OPTIMUM DESIGN OF REINFORCED CONCRETE RECTANGULAR BEAMS USING SIMULATED ANNEALING

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ABSTRACT:

This paper presents the application of Simulated Annealing optimization method (SA) for solving the problem of the optimum design of reinforced concrete beams based on the recommendations of American Building Code Requirements for structural concrete (ACI 318-05) and the ultimate strength design method. Cost of concrete, cost of steel reinforcement and cost of formworks are considered. The constraints of the problem included the concrete beam strength, width-height ratio, minimum width, and deflection constraints. This optimization problem is implemented by constructing a computer program using *Matlab*. A number of examples are solved using the developed program and proved that the produced design is economical; also it is proved that the developed program is efficient and versatile.

Keywords: optimum, design, reinforced concrete, beams, simulated annealing

الخلاصة :

يقدم هذا البحث تطبيق طريقة (اثلة التلدين) ل مسألة التصميم الامثل للعتبات ا رسانية ا سلحة طبقا لطريقة ا دونة الامريكية وباستخدام طريقة ا قاومة القصوى التصميم. دالة ا دف ا سألة اخذت بنظر الاعتبار كلفة متر طول من العتبة وتتضمن كلفة ا رسانة وكلفة حديد التسليح وكلفة القالب. ددات التصميم وضعت لتحقيق متطلبات ا قاومة ونسبة العرض ا العمق و ا د الادنى للعرض وا طول. كتب برنامج باستخدام *Matlab* ل مسالة التصميم الامثل. عدد من ا سائل التطبيقية حلت باستخدام ال نامج ا كتوب و ساعدة حقيبة ال يات ا اهزة برنامج *Matlab* وال استنتج منها ان التصميم الناتج من حل ا سألة باستخدام ال نامج تصميم اقتصادي.

INTRODUCTION:

Optimum design of reinforced concrete elements plays an important role in economical design of reinforced concrete structures. Structural design requires judgment, intuition and experience, besides the ability to design structures to be safe, serviceable and economical. The design codes do not directly give a design satisfying all of the above conditions. Thus, a designer has to execute a number of design-analyze cycles before converging on the best solution. The intuitive design experience of an expert designer can give a good initial solution, which can reduce the number of design-analyze cycles.

Cost optimum design of reinforced concrete beams is receiving more and more attention from the researchers. Balagura [2], Brown [3], Friel [4], and Traum [5] have presented algorithms dealing with optimum design of single reinforced concrete beams and one way slabs. In 1980 Balaguru[6] used Lagrangian multiplier technique to calculate the optimum dimensions and the amount of reinforcements for a reinforced rectangular beam. AL-Nassiry [7] used, in 2001, linearization techniques to solve the nonlinear optimization problem of reinforced rectangular beams. In this paper, the optimum design of reinforced concrete beams, subjected to strength, width-height ratio, minimum width, and deflection constraints, is sought using Simulated Annealing. As such a computer program using *Matlab* has been developed. The problem is formulated based on the requirements of ACI code and the ultimate strength design method.

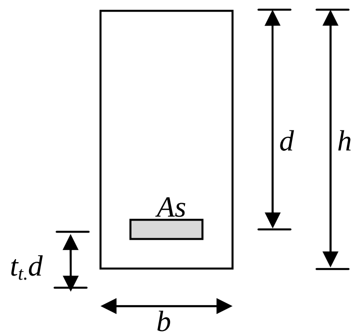

Fig.(1) Singly Reinforced Concrete Beam Cross-Section Details

STATEMENT OF THE PROBLEM

The optimization problem can be stated as: obtain the optimum dimensions and the amount of tension reinforcement for the reinforced rectangular beam shown in Fig.(1), given:

1. The ultimate External bending moment (M_u).
2. Cost of concrete per unit volume (c_c).
3. Cost of steel per unit weight (C_s).
4. Cost of formwork per unit area (C_f).
5. Ratio of effective cover thickness of beam (t_t).
6. Compressive strength of concrete (f_c').
7. Yield strength of steel (fy).
8. Unit weight of concrete (γ_c).
9. Unit weight of steel (γ_s).

ASSUMPTIONS

The procedure recommended for the optimum design of reinforced rectangular beam in this paper is based on the following assumptions:

1. Plane sections before bending remain plane after bending.
2. At ultimate capacity, strain and stress are not proportional.
3. Strain in the concrete is proportional to the distance from the neutral axis.
4. Tensile strength of concrete is neglected in flexural computations.
5. The ultimate concrete strain is 0.003.
6. The modulus of elasticity of the reinforcing steel is 200,000 MPa.
7. The average compressive stress in the concrete is 0.85 fc'.
8. The average tensile stress in the reinforcement does not exceed f_y.
9. Only bending and deflection effects on the critical cross section are considered. So, the beam has to be checked for shear considerations.

FORMULATION OF THE PROBLEM

Considering the cost of unit length of the beam, the problem can be mathematically stated as:

Minimize,

$$(C)=C_c.b.h+C_{s.}(A_s).\gamma_s+C_f[b+2h] \quad .....(1)$$

subject to the following constraints:

1) Resisting moment of the cross section $\geq$ given external moment.
2) The maximum deflection $\leq$ the allowable deflection

The following notations and definitions are to be used

$\alpha=(1+t_t)$ ….(2)

$$\beta_1 = \frac{b}{d} \quad ….(3)$$

Using equations (2) and (3), eq.(1) can be written as:

Minimize $C=C_c \quad \alpha\,\beta \quad d^2+ \quad C_s \quad A_s.\gamma_s+C_f(2\alpha d+\beta d)$ ...(4)

Using the ACI code, the resisting moment capacity of the reinforced concrete section can be written as:

$$\text{Mu}=\phi\, As.\beta_1^{\,2} d^5.f_y\left(1-0.59\,As.\beta_1.d^2\,\frac{.f_y}{f_c'}\right) \quad ….(5)$$

where ϕ is the strength reduction factor (ϕ=0.9)[7]

Self weight moment $=M_s = 1.4*\gamma_c.\beta_1.d.\alpha\ d.\ \tau.\ l^2$ ….(6)

where γ_c is the unit weight of reinforced concrete and τ is the multiplying factor depending on the beam support conditions (τ=1/8 for simply supported beam).

Using eqs.(5) and (6), the first constraint (g_1) can be written as:

$$\phi As.\beta_1^{\,2} d^5.f_y\left(1-0.59As.\beta_1.d^2\,\frac{.f_y}{f_c'}\right) \ge M_u + 0.175\,\gamma_c.\alpha.\beta_1.l^2.d^2 \quad ….(7)$$

or

$$g_1=(M_u + 0.175\,\gamma_c.\alpha.\beta_1.l^2.d^2) - \phi As.\beta_1^{\,2} d^5.f_y\left(1-0.59As.\beta_1.d^2\,\frac{.f_y}{f_c'}\right) \le 0$$

DEFLECTION COMPUTATION:

Immediate deflection shall be computed with the modulus of elasticity E_c for concrete as ($4700\sqrt{f_c'}$) and with the effective moment of inertia, I_e, as follows, but not greater than I_g [7].

$$I_e = \left(\frac{M_{cr}}{M}\right) I_g + \left[1-\left(\frac{M_{cr}}{M}\right)^3\right] I_{cr} \quad …(8)$$

where

$M_{cr} = \dfrac{f_r.I_g}{Y}$, cracking moment

M=unfactored external bending moment,

Y= distance from centroid of gross section to extreme fiber in tension.

I_g= moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.

I_{cr}= moment of inertia of cracked section transformed to concrete.

For normal weight concrete f_r, modulus of rapture of concrete is taken as ($0.62\sqrt{f_c'}$).

For simply supported rectangular beams subjected to uniformly distributed load, the maximum deflection (at mid span) is equal to:

$$\Delta = \frac{5wl^4}{384E_c.I_c} \qquad .....(9)$$

where

w=uniformly distributed live load over span length.

The allowable deflection is taken as (l/360). The cracking moment can be computed as:

$$M_{cr} == f_r.\frac{\beta_1\alpha^2 d^3}{6} \qquad ...(10)$$

DEPTH OF NEUTRAL AXIS

As shown in Fig.(2), the neutral axis depth (x) for cracked section can be computed as follows:

Taking moment about the neutral axis:

$$b(x^2/2)=n.As.(d-x) \qquad ......(11)$$

$$\beta_1 d(x^2/2)=n.\ As.(d-x\ ) \qquad ......(12)$$

where

$n=E_s/E_c$

E_s= steel modulus of elasticity.

Solving eq.(12) gives the depth of neutral axis (x).

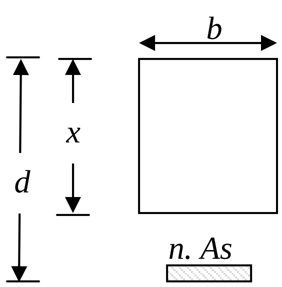

Fig. (2) Concrete Cracked Section Details

MOMENT OF INERTIA OF CRACKED SECTION

The moment of inertia of cracked section with respect to the neutral axis can be computed as:

$$I_{cr}=\beta_1 d(x^3/12)+n.A_s\,(d-x)^2 \qquad \ldots(13)$$

DEFLECTION CONSTRAINTS

After determination of neutral axis depth using eq.(12) and the cracked section moment of inertia using eq. (13), the deflection constraint can be formulated as:

$$\frac{5wl^4}{384E_c.I_e}\leq\frac{l}{360} \qquad \ldots\ldots..(14)$$

or

$$g_2=\frac{5wl^4}{384E_c.I_e}-\frac{l}{360}\leq 0$$

SIDE CONSTRAINTS

In order to get a reasonable design of reinforced concrete beam, the design variables of the problem (d, As) must satisfy the following side constraints:

1) $As_{min}\leq As\leq As_{\max}$
2) $d_{min}\leq d\leq d_{max}$

SIMULATED ANNEALING

Simulated Annealing (SA) is an artificial intelligence based optimization technique that resembles the cooling process of molten metal through annealing. At high temperature, atoms in molten metal can move freely with respect to each another, but as the temperature is reduced, the movement of atoms gets restricted. The atoms start to get ordered and finally form the crystals having the minimum possible energy. However, the formation of the crystal mostly depends on the cooling rate. If the temperature is reduced at a very fast rate, the crystalline state may not be achieved at all; instead, the system may end up in a polycrystalline state, which may have a higher energy state than the crystalline state. Therefore, in order to achieve the absolute minimum energy state, the temperature needs to be reduced at a slow rate. Controlling a temperature-like parameter introduced with the concept of the Boltzmann probability distribution simulates the cooling phenomenon. According to the Boltzmann probability distribution, a system in thermal equilibrium at a temperature T has its energy distributed probabilistically according to P(E)=exp(-E/KT), where K is the Boltzmann constant. Simulated annealing is a point-by-point method. The algorithm begins with an initial point and a high temperature (T). A second point is created at random in the vicinity of the initial point and the difference in function values (ΔE) at these two points is calculated. If the second point has a smaller function value, the point is accepted; otherwise the point is accepted with a probability exp (-ΔE/T).

This completes the one iteration of simulated annealing procedure. In the next generation, another point is created at random in neighborhood of current point and Metropolis algorithm is used to accept or reject the point. The algorithm is terminated when a sufficiently small temperature is obtained or a small enough change in function values is found.

SIMULATED ANNEALING ALGORITHM

1- Choose random initial values for the design variables (d_i,As_i), select the initial system temperature, and specify the cooling i.e. annealing schedule (T=*k*T) where *k* is system parameter.
2- Evaluate C(d_i, As_i) using eq.(1).
3- Perturb (*di*, As_i) to obtain a neighboring design vector (d_{i+1}, As_{i+1}).
4- Evaluate C(d_{i+1}, As_{i+1}).
5- If C(d_{i+1}, As_{i+1})< C(d_i, As_i), then, (d_{i+1}, As_{i+1}) is the new current solution.
6- If C(d_{i+1}, As_{i+1})> C(d_i, As_i), then accept (d_{i+1}, As_{i+1}) as the new current solution with a probability $e^{(-\Delta E/T)}$ where ΔE=C(d_{i+1}, As_{i+1})-C(d_i, As_i).
7- The current solution is checked against the constraints and it is adopted as the new current solution when it is feasible, i.e. when it satisfies the constraints. On the other hand, the current solution is discarded when it does not satisfy the constraints.
8- Reduce the system temperature (T) according to the cooling schedule (T=*k*.T).
9- The algorithm stops when T is a small percentage of the initial temperature.

APPLICATION EXAMPLE

In this work, optimum design of reinforced concrete beam has been done.

A number of examples were run to evaluate the validity of the developed formulation and computer program.

A reinforced concrete rectangular simply supported beam with the following values is considered:

C_c= 175000 (I.D/m^3)

C_s= 1250000 (I.D/ton)

C_f= 470000 (I.D/m^2)

γ_c=24 (kN/m^3)

γ_s=7850 (kg/m^3)

t_t=t_c=0.1

f_c'=21MPa

f_y=414 MPa

β=0.25

l=9m

d_{min}=230mm

d_{max}=800mm

w=10kN/m

E_s=200000 MPa

$E_c=4700\sqrt{f_c'}$=26120MPa

Initial temperature (T=100)

Cooling parameter (k=0.95).

The optimization problem can be stated as follows:

Find the values of the design variables (d and As) which minimize the cost function (C) under the constraints g_1, g_2 and the side constraints stated above.

The solution of this problem by the simulated annealing method is shown in Table (1).

TABLE (1) RESULTS OF R.C.B. BY SIMULATED ANNEALING METHOD

solution	Initial	Final
d(mm)	500	600
As(mm2)	250	347
C(I.D)	853721	710038

Table (2) shows the optimum values of the design variables for various span length.

TABLE (2) OPTIMUM VALUES OF DESIGN VARIABLES FOR VARIOUS SPAN LENGTH

l (m)	*d(mm)*	*As(mm²)*	*C(I.D)*
6	582	296	514296
7	567	372	570338
8	588	392	595446
9	600	394	610218
10	680	491	707229

It is to be noted that the results of the effective depth (*d*), obtained in the two tables above, are larger than the minimum thickness values of the ACI-code, which is for simply supported beam is $l/16$.
In order to investigate the effect of the unit costs of the concrete and steel, the cost function can be written in the following form:

$$C/Cs = C_c/C_s\ \alpha\ \beta\ d^2 + A_s.\gamma_s + C_f/C_s(2\alpha d + \beta d)$$

Table (3) shows that the increase of the ratio *Cc/Cs* leads to increase the compression steel area (*As*) and decrease the effective depth of beam (*d*).

TABLE (3) EFFECT OF INCREASING THE CONCRETE UNIT COST (CC) ON THE OPTIMUM DESIGN

Cc/Cs	*d(mm)*	*As(mm²)*	*C(I.D)*
0.14	600	374	610688
0.16	589	393	611233
0.18	574	379	610255
0.20	561	366	611232
0.22	550	356	611119
0.24	532	339	611343
0.26	520	329	610156
0.28	502	313	611411
0.30	488	301	611051

CONCLUSIONS

1. The simulated annealing optimization method has been presented. The developed procedure, wherein the optimal cost of a reinforced concrete beam has been formulated as a nonlinear programming problem, has been found to be quiet efficient in isolating the optimal solution.
2. The increase of the ratio *Cc/Cs* leads to increase the steel area (*As*) and decrease the effective depth of beam (*d*).

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