Maximum Arcs in a Projective Plane PG(2,9) Over Galois Field

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المستخلص

في هذا البحث، قمنا بإنشاء الاقواس العظمى الكاملة في المستوى الاسـقاطي (k,n) على حقـل كـالوا GF(9) ، وجـدنا الاقـواس (10,2) و (16,3) و(23,4) و(32,5) و(40,6) و(48,7) و (59,8) و (71,9) جميـع هـذه الاقواس عظمى في المستوي الاسقاطي PG(2,9) على GF(9) .

Abstract

In this work, we construct complete and maximum (k,n)-arcs in projective plane over Galois field GF(9), where $2 \leq n \leq 9$, by using geometrical method. We found (10,2)-arc, (16,3)-arc, (23,4)-arc, (32,5)-arc, (40,6)-arc, (48,7)-arc, (59,8)-arc and (71,9)-arc, all of them are maximum arc in PG(2,9) over GF(9).

Introduction

Ban in 2001 (5) gave the maximum of (k,n)-arcs and Ahmed, A.M., Al-Mukhtar, A.Sh. and Kadhum, S.J. in 2002 (6) gave the maximum arcs in the projective plane PG(2,5) over Galois field GF(5).

The aim of this paper is to find the maximum arcs in a projective plane PG(2,9) over Galois field GF(9).

This paper is divided into ten section, section one consists of the basic theorems and definition of projective plane. The additions and multiplications operations of GF(9) explained in section two. In section three to section ten, the construction maximum complete (k,n)-arcs for n=2,3,4,5,6,7,8,9 in PG(2,9).

1.1 Definition "Projective plane" (1)

A projective plane PG(2,9) over Galois field GF(9), where p is prime number, consists of $p^2 + p + 1$ points and $p^2 + p + 1$ lines, every line contains $p + 1$ points and every point is on $p + 1$ lines. Any point of the plane has the form of a triple (x_0,x_1,x_2), where x_0,x_1,x_2 are elements in GF(p) with the exception of a triple consisting of three zero elements. Two triples (x_0,x_1,x_2) and (y_0,y_1,y_2) represent the same point if there exists λ in GF(p) \ {0}, s.t.$(y_0,y_1,y_2) = \lambda\ (x_0,x_1,x_2)$.

There exists one point of the form (1,0,0), there exists p points of the form (x,1,0), there exists p^2 points of the form (x,y,1), similarly for the lines.

A point $P(x_0,x_1,x_2)$ is incident with the line $[y_0,y_1,y_2]$ iff: $x_0\ y_0 + x_1\ y_1 + x_2\ y_2 = 0$.

The projective plane PG(2,p) satisfyies the following axioms:

a) Any two distinct lines intersected in a unique point.

b) Any two distinct points are contained in a unique line.

c) There exists at least four points such that no three of them are collinear.

The projective plane PG(2,9) contains 91 points, 91 lines, 10 points on every line and 10 lines through every point. Let p_i and L_i, i = 1,2,…,91 be the points and lines of PG(2,9) respectively, all the points and lines of PG(2,9) are given in table (1).

1.2 Definition (3)

A (K,n)-arc K in PG(2,p) is a set of K points such that some n, but no n + 1 of them are collinear.

1.3 Definition (3)

A (K,n)-arc K in PG(2,p) is complete if it is not contained in a (K+1,n)-arc.

1.4 Definition (1)

The i-secant of a (k,n)-arc K is a line intersects the arc K in exactly i points, a 0-secant is called an external line of K, a 1-secant is called a unisecant line, a 2-secant is called a bisecant line and 3-secant is called a trisecant line.

1.5 Definition (1)

A point N which is not on a (k,n)-arc K has index I denoted by N_i, if there are exactly i (n-secant) of K thought N. Let $C_i = | N_i |$ be the number of the points N_i of index i.

1.6 Remark (2)

The (k,n)-arc K is complete if and only if $C_0 = 0$, thus K is complete if every point of PG(2,p) lies on some n-secant of K.

1.7 Definition (3)

A (k,n)-arc K in PG(2,p) is maximual arc if $k = (n - 1)p + n$.

1.8 Definition (1)

The maximum number of points that came (k,2)-arc have is m(2,p)-this arc called an oval

1.9 Definition (3)

A polynomial F in $K[x_1,x_2,\ldots,x_n]$ is homogenous or form of degree d if all its terms have the same degree g. A subset V of PG(n,k) is a variety over K if there exists forms $F_1, F_2,\ldots, F_r$ in $K[x_1,x_2,\ldots,x_n]$ such that

$V=\{P(A) \text{ in } PG(n,k)/F_1(A)=F_2(A)=\ldots=F_r(A)=0\}=V(F_1,F_2,\ldots,F_r)$.

1.10 Definition (3)

A variety V(F) in PG(n,k) is called a primal. The order or degree of a primal V(F) is the degree of F.

1.11 Definition (3

A quadric Q in PG(n – 1,p) is a primal of order two, so if Q is a quadric, then Q=V(F) where F is quadric form, that is

$$=a_{11}x_1^2+a_{12}x_1x_2+\ldots+a_{nn}x_n^2 \sum_{\substack{i\le j\\ i,j=1}}^{n} a_{ij}x_ix_j\, F=$$

1.12 Definition (3) (Conic)

Let Q(2,p) be the set of quadrics in PG(2,p) that is the varieties V(F), where

$$=a_{11}x_1^2+a_{22}x_2^2+a_{33}x_3^2+a_{12}x_1x_2+a_{13}x_1x_3+a_{23}x_2x_3 F$$

is non-singular, then the quadric is a conic. $A=\begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ If

1.13 Theorem (1)

In PG(2,p), with p odd, every oval is a conic.

1.14 Theorem (4)

Let m be a point of a (k,2)-arcK and let t(m) be the number of unisecants through m in PG(2,p) then t = t(m) = p + 2 – k.

1.15 Corollary (4)

If k is an oval then t(m) = 1.

1.16 Theorem (3)