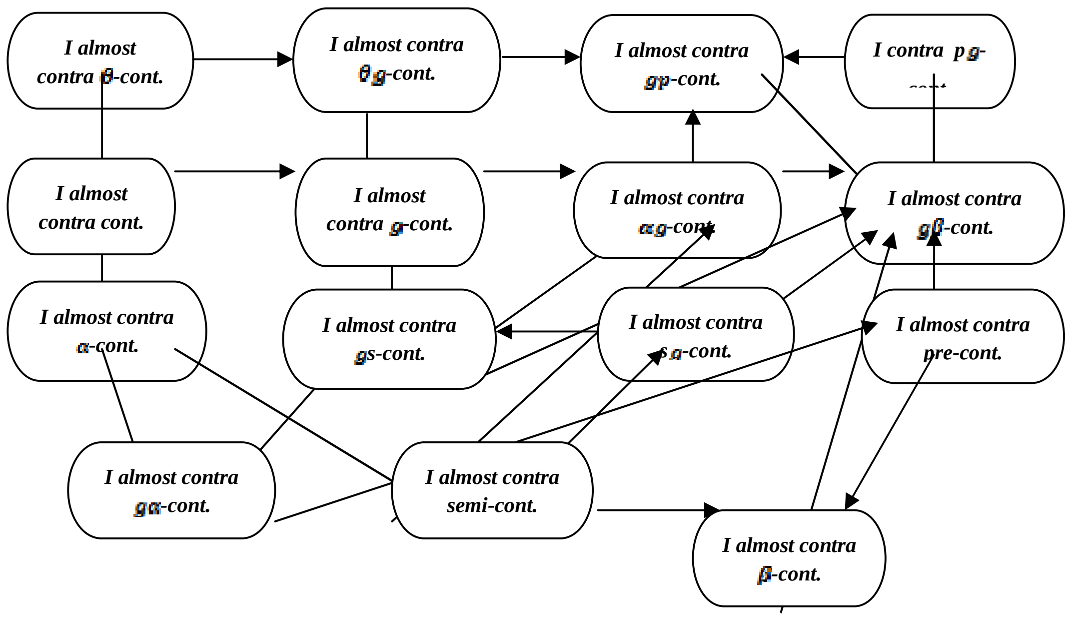
2- I almost contra $g\alpha$-cont. is not imply I almost contra g-cont.

Example 3.19. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a, c\}, \{b\}\rangle$ and $B = \langle x, \{c\}, \{a, b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle x, \{1\}, \{2\}\rangle$ and $D = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = f(c) = 2$.

$\alpha OX = \{\tilde{\emptyset}, \tilde{X}, A, B, E, F, M, L, K, N\}$ where $E = \langle x, \{c\}, \{b\}\rangle, F = \langle x, \{c\}, \emptyset\rangle, M = \langle x, \{c\}, \{a\}\rangle, L = \langle x, \{b, c\}. \emptyset\rangle, K = \langle x, \{b, c\}, \{a\}\rangle$ and $N = \langle x, \{a, c\}, \emptyset\rangle$. Now let $G = f^{-1}(C) = \langle x, \{a\}, \{b, c\}\rangle$ then G is Iαg-closed and I$g\alpha$-closed set in X since the only IOS containing G in X are A and N so $\alpha clG = \bar{K} = G \subseteq A$ and N. but G is not Ig-closed set in X since $clG = \bar{B} \nsubseteq A$. Then the inverse image of each IOS in Y is I$g\alpha$-closed and Iαg-closed set in X and hence f is I almost contra $g\alpha$-cont. function and I almost contra αg-cont. function but it's not I almost contra g-cont. function.

We summarized the above result by the following diagram.

Diagram 3.20. The following implications are true and not reversed:



4-Some relations among almost contra continuous functions and another kinds of continuity.

We introduce the following definitions.

Definition 4.1. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then f is said to be I almost SR-cont. (resp. I almost cont., I almost RC-cont. and I regular irresolute) if the inverse image of each IROS in Y is ISRS (resp. IRCS and IROS) in X.

Remark 4.2. The notions I almost contra cont. function and I almost cont. function are independent.

The following examples shows this cases.

Example 4.3. Let $X = \{a, b, c\}$ and let $T = \{\tilde{\emptyset}, \tilde{Y}, A, B\}$ where $A = \langle x, \{b\}, \{c\}\rangle$ and $B = \langle x, \emptyset, \{a, c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \{3\}, \{1\}\rangle$ and $D = \langle y, \emptyset, \{1,3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 2, f(b) = 1$ and $f(c) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, C\}$. It's easy to verify f is I almost contra cont. function but not I almost cont. function.

Example 4.4. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle$ and $B = \langle x, \emptyset, \{a, b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \emptyset, \{1,2\}\rangle$ and $D = \langle y, \{1\}, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, D\}$. Then f is I almost cont. function since the inverse image of each IROS in Y is IOS in X but f is not I almost contra cont. function.

Proposition 4.5. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1. f is I almost RC-cont. function.
2. f is I almost β-cont. function and I almost contra cont. function.

Proof 1⇒2 Let V be IROS in Y then $f^{-1}(V)$ is IRCS in X (since f is I almost RC-cont. function) then $clintf^{-1}(V) = f^{-1}(V)$ hence $f^{-1}(V)$ is ICS in X so $clintf^{-1}(V) \subseteq f^{-1}(V)$ and $f^{-1}(V) \subseteq clintf^{-1}(V)$ imply $f^{-1}(V) \subseteq clintclf^{-1}(V)$. There fore, $f^{-1}(V)$ is IβCS and hence f is I almost contra cont. function and I almost β-cont. function.

$2 \Rightarrow 1$ Let U be IROS in Y then $f^{-1}(U)$ is IβOS and ICS in X (by hypothesis) then $f^{-1}(U) \subseteq clintclf^{-1}(U)$ and $clf^{-1}(U) = f^{-1}(U)$ imply $f^{-1}(U) \subseteq clintf^{-1}(V)$ and $clintf^{-1}(U) \subseteq f^{-1}(U)$ imply $f^{-1}(U) = clintf^{-1}(U)$. There fore, $f^{-1}(U)$ is IRCS in X. Hence f is I almost RC-cont. function.♦

Proposition 4.6. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1- f is I almost SR-cont. function.

2- f is I almost β-cont. function and I almost contra semi-cont. function.

Proof 1⇒2 Suppose that V be any IROS in Y then $f^{-1}(V)$ is ISRS in X (by hypothesis) then $f^{-1}(V)$ is ISOS and ISCS so $f^{-1}(V) \subseteq clintf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$. Now since $f^{-1}(V) \subseteq clintf^{-1}(V)$ imply $f^{-1}(V) \subseteq clintclf^{-1}(V)$. There fore, $f^{-1}(V)$ is IβOS and ISCS in X. Hence f is I almost β-cont. and I almost contra semi-cont. function.

2 ⇒1 Suppose that U be IROS in Y then $f^{-1}(U)$ is IβOS and ISCS in X (by hypothesis) then $f^{-1}(U) \subseteq clintclf^{-1}(U)$ and $intclf^{-1}(U) \subseteq f^{-1}(U)$. Now we have $intclf^{-1}(U) \subseteq f^{-1}(U)$ $\subseteq clintclf^{-1}(U)$. then $f^{-1}(U)$ is ISOS in X also $f^{-1}(U)$ is ISCS in X. There fore, $f^{-1}(U)$ is ISRS in X and hence f is I almost SR-cont. function.♦

Corollary 4.7. Every I almost contra cont. function and I almost β-cont. function is I almost semi-cont. function.

Proof: Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ an I almost contra cont. function and Iβ-cont. function, so for any IOS V in Y then $f^{-1}(V)$ is ICS and IβOS in X imply $f^{-1}(V) = clf^{-1}(V)$ and $f^{-1}(V) \subseteq clintclf^{-1}(V)$ imply $f^{-1}(V) \subseteq clintf^{-1}(V)$. There fore, $f^{-1}(V)$ is ISOS in X. hence f is I almost semi-cont. function.♦

Proposition 4.8. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1- f is I R-irresolute function.

2- f is I almost pre-cont. function and I almost contra semi-cont. function.

Proof 1⇒2 Let V be IOS in Y then $f^{-1}(V)$ is IROS in X (since f is I R-irresolute cont. function) then $f^{-1}(V) = intclf^{-1}(V)$ imply $f^{-1}(V) \subseteq intclf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$. There fore, $f^{-1}(V)$ is IPOS and ISCS in X. Hence f is I almost pre-cont. function and I almost contra semi-cont. function.

2 ⇒ 1 Let U be IOS in Y then $f^{-1}(U)$ is IPOS and ISCS in X (by hypothesis) then $f^{-1}(U) \subseteq intclf^{-1}(U)$ and $clintf^{-1}(U) \subseteq f^{-1}(U)$ imply $f^{-1}(U) = intclf^{-1}(U)$. There fore, $f^{-1}(U)$ is IROS in X. Hence f is I R-irresolute cont. function.♦

Proposition 4.9. Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1. f is I almost contra semi-cont. function.
2. f is I almost B-cont. function and I almost contra gs-cont. function.

Proof 1⇒2 Suppose that V be any IOS in Y then $f^{-1}(V)$ is ISCS in X (by hypothesis). Now let A be IOS in X and $f^{-1}(V) \subseteq A$ then $f^{-1}(V) = A \cap f^{-1}(V)$ imply $f^{-1}(V)$ is IBS, so $f^{-1}(V) = sclf^{-1}(V)$ since $sclf^{-1}(V) = f^{-1}(V) \cup intclf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$. Hence for each IOS A in X and $f^{-1}(V) \subseteq A$ then $sclf^{-1}(V) \subseteq A$. There fore, $f^{-1}(V)$ is Igs-closed set and IBS in X, so f is I almost contra gs-cont. function and I almost B-cont. function.

2 ⇒1 Suppose that U be any IOS in Y then $f^{-1}(U)$ is IBS and ISCS in X (by hypothesis). Then $f^{-1}(U) = A \cap G$ where A is IOS containing $f^{-1}(U)$ in X and G is ISCS in X. So $sclf^{-1}(U) \subseteq A$ since $f^{-1}(U)$ is Igs-closed set. Now $intclf^{-1}(U) = intcl\,(A \cap G) \subseteq int(clA \cap clG) = intclA \cap intclG \subseteq intclA \cap G$

since G is ISCS. So $intclf^{-1}(U) \cap A \subseteq intclA \cap A \cap G$ since $intclf^{-1}(U) \cup f^{-1}(U) = sclf^{-1}(U) \subseteq A$

and $A \subseteq intclA$. We have $intclf^{-1}(U) \subseteq A \cap G = f^{-1}(U)$. There fore, $f^{-1}(U)$ is ISCS in X and hence f is I almost contra semi-cont. function.♦

Corollary 4.10. Let (X,T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then the following statements are equivalent:

1. f is I almost SR-cont. function.
2. f is I almost β-cont. function, I almost B-cont. function and I almost contra gs-cont. function.

Proof 1⇒2 Let V be IOS in Y then $f^{-1}(V)$ is ISRS in X (since f is I almost contra SR-cont. function). Then $f^{-1}(V)$ is ISCS and ISOS in X, that is $intclf^{-1}(V) \subseteq f^{-1}(V)$ and $f^{-1}(V) \subseteq clintf^{-1}(V)$ imply $f^{-1}(V) \subseteq clintclf^{-1}(V)$ then $f^{-1}(V)$ is IβOS. Now let A be IOS in X and $f^{-1}(V) \subseteq A$ imply $f^{-1}(V) = A \cap f^{-1}(V)$ then $f^{-1}(V)$ is IBS. So $f^{-1}(V) = sclf^{-1}(V)$ since $sclf^{-1}(V)= f^{-1}(V) \cup intclf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$. Hence for each IOS A in X and $f^{-1}(V) \subseteq A$ then $sclf^{-1}(V) \subseteq A$. There fore, $f^{-1}(V)$ is Igs-closed set, IBS and IBOS in X and hence f is I almost B-cont. function, I almost β-cont. function and I almost contra gs-cont. function.

2 ⇒1 Let U be IOS in Y then $f^{-1}(U)$ is IβOS, IBS and Igs-closed set in X (by hypothesis) then $f^{-1}(U) \subseteq clintclf^{-1}(U)$ and $f^{-1}(U) = A \cap G$ where A is IOS containing $f^{-1}(U)$ in X and G is ISCS in X so $sclf^{-1}(U) \subseteq A$ since $f^{-1}(U)$ is Igs-closed set. Now $intclf^{-1}(U) = intcl(A \cap G) \subseteq int(clA \cap clG) = intclA \cap intclG \subseteq intclA \cap G$ since G is ISCS. So $intclf^{-1}(U) \cap A \subseteq intclA \cap A \cap G$ since $intclf^{-1}(U) \cup f^{-1}(U) = sclf^{-1}(U) \subseteq A$ and $A \subseteq intclA$ then $intclf^{-1}(U) \subseteq A \cap G = f^{-1}(U)$.Hence $f^{-1}(U)$ is ISCS in X. Now since $f^{-1}(U) \subseteq clintclf^{-1}(U)$ and $intclf^{-1}(U) \subseteq f^{-1}(U)$ imply $intclf^{-1}(U) \subseteq f^{-1}(U) \subseteq clintclf^{-1}(U)$. Then we have $f^{-1}(U)$ ISOS and ISCS in X, so $f^{-1}(U)$ is ISRS in X and hence f is I almost SR-cont. function.♦

Corollary 4.11. Let (X,T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then the following statements are equivalent:

1. f is I R-irresolute function.
2. f is I almost pre-cont. function, I almost B-cont. function and I almost contra gs-cont. function.

Proof 1⇒2 Suppose that V is IOS in Y then $f^{-1}(V)$ is IROS in X (by hypothesis). That is $f^{-1}(V) = intclf^{-1}(V)$ imply $f^{-1}(V) \subseteq intclf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$ then $f^{-1}(V)$ is IPOS and ISCS in X. Now let A be IOS in X and $f^{-1}(V) \subseteq A$ then $f^{-1}(V) = A \cap f^{-1}(V)$ imply $f^{-1}(V)$ is IBS. So $f^{-1}(V) = sclf^{-1}(V)$ since $sclf^{-1}(V) = f^{-1}(V) \cup intclf^{-1}(V)$ and $intclf^{-1}(V) \subseteq f^{-1}(V)$ and hence for each IOS A in X and $f^{-1}(V) \subseteq A$ then $sclf^{-1}(V) \subseteq A$. There fore, $f^{-1}(V)$ is Igs-closed set, IBS and IPOS in X. hence f is I almost B-cont. function, I almost pre-cont. function and I almost contra gs-cont. function.

2 ⇒1 Suppose that U is IOS in Y then $f^{-1}(U)$ is IPOS, IBS and Igs-closed set in X (by hypothesis) then $f^{-1}(U) \subseteq intclf^{-1}(U)$ and $f^{-1}(U) = A \cap G$ where A is IOS containing $f^{-1}(U)$ in X and G is ISCS in X so $sclf^{-1}(U) \subseteq A$ since $f^{-1}(U)$ is Igs-closed set. Now $intclf^{-1}(U) = intcl(A \cap G) \subseteq int(clA \cap clG) = intclA \cap intclG \subseteq intclA \cap G$ since G is ISCS so $intclf^{-1}(U) \cap A \subseteq intclA \cap A \cap G$ since $intclf^{-1}(U) \cup f^{-1}(U) = sclf^{-1}(U) \subseteq A$ and $A \subseteq intclA$ then $intclf^{-1}(U) \subseteq A \cap G = f^{-1}(U)$. Hence

$f^{-1}(U)$ is ISCS in X, then we have $intclf^{-1}(U) \subseteq f^{-1}(U)$ and $f^{-1}(U) \subseteq intclf^{-1}(U)$ imply $f^{-1}(U) = intclf^{-1}(U)$. There fore, $f^{-1}(U)$ is IROS in X and hence f is I R-irresolute function.♦

The following definition is given in [1] by general topology, we generalized it on ITS's.

Definition 4.12. Let (X,T) and (Y,σ) be two ITS's then a function $f: X \to Y$ is said to be intuitionistic contra continuous (resp. intuitionistic contra semi-continuous if the inverse image of each IOS in Y is ICS (resp. ISCS) in X.

The following definition is given in [5] by general topology, we generalized it on ITS's.

Definition 4.13. Let (X,T) and (Y,σ) be two ITS's then a function $f: X \to Y$ is said to be intuitionistic regular set connected if the inverse image of each IOS in Y is clopen in X.

Proposition 4.14. Let (X,T) and (Y,σ) be two ITS's then and let $f: X \to Y$ be a function then:

1- If f is 1 perfectly cont. function then f is ISR-cont. function.
2- If f is ISR-cont. function then f is I contra semi-cont. function.
3- If f is I contra cont. function then f is I contra semi-cont. function.
4- If f is I perfectly cont. function then f is I regular set connected function.
5- If f is I regular set connected function then f is I almost contra cont. function.
6- If f is I contra cont. function then f is I almost contra cont. function.
7- If f is I almost contra cont. function then f is I almost contra semi-cont. function.
8- If f is I contra semi-cont. function then f is I almost contra semi-cont. function.

Proof:

1- Let V be IOS in Y then $f^{-1}(V)$ is clopen set in X (since f is I perfectly cont. function) so $f^{-1}(V) = cl(f^{-1}(V))$ and $f^{-1}(V) = int(f^{-1}(V))$ imply $intcl(f^{-1}(V)) \subseteq f^{-1}(V)$ and $f^{-1}(V) \subseteq clintf^{-1}(V)$ then $f^{-1}(V)$ is ISCS and ISOS. There fore, $f^{-1}(V)$ is ISR-cont. function.♦

2- Suppose that V be IOS in Y then $f^{-1}(V)$ is ISRS in X (since f is ISR-cont. function) so $f^{-1}(V)$ is ISOS and ISCS. There fore, f is I contra semi-cont. function.♦

3- For any IOS V in Y then $f^{-1}(V)$ is ICS in X (since f is I contra cont. function) so $clf^{-1}(V) = f^{-1}(V)$ imply $intcl(f^{-1}(V)) \subseteq f^{-1}(V)$. There fore, $f^{-1}(V)$ is ISCS in X and hence f is I contra semi-cont. function.♦

4- Let V be IOS in Y then $f^{-1}(V)$ is clopen set in X (since f is I perfectly cont. function). Now since every IROS is IOS that is the inverse image of each IROS in Y is clopen set in X, so f is I regular set connected function.♦

5- Suppose that V be IROS in Y then $f^{-1}(V)$ is clopen set in X (by hypothesis). That is $f^{-1}(V)$ is IOS and ICS in X and hence f is I almost contra cont. function.♦

6- Let V be IOS in Y then $f^{-1}(V)$ is ICS in X (since f is I contra cont. function). Now since every IROS is IOS that is the inverse image of each IROS in Y is ICS in X. Hence f is I almost contra cont. function.♦

7- For any IROS in Y then $f^{-1}(V)$ is ICS in X (by hypothesis) then $f^{-1}(V) = cl(f^{-1}(V))$ imply $f^{-1}(V) \subseteq intcl(f^{-1}(V))$, so $f^{-1}(V)$ is ISCS in X and hence f is I almost contra semi-cont. function.♦

8- Let V be IOS in Y then $f^{-1}(V)$ is ISCS in X (since f is I contra semi-cont. function). Now since every IROS is IOS then the inverse image of each IROS in Y is ISCS in X. Hence f is I almost contra semi-cont. function.♦

We start with example to show that ISR-cont. function is not imply I perfectly cont. function.

Example 4.15. Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\} \rangle$ and $B = \langle x, \emptyset, \{b,c\} \rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{2\}, \{3\} \rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. Now let $G = f^{-1}(C) = \langle x, \{b\}, \{c\} \rangle$ then $intclG = B \subseteq G$ and $G \subseteq clintG = \bar{B}$, that is G is ISCS and ISOS imply G is ISRS in X but G is not clopen set in X since $intG = B \neq G$ so $clG = \bar{B} \neq G$. Then f is ISR-cont. function but f is I perfectly cont. function.

The next example shows that:

1- I contra semi-cont. is not imply ISR-cont.
2- I contra semi-cont. is not imply I contra cont.

Example 4.16. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a\}, \{b\}\rangle, B = \langle x, \{a, b\}, \{c\}\rangle, C = \langle x, \{a, b\}, \emptyset\rangle$ and $D = \langle x, \{a\}, \{b, c\}\rangle$ and let $Y = \{a, b, c\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E\}$ where $E = \langle y, \{2\}, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 3$ and $f(c) = 2$. Now let $G = f^{-1}(E) = \langle x, \{c\}, \{a\}\rangle$ then G is ISCS in X since $intclG = \emptyset \subseteq G$ but G is not ISOS since $G \not\subseteq clintG = \emptyset$ so G is not ISRS in X as well as G is not ICS in X since $clG = \overline{D} \neq G$, then the inverse image of each IOS in Y is ISCS in X.

We are going to show that I regular set connected function is not imply I perfectly cont. function.

Example 4.17. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a\}, \{B\}\rangle, B = \langle x, \emptyset, \{b, c\}\rangle, C = \langle x, \{b, c\}, \emptyset\rangle$ and $D = \langle x, \emptyset, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E, F\}$ where $E = \langle y, \{1\}, \{2\}\rangle$ and $F = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, F\}$. Now f is I regular set connected but f is not I perfectly cont. function.

The following example shows that I almost contra cont. is not imply I regular set connected.

Example 4.18. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y, \emptyset, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. $ROY = \sigma$. Now let $G = f^{-1}(B) = \langle x, \emptyset, \{a\}\rangle$ then G is ICS in X but G not IOS so it's not clopen in X. There fore, f is I almost contra cont. function but not I regular set connected.

The next example shows I almost contra cont. is not imply I contra cont.

Example 4.19. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{a\}\rangle$ and $B = \langle x, \{a, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \{3\}, \{1\}\rangle$ and $D = \langle y, \emptyset, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 3$ and $f(c) = 2$. $ROY = \{\tilde{\emptyset}, \tilde{Y}, D\}$. We have a set $H = f^{-1}(C) = \langle x, \{b\}, \{a\}\rangle$ is not ICS in X since $clH = X \neq H$, then f is not I contra cont. function but f is I almost contra cont. function.

The following example shows that I almost contra semi-cont. is not imply I almost contra cont.

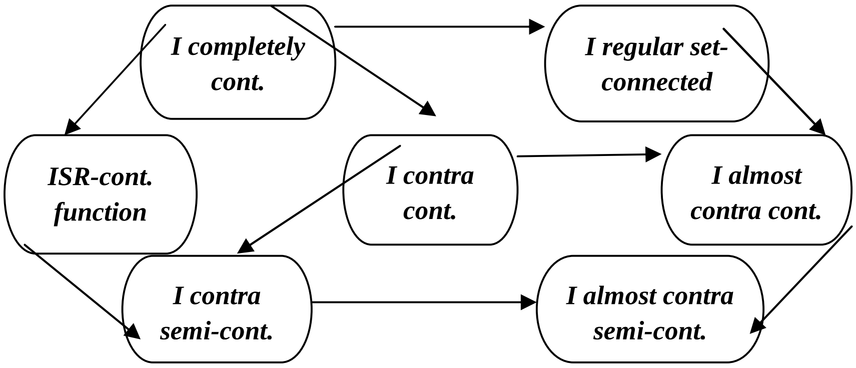
Example 4.20. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle$ and $B = \langle x, \emptyset, \{a, b\}\rangle$ and $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \emptyset, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. Now a set $G = f^{-1}(C) = \langle x, \emptyset, \{a\}\rangle$ then G is ISCS in X since $intclG = B \subseteq G$ but G is not closed since $clG = \overline{A} \neq G$, hence f is I almost contra semi-cont. function but not I almost contra cont. function.

In the last example we show I almost contra semi-cont. is not imply I contra semi-cont.

Example 4.21. Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a, c\}, \emptyset\rangle$ and $B = \langle x, \{c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{3\}, \{1\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. $ROY = \{\tilde{\emptyset}, \tilde{Y}\}$. We have that f is I almost contra semi-cont. function but f is not I contra semi-cont. function.

We summarized the above result by the following diagram.

Diagram 4.22. The following implications are true and not reversed:



Proposition 4.23. Let (X,T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then I contra cont. and I almost contra cont. are equivalent if:

1. (Y,σ) is discrete.
2. (Y,σ) is indiscrete.
3. (Y,σ) is disconnected.

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الدوال المستمرة المتعاكسة تقريبا بكافة انواعها وعلاقتها مع بعضها وتعميمها على الفضاءات التوبولوجية الحدسية

يونس جهاد ياسين   علي محمد جاسم

E.mail: scianb@yahoo.com

الخلاصة

سندرس في هذا البحث مفهوم الدوال المستمرة المعاكسة تقريباً (almost contra continuous) بكل انواعها (almost contra semi continuous, almost contra g-continuous,...) وتعميمها بين الفضاءات التبولوجية الحدسية وكذلك سندرس علاقة هذه الدوال مع بعضها عن طريق بعض المبرهنات والأمثلة وتوضيحها بمخطط سمي وكذلك سندرس علاقة هذه الدوال مع أنواع أخرى من الدوال المستمرة منها الدوال المستمرة المعاكسة وغيرها من الدوال المستمرة.