Evaluation of Correlations for Mixed Convective Heat Transfer in a Porous Vertical Cylindrical Annulus

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Abstract:

This paper presents a numerical investigation of mixed convection heat transfer in vertical packed annulus with air as the working fluid . The inner cylinder is at constant temperature and the outer cylinder is insulated . The flow in the packed annulus is governed by the usual Darcy model . The analyses were carried out in the range of 10 < Ra^* < 100 , 1 < Pe < 10 and 0.2 < A < 1 . The effect of each parameter on Nusselt number was explained and shown that it is increased as (Ra^*) increased and (A) increased . The best analysis correlation obtained for average Nusselt number was as follows :

$$\overline{Nu} = 3.391\ (A)^{1.024}\ (Ra^* / Pe)^{0.02827}$$

دراسة عددية في انتقال الحرارة بالحمل المختلط خلال حلقة عمودية مملوءة بوسط مسامي

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المستخلص:

هذا البحث يبّين اجراء دراسة عددية، لانتقال الحرارة بالحمل المختلط للهواء المارخلال حلقة عمودية مملوءة بوسط مسامي ، الجدار الداخلي للحلقة ذو درجة حرارة ثابتة، بينما الجدار الخارجي للحلقة كان معزولاً .

ان المعادلة المستخدمة في الجريان خلال الوسط المسامي هي معادلة دارسي . وقد تمت الدراسة لمدى رقم رالي المطور 10 < Ra^* < 100 ورقم بكلت 1 < Pe < 10 ونسبة باعية 0.2 < A < 1 وبينت تأثير كل منها على رقم نسلت ، حيث انه يزداد بزيادة رقم رالي المطور (Ra^*) والنسبة الباعية (A) . ان افضل علاقة تم الحصول عليها من التحليلات كانت كما يأتي :

$$\overline{Nu} = 3.391\ (A)^{1.024}\ (Ra^* / Pe)^{0.02827}$$

Nomenclature

A : Aspect ratio = (D/L)
D : Annulus gap = ($r_0 - r_i$) (m)
Da : Darcy number = K / D^2
g : Gravitational acceleration (m/s^2)
ke : Effective thermal conductivity , ($W/m.k^0$)
K : Permeability of porous media (m^2)
L : Annulus length (m)
Nu : Nusselt number = hL / ke
P : Pressure (Pa)
Pe : Peclet number = $\frac{W_i D}{\alpha_e}$
R : Dimensionless distance in radial direction
Ra^* : Modified Rayleigh number = (g β ke D $\Delta T \ / \upsilon \alpha_e$)
r_i : Inner cylinder radius (m)
r_0 : Outer cylinder radius (m)
T_W : Inner cylinder temperature (k^0)
T_i : Inlet temperature of fluid flow (k^0)
u : Fluid velocity in r- direction (m/s)
v : Fluid velocity in z – direction (m/s)
U : Dimensionless velocity in r- direction
V : Dimensionless velocity in z – direction
z : Axial coordinate (m)
Z : Dimensionless distance in the axial direction
W_i : Inlet velocity (m/s)

Greek Symbols

ρ : **Fluid density (kg / m^3)**
α_e : **Effective thermal diffusivity (m^2 / s)**
β : **Coefficient of thermal expansion (k^{-1})**
υ : **Kinematic viscosity of the fluid (m^2 / s)**
μ : **Dynamic viscosity of the fluid ($N.S / m^2$)**
θ : **Dimensionless temperature**
ψ : **Stream function (m^3 /s)**
$\underline{\varphi}$: **Dimensionless stream function**

Introduction

The problem of fluid flow and convection heat transfer through porous media is frequently encountered in many fields of science and engineering , such as hydrology , civil , and mechanical engineering , chemical and petroleum engineering , thermal management of electronic cooling and improvement of performance of heat transfer systems . [1,2]

Most pervious work has been devoted to the studies either natural or forced convection in a porous cylindrical annulus . For example, Ref. [3] have which has been studied theoretically forced convection in porous channel and investigated the effect of thickness of porous region adjacent to the wall , the Darcy number , and the ratio of the effective thermal conductivity of the porous medium of the thermal conductivity on the heat and fluid flow . Prasad and kulack [4] have investigated numerically steady free convection in a vertical porous annulus whose vertical walls are at constant temperatures , the horizontal walls being insulated , they found that the convective velocities are higher in the upper half than the lower half of the annulus , the local rate of heat transfer is much higher near the top edge of the cold wall , and the average Nusselt number always increase as the radius ratio increases . The same configuration was solved numerically by Thansekher *etal* [5] , for 0.1 $\leq$ an isotropic permeability ratio $\leq$ 10 , 0.1 $\leq$ thermal diffusivity $\leq$ 10 , 1 $\leq$ aspect ratio $\leq$ 20 , 2 $\leq$ radius ratio $\leq$ 20 and Darcy modified Rayleigh number $\leq$ 10000 . Also , Barbosa Mota and Saatdjan [6] have analyzed the two dimensional Boussinesq equations numerically natural convection in a porous medium bounded by two horizontal cylinder , which are held at the constant , for radius ratio above 1.7 and for Rayleigh number above a critical value .

The Brinkman extended Darcy model which allows the no – slip boundary condition on a solid wall has been by Bennacer *et al* [7] to study numerically natural convection within a vertical circular porous annulus . Motions are driven by the externally applied constant temperature concentration differences imposed across the vertical walls of the enclosure . The effect of both the Darcy number , and the radius ratio on Nusselt and Sherwood number are obtained . In addition , Jha [8)] have studied the Brinkman extended Darcy model (Brinkman flow) of a laminer free – convective flow in an annular porous region closed from expressions for velocity field , temperature field , skin – friction and mass flow rate are given ,

under a thermal boundary condition of mixed kind at the outer surface of the inner cylinder while the inner surface of the outer cylinder is isothermal . Mixed convective processes of fluid flow and heat transfer in porous media for different geometrical parameters aspect ratio and heating mode have been studied numerically and theoretically by [9,10,11] .

For porous annulus , PU *et al* [12] conducted experimental study in the range of $2 < pe < 2200$ and $700 < Ra^* < 1500$ of mixed convection heat transfer in vertical packed channel . It is found that three convection regimes exist : natural convection regime $105 < Ra^* / Pe$ mixed convection regime $1 < Ra^* / Pe < 105$ and forced convection regime : $Ra^* / Pe <1$.

This paper deals with the numerical study of mixed convection flow in porous vertical annulus to predict heat transfer for $10 < Ra^*<100$, $1< Pe < 10$, $0.2<A < 1$ and $Ra^* / Pe=1$ to achieve the above classification for mixed convection heat flow which is obtained by PU *et al* [12] .

Mathematical Formulations

Consider a porous layer bounded between two vertical concentric cylinders of radii (r_i) and (r_0) as are shown in fig (1) . The inner cylinder surface is at constant temperature (T_W) and the outer surface is insulated . Air flow through porous layers at initial uniform velocity (W_i) and temperature (T_i) . The flow is assumed to be two dimensional and the usual Darcy model is adopted .

The coordinate along the radius is denoted by (r) and that perpendicular to it is denoted by (Z) .

The following assumptions are made in this study for the formulation . The air and porous media are in local thermal equilibrium , the air properties are constant , except for the density variation in producing the buoyancy force . Neglecting viscous drag (Brinkman model) and inertia terms , also the porous media is homogenous [10].

Using these assumptions , the governing equations are :

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial z}=0 \qquad \textbf{(1)}$$

$$\frac{\partial P}{\partial r} = -\frac{\mu u}{K} \tag{2}$$

$$\frac{\partial P}{\partial Z} = \frac{\mu v}{K} + \rho g \tag{3}$$

$$u\frac{\partial T}{\partial r} + V\frac{\partial T}{\partial z} = \alpha_e \left[\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] \tag{4}$$

Where

$$\rho = \rho_0\,[1 - \beta\,(T - T_0)] \tag{5}$$

Defining stream function ψ as :

$$u = -\frac{1}{r} \cdot \frac{\partial \psi}{\partial z} \tag{6}$$

$$v = \frac{1}{r} \cdot \frac{\partial \psi}{\partial r} \tag{7}$$

The two components of the momentum equation can be simplified using the Boussineq approximation in the body force term (ρg) in equation (3) , to eliminate the pressure term by cross differenting equation (2) & (3) and then using stream function definition (6) & (7) the momentum equation becomes :

$$\frac{1}{r} \cdot \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r} \cdot \frac{\partial \psi}{\partial r}\right) = \frac{K}{\mu}\,\rho_0\,g\,\beta\,\frac{\partial T}{\partial r} \tag{8}$$

The governing equations are made dimensionless by adopting the following dimensionless quantities :

$$R = \frac{r - r_i}{r_0 - r_i} \qquad \theta = \frac{T - T_i}{T_W - T_i} \qquad \varphi = \frac{\psi}{W_i\,DL}$$

$$Z = \frac{z}{L} \qquad U = \frac{u}{W_i} \qquad V = \frac{v}{W_i} \tag{9}$$

The dimensionless momentum equation form is :

$$A^2\left(\frac{\gamma}{\gamma R+1}\right).\frac{\partial^2 \underline{\varphi}}{\partial Z^2}+\frac{\partial}{\partial R}\left(\frac{\partial}{(\partial R+1)}.\frac{\partial \underline{\varphi}}{\partial R}\right)=A.\frac{Ra^*}{Pe}.\frac{\partial\theta}{\partial R} \qquad \textbf{(10)}$$

And the dimensionless energy equation is :

$$\frac{\partial \underline{\varphi}}{\partial Z}.\frac{\partial\theta}{\partial R}-\frac{\partial \underline{\varphi}}{\partial R}.\frac{\partial\theta}{\partial Z}=\frac{1}{Pe}\left[\frac{\partial}{\partial R}\left(\frac{\gamma R+1}{\gamma}\right)\left(\frac{\partial\theta}{\partial R}\right)+A^2\left(\frac{\gamma R+1}{\gamma}\right)\left(\frac{\partial^2\theta}{\partial Z^2}\right)\right] \qquad \textbf{(11)}$$

In the above system the following dimensionless numbers appear :

$$Ra=\frac{gB\Delta TD^3}{\upsilon\,\alpha_e} \qquad Da=\frac{K}{D^2} \qquad Ra^*=Ra.Da$$

$$Ra^*=\frac{gB\Delta TDK}{\upsilon\,\alpha_e} \qquad Pe=\frac{W_i\;D}{\alpha_e} \qquad \textbf{(12)}$$

The non – dimensional boundary condition of the problem are :

At R = 0 $\theta=1$ $\underline{\varphi}=0$

(13)

At R = 1 $\frac{\partial\theta}{\partial R}=1$ $\underline{\varphi}=0$

The problem described here as equations (10) , (11) and (13) was solved numerically based on an algorithm described in details in the next section .

After knowing the temperature profiles from the numerical solution , the mixing cup temperature and local Nusselt number at any cross – section can be calculated .

$$Nu_i=\frac{q\;D}{K\,(t_{si}-t_{mi})} \qquad \textbf{(14)}$$

or

$$Nu_i=\frac{\left(\frac{\partial\theta}{\partial R}\right)_1}{\theta_{si}-\theta_{mi}} \qquad \textbf{(15)}$$

Where

θ_{si} **= Dimensionless local inner cylinder surface temperature**

θ_{mi} **= Dimensionless local mixing cup temperature .**

$\left(\frac{\partial\theta}{\partial R}\right)_1$ **= was calculated using the following third – order difference schemes :**

$$\left(\frac{\partial\theta}{\partial R}\right)_1 = \frac{1}{6\,\Delta\, r}\,(-11\,\theta_{1,J}\ +18\,\theta_{2,J}\ -9\theta_{3,J}\ +2\theta_{4,J}) \qquad \textbf{(16)}$$

The mixing cup temperature at any cross section is defind by :

$$t_m = \frac{\int_{r_i}^{r_0} t\,u\,r\;dr}{\int_{r_i}^{r_0} u\,r\;dr} \qquad \textbf{(17)}$$

or in a dimensionless form

$$\theta_m = \frac{\int_0^1 \theta\;U\;\;R\;dR}{\int_0^1 U\;\;R\;dR} \qquad \textbf{(18)}$$

Then , The average Nusselt number is given by :

$$\overline{Nu} = \frac{1}{Z}\int_0^Z Nu_i\;\;d\;\;Z \qquad \textbf{(19)}$$

Numerical Method

Equations (10) and (11) are transformed into difference equations by using a finite – difference scheme . The system of algebraic equation will be obtained and solved by using Gauss – sidel method which makes use of the new values as soon as they are available .

The sequence of operation can be briefly stated as follows :

1- Use the known stream function and temperature at step (n) as initial guesses for the solution of step (n+1) .

2- The above process is repeated until convergence is achieved with the prescribed errors (10^{-4}) .

3- The known values of stream function and temperature at this axial step were used to solve the next axial step .

4- Final stream function and temperature at each grid in flow domain are guessed .

Results and Discussions

Numerical results for a range of modified Rayleigh number 10 < Ra^* < 100 , aspect ratio 0.2 < A < 1 and peclet number 1 < Pe < 10 are reported to investigate the behavior of heat transfer through the porous annulus .

Fig. (2) shows sample of results of dimensionless temp. distribution with dimensionless radius annulus at various Ra^* , A and Pe . It can be seen that the temp. profile decreases a way from the inner heated cylinder . Because of the high porosity at the fixed surface and channeling effect , convection effect enhances close to inner heated cylinder .

Fig. (3) illustrates the dimensionless temp. line at mid – length of annulus , it is shown the gradient of the lines increase as the modified Rayleigh number increases and the difference between them decreases as aspect ratio increases .

In Fig. (4) the effect of aspect ratio at the same Ra^* and Pe is shown . It is clear that the decreasing in dimensionless temp . through porous media decreases grading and the influence take more distance from the inner cylinder to outer , as the aspect ratio decreased .

Fig. (5) shows that the variation in stream function increases near the inner surface to the maximum value due to the convection and channeling effects then decreases sharply and then becomes steady at the core of annulus , after that decrease until it reaches the outer cylinder equaling zero .

To investigate the effect of modified Rayleyh number (Ra^*) on stream function , Fig.(6) is plotted at various (A) and (Pe) , stream function have the same behavior but the value its increases as (Ra^*) increases and (A) decreases . Also fig. (7) appears that the value of stream function increases as aspect ratio (A) decreases at the same (Pe) and (Ra^*) number .

The relation between local Nusselt number with Dimensionless length at a certain (Ra^*) is shown in Fig. (8) . It is noted that the highest value occurred at the (Z) < 0.4 and become approximately the same value at (Z) > 0.4 . The variation of (Pe) number slightly influences on (Nu) .

The average Nusselt number $\overline{Nu}$ is calculated from eq. (19) and plotted with variation of (Ra/Pe) for various aspect ratio (A) in

Fig (9) . It is shown that ($\overline{Nu}$) decreased as (A) decreases and as (Ra^*) decreases .

Based upon the heat transfer results , the following correlation is presented for average ($\overline{Nu}$) :

$$\overline{Nu} = 3.391\,(A)^{1.024}\ \ (Ra^* / Pe)^{0.02827} \qquad \textbf{(20)}$$

The correlation obtained is based on the problem parameters range modified Rayleigh number (10-100) , peclet number (1-10) and aspect ratio (0.2-1) .

To compare Eq.(20) which is obtained in this study with the equation of the previous work[12] :

$$\overline{Nu} / Pe^{1/2} = 0.383\left[1 + 0.098\left(Ra / Pe\right)\right]^{0.488} \qquad \textbf{(21)}$$

In Ref .[12] the packed channel were used for that the parameter(A) which was defined as (annulus gap / length of channel) are assumed to be equal(1) to approximate the results. Fig(10) shows the plots of average Nusselt number values from Ref. [12] for (A=1 , Pe=8) , (A=1 , Pe=10) and the calculated average Nu obtained in the present work . It is shown that all plots have the same behavior of Nu at different (Ra^* / Pe). The average Nu at the packed annulus are higher than in the packed channel due to varition in shape ,which enhanced the channeling effect and then increased heat transfer at the cylinder wall.

Conclusions

This paper present a numerical investigation of mixed convection in a cylindrical annulus filled with fluid – saturated porous material . It was found that the dimensional temp. increased as (Ra^*) increase and (A) decreased , also Nusselt number was increased as (Ra^*) increased and aspect ratio increased . Useful correlation reporting the dependence of the convection heat transfer on the problem parameters were obtained in this study .

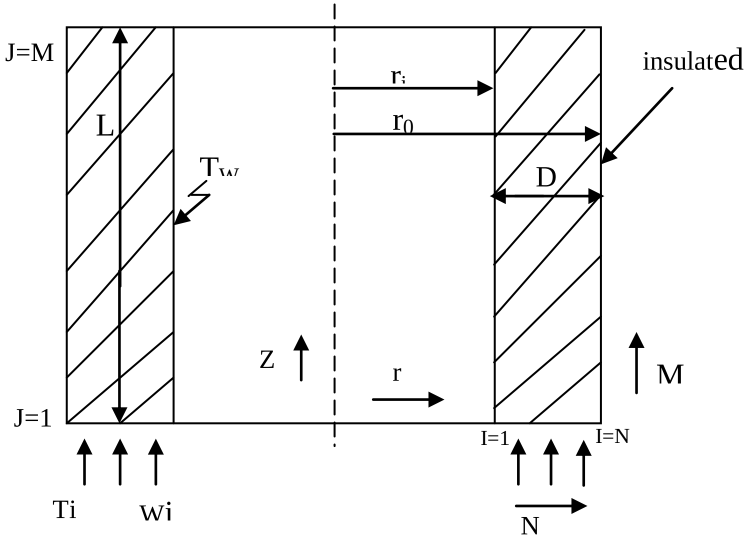
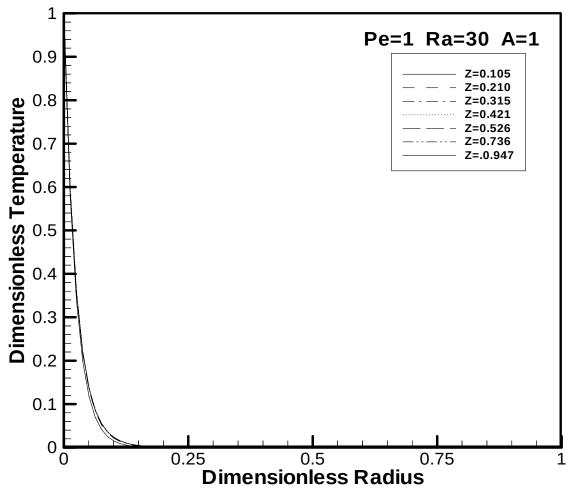
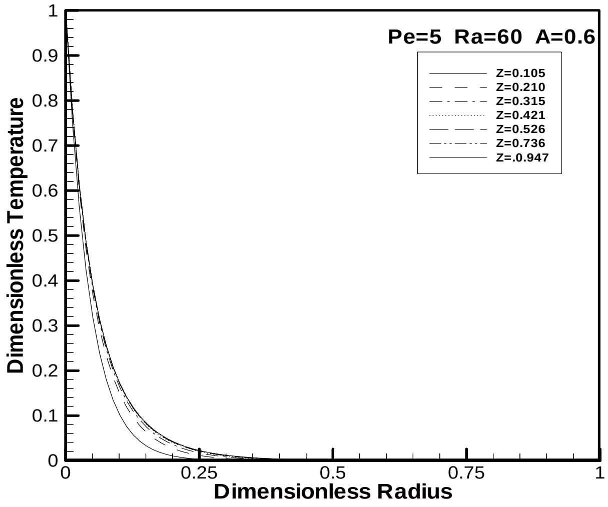
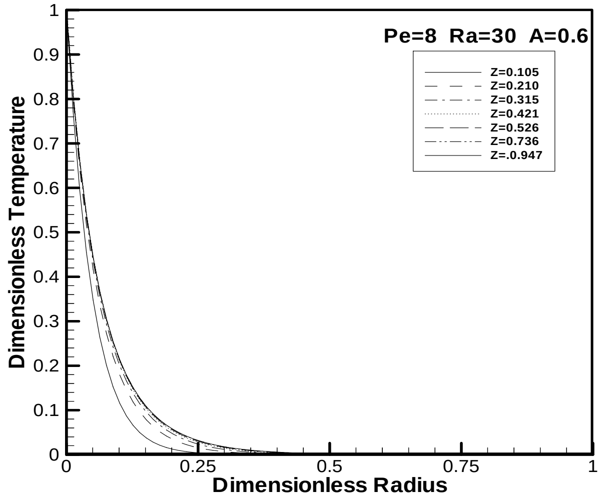


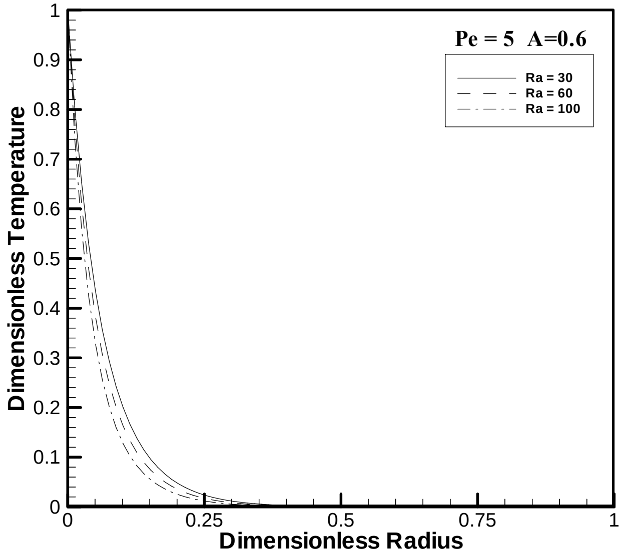
Fig. (1) Physical Model

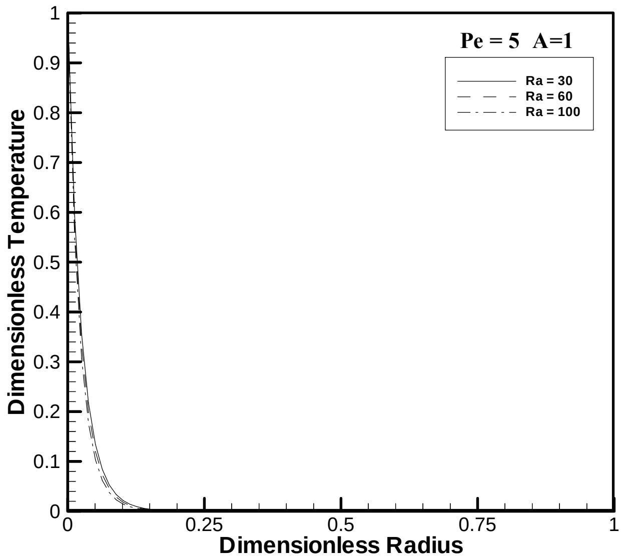




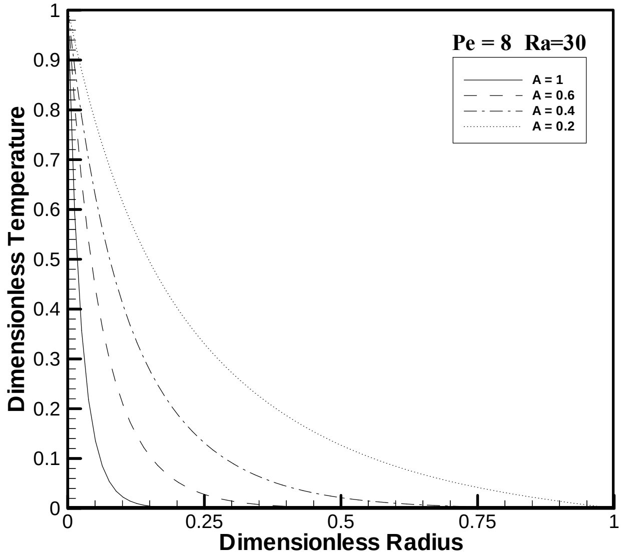


Fig(2) Dimensionless temp. vs Dimensionless radius for various Ra,Pe &A

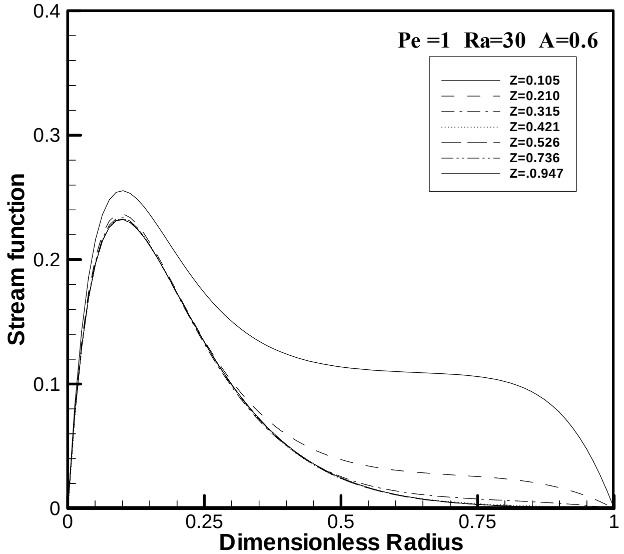


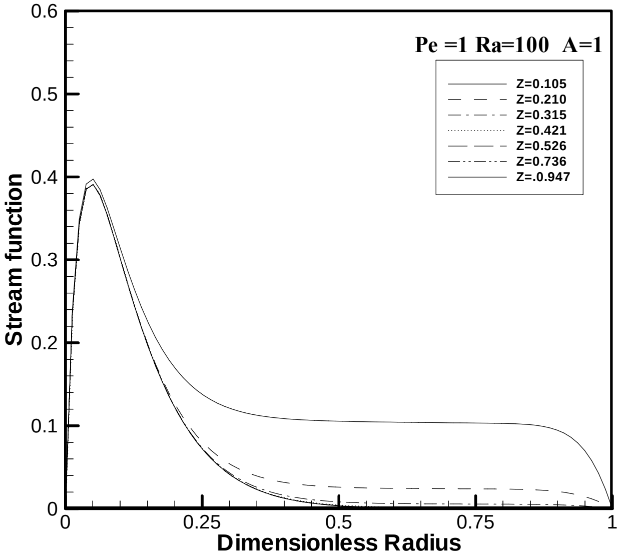


Fig(3) Dimensionless temp. vs Dimensionless radius at mid-length

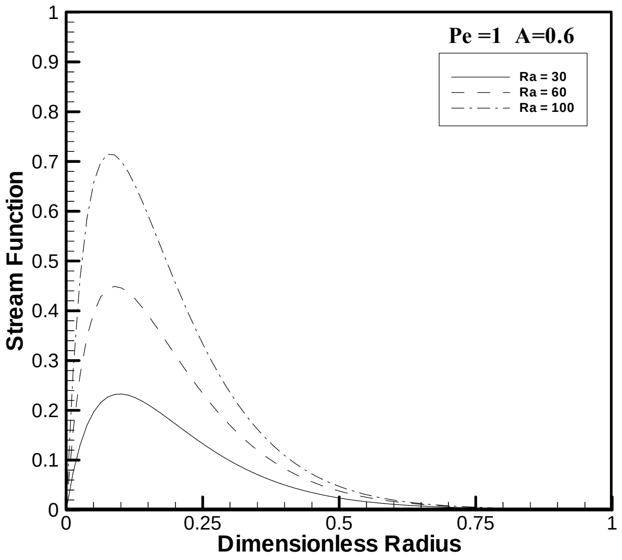


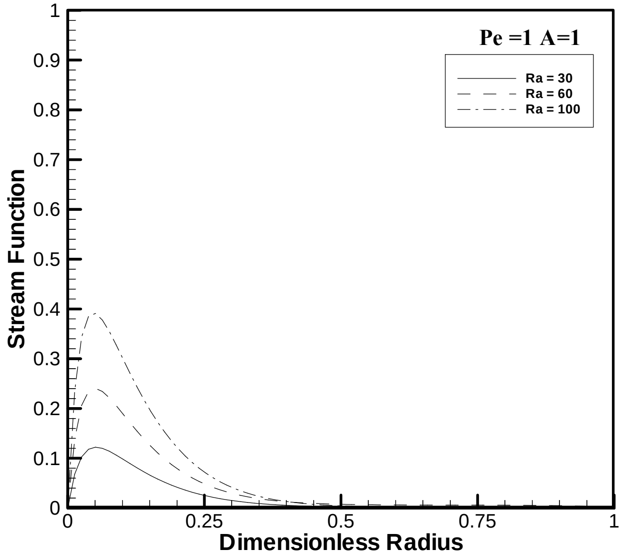
Fig(4) Dimensionless temp. vs Dimensionless radius at mid-length



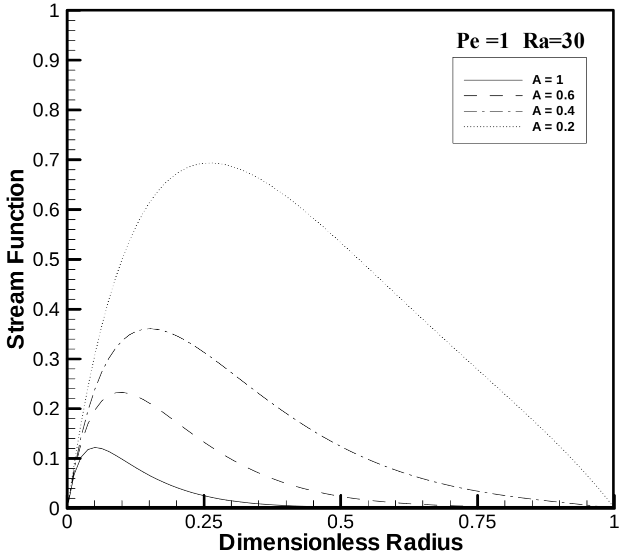


Fig(5) Stream function vs Dimensionless radius for various Ra , Pe & A

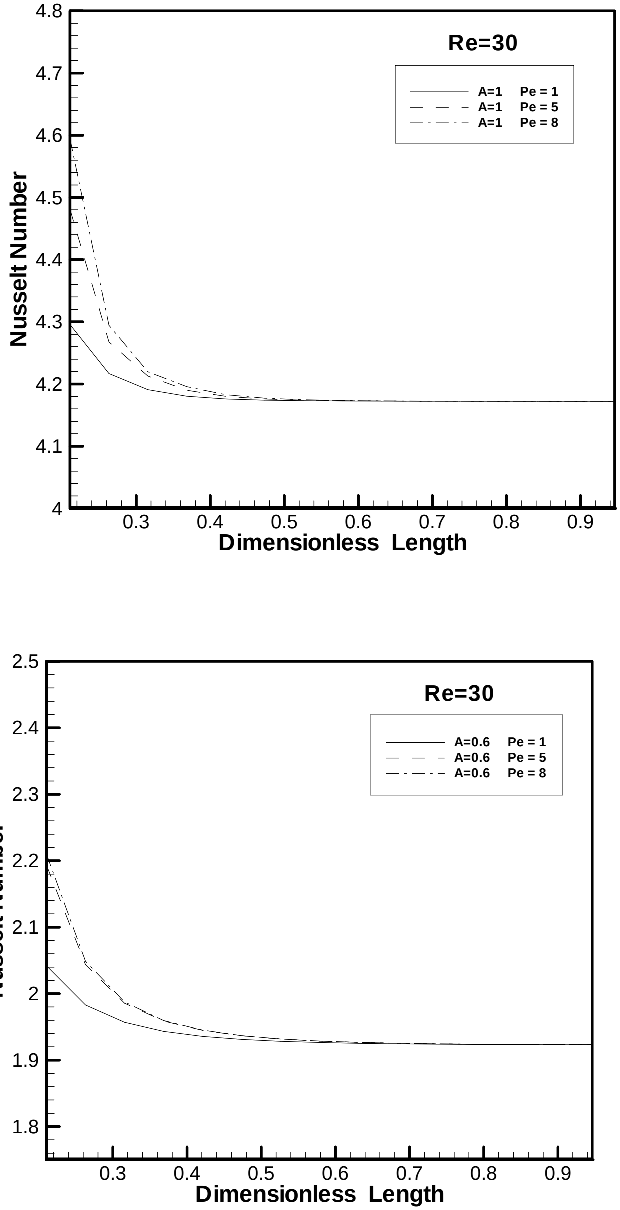




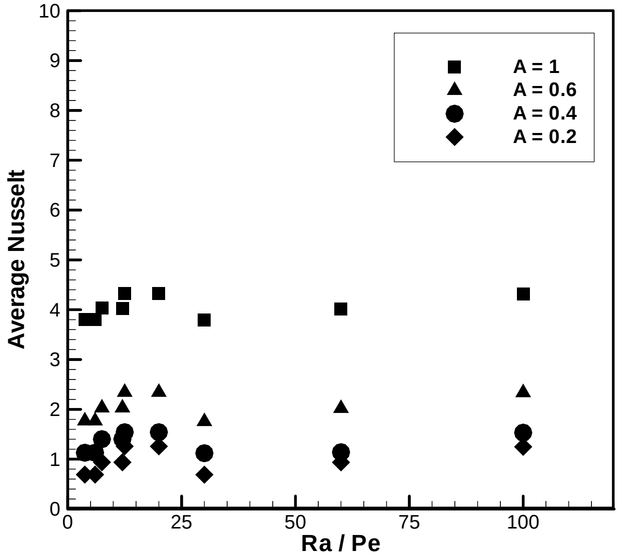
Fig(6) Stream function vs Dimensionless radius at mid-length



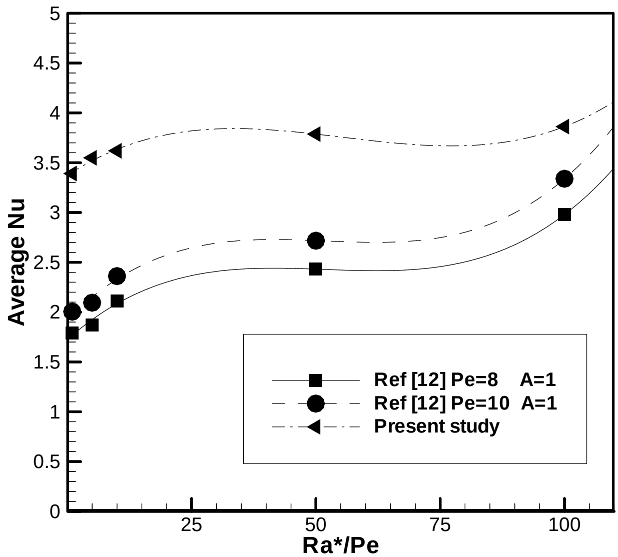
Fig(7) Stream Function vs Dimensionless radius at mid-length



Fig(8) Nusselt Number vs Dimensionless Length



Fig(9) Average Nusselt Number vs Ra/Pe at various A



Fig(10) Comparison of Average Nusselt Number with Ref[12]

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