

The effect of temperature of operation on the Turn-on Time Delay of Semiconductor Lasers

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Abstract:

In this paper, the effect of temperature of operation on the turn-on time delay, t_d , of semiconductor laser has been re-studied theoretically. We proposed a model to calculate the temperature dependence (TD) of t_d according to the TD of laser cavity parameters. An accurate analytical expression of t_d as a function of temperature has been driven in term of TD of threshold carrier density. The TD of threshold carrier density was calculated according to the TD of cavity parameters and not by the well-known exponential Pankove relation via the use of characteristics temperature and current. A very good agreement between the values of t_d calculated by the proposed model and the numerical method through the shown simulation results. We also re-derived approximated expressions of t_d as a function of temperature according to this model.

Key words: semiconductor laser, turn-on time delay, temperature effect

Introduction:

Direct modulation (DM) of semiconductor lasers is used in most optical communication systems because its implementation is simpler than external modulation [1]. Because of physical limitations, the output light under direct modulation cannot respond to the input signal instantaneously. Instead, there can be delay and oscillation when input has large and rapid variations [1-3]. DM involves changing the current input of the laser, i.e. modulating it around the bias current, in order to produce time independent output in optical intensity [2]. When the laser source is injected by biased current below threshold, the optical gain of the semiconductor laser active region is insufficient for stimulated emission and output optical power is very low. To start lasing, the threshold carrier concentration should be reached in the laser active region. In direct digital modulation, when the laser is turned on by increasing the

device current from its initial value, I_0 , to a specified value, I , greater than threshold, I_{th} i.e. at rising edge of the input pulse, stimulated emission is delayed by t_d , and the photon density, N_{ph} , will stay essentially zero until the carrier density, N , reaches its threshold value, N_{th} [1, 3, 4]. Once N be greater than N_{th} , i.e. $N > N_{th}$, the optical gain becomes positive, consequently, N_{ph} increases rapidly and quickly exceeds its steady state value. This brings N down and even below steady state value. The interaction between carrier and photon densities forms an oscillation called *Relaxation oscillation*. Finally the drop in current i.e at the falling edge of input pulse, brings N down, as a result, the laser medium has negative optical gain, and the output drops to zero. The above explanation suggests analyzing the input pulse response by dividing the pulse modulation into three stages [1]:

- 1- $0 < t < t_d$; In this period $N_{ph} \approx 0$ and t_d occurs until N reaches N_{th} .
- 2- $t_d < t < T_b$, where T_b is the pulse duration. In this period N_{ph} and N experience a relaxation oscillation.
- 3- And at $t > T_b$; I drops back to the low value.

In our theoretical analysis, as first part, we are interested with the first stage i.e. t_d analysis.

Many studies have been presented for determining t_d by solving the carrier density rate equation [1, 3, 4]. To our knowledge, most of these studies were based on either numerical or approximation formula due to the difficulty of integrating the nonlinear rate equation analytically. The nonlinearity exists by the functional form of recombination rate which can be written as a function of carrier density N as [1]

$$r(N) = AN + BN^2 + C_{Auger}N^3 \dots\dots(1)$$

where A , B and C_{Auger} are the nonradiative, radiative and Auger recombination coefficients respectively. In [4], $r(N)$ is assumed as $r(N) \approx AN$ and also in [4], an analytical expression has been obtained for t_d when $r(N) \approx AN + BN^2$. The nonlinear rate equation has been integrated numerically [4] in order to study the effect of Auger coefficient and the shown results were at special case. In [3], the coefficients A and B have been set to zero and t_d was calculated according to that assumption. In [1] $r(N)$ was assumed as constant, i.e. $r(N) = AN_{th} + BN_{th}^2 + C_{Auger} N_{th}^3$. However, as mentioned above, a full

analytical expression in terms of all main parameters (i.e. N , A , B , C_{Auger} , I_{th} , I and dimensions of cavity) which can be applied to most types of laser sources and at different wavelengths operation appears mathematically difficult to obtained. In addition, most of the above analysis was taken at fixed temperature degree i.e. room temperature.

Also, the improvements in laser manufacture allow today operating uncooled direct modulated lasers (DMLs) over abroad temperature range. Given that a strong cost reduction in optical systems can be achieved by eliminating costly laser temperature control. Uncooled DMLs have been considered as a key technology for future optical networks systems [6].

It is well known that temperature variation affects t_d where it increases with increasing of temperature due to the temperature dependence (TD) of threshold current and other cavity parameters. The TD of threshold current has been empirically described by the well known exponential Pankove relation via the use of two parameters T_o and I_o , known as the characteristics of both temperature and current [5] but it is known that T_o is not strictly constant with temperature [5]. In this study, we derived an accurate expression to determine the TD of t_d according to the TD of laser cavity parameters. We also present mathematical software (MathCAD) in order to compute t_d theoretically for most laser parameters.

Mathematical description:

In the time period ($0 < t < t_d$), the stimulated emission rate can be neglected and the carrier density rate equation of single mode semiconductor laser can written as [1, 3-5]

$$\frac{dN}{dt} = \frac{I}{qV} - (AN + BN^2 + C_{Auger} N^3) \dots\dots(2)$$

where q is the electron charge and V is the volume of laser active region. Recalling that t_d is calculated as [1,4]

$$t_d = \int_{N_i}^{N_{th}} \frac{qV}{I - qV(AN + BN^2 + C_{Auger} N^3)} dN \dots\dots(3)$$

The above equation can be rewritten as

$$t_d = t(N)|_{N=N_{th}} - t(N)|_{N=N_i} \dots\dots(4)$$

where $t(N)$ is the solution of integration of eqn.3 in its unlimited form.

To include the effect of temperature and initial carrier density, eqn. 4 may be

$$t_d = t(N, \sigma) \Big|_{N=\sigma N_{th}(T), \sigma=1} - t(N, \sigma) \Big|_{N=N_i(T)=\sigma N_{th}(T), 0 \leq \sigma \leq 1} \dots (5)$$

where σ is an integer ($0 \leq \sigma \leq 1$) which represents the ratio of TD of initial carrier density, $N_i(T)$, to $N_{th}(T)$ where $N_{th}(T)$ is the TD of threshold carrier density which after modification can be written depending on [1, 5] as

$$N_{th}(T) = N_o(T) + \frac{1}{\Gamma v_g g_o(T) \tau_{ph}(T)} \dots (6)$$

where Γ and v_g are the optical confinement factor and group velocity respectively. In eqn.6, $N_o(T)$, $g_o(T)$ and $\tau_{ph}(T)$ are the TDs of carrier density at transparency, gain parameter and photon lifetime for uncooled Quantum well distributed feedback (QWDFB) laser and they can be written respectively as[6]

$$N_o(T) = \frac{T}{T_o} N_o(T_o) \dots (7a)$$

$$g_o(T) = \frac{T_o}{T} g_o(T_o) \dots (7b)$$

$$t(N) = N_1 \left[\ln|N - N_2| - 0.5 \ln \left[(N + 0.66N_2)^2 + (N_2)^2 \right] - 1.6 \operatorname{atan2}(N + 0.66N_2, -N_2) \right] \dots (9)$$

where $N_1 = -4.951 \cdot 10^{-1}$ and $N_2 = 2.277 \cdot 10^{18}$ are numbers related to the integration result and $\operatorname{atan2}(x,y)$ is built in software function returns the angle (in radians) from the x-axis to a line containing the origin (0, 0) and the point (x, y) and both x and y must be real, (see Appendix A). Replacing each N by $\sigma N_{th}(T)$ in eqn.9 to obtain $t(T, \sigma)$ then substituting the latter in eqn.8 to obtain t_d . In spite of the resulting expression is for special case of device current and coefficients but it gives us very accurate and direct mathematical expression i.e. mathematical relationship picture of t_d as function of $N_{th}(T)$ and ,in turn, T .

From the integration result which it is in term of N , we note that the values of N_1 and N_2 depend on I and laser parameters q , V , A , B and C_{Auger} . In the analysis, we assumed the value of I corresponding to

rewritten as

and

$$\tau_{ph}(T) = \frac{1}{v_g \left[\alpha_{int}(T) + \frac{1}{L} \ln \frac{1}{r_1 r_2} \right]} \dots (7c)$$

where L is the laser cavity length, r_1 and r_2 are the field reflectivities of the laser facets ($r_1 r_2 = 0.32$) and $\alpha_{int}(T)$ is the TD of internal cavity loss which can be written as [6]

$$\alpha_{int}(T) = \frac{T}{T_o} \alpha_{int}(T_o) \dots (7d)$$

Finally, t_d can be written as function of T and σ as

$$t_d(T, \sigma) = t(T, \sigma) \Big|_{\sigma=1} - t(T, \sigma) \Big|_{0 \leq \sigma \leq 1} \dots (8)$$

After solving the integration in eqn.5 directly for fixed device current, $I = 2 I_{th}(T_o)$, A , B and C_{Auger} took the values as indicated in table I, the result can be written as

the fixed value of threshold current, $I = u I_{th}(T_o)$. The value of $I_{th}(T_o)$ also depends only on the laser parameters at T_o , therefore, for the same device i.e. same parameters, the values of N_1 and N_2 will change if u changes only. According to that, we calculated the values of N_1 and N_2 by re-integrating the rate equation for different values of I , i.e different values of u , then by curve fitting method we obtained an expressions for N_1 and N_2 in term of u as (see appendix B for details)

$$N_1(u) = [K_{10} \ln(u) + K_{11}] 0.984 \dots (10a)$$

$$\text{where } K_1 = \begin{pmatrix} -2.993 \\ 7.343 \end{pmatrix}$$

and

$$N_2(u) = [K_{20} \ln(u) + K_{21}] 0.997 \dots (10b)$$

where $K_2 = \begin{pmatrix} 0.969 \\ 1.647 \end{pmatrix}$

According to that, we can modify the

expression in eqn.9 to be in term of u in addition to N as

$$t(N, u) = N_1(u) \left[\ln|N - N_2(u)| - 0.5 \ln \left[(N + 0.66N_2(u))^2 + (N_2(u))^2 \right] - 1.6 \operatorname{atan2}(N + 0.6N_2(u), -N_2(u)) \right] \dots(11)$$

Where $u (= I/I_{th}(T_o))$ again is the ratio of device to threshold currents.

To obtain more flexible expression forms of t_d i.e. more analytic form, some of approximations have been made and then the integration in eqn.3 can be solved

analytically, consequently, $t(N)$ can be written as

1- When $B \approx 0$ and $C_{Auger} \approx 0$

$$t(N) \approx \frac{-1}{A} \ln \left[\frac{I - qVAN}{A} \right] \dots (12)$$

2- when $C_{Auger} \approx 0$, then

$$t(N) = \frac{2}{(4IqVB + (qV)^2 A^2)^{\frac{1}{2}}} \tanh^{-1} \left(\frac{2qVBN + qVA}{(4IqVB + (qV)^2 A^2)^{\frac{1}{2}}} \right) qV$$

3- when $A \approx 0$ and $B \approx 0$, then

$$t(N) = \frac{1}{G} \left[-\ln(N - F) + \frac{1}{2} \ln(N^2 + NF + F^2) + \sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) \left(\frac{2}{F} N + 1 \right) \right]$$

where

$$G = 3C_{Auger} F^2 \quad \text{and} \quad F = \left(\frac{I}{qVC_{Auger}} \right)$$

Also t_d can be obtained as function of T corresponding to the above eqns. by substitute $\sigma N_{th}(T)$ instead of N then substitute the resulting expression at eqn.8. A slight difference in simulation results is expected due to the approximations but these relationships give more simple and analytical form to determine t_d . Eqn. 14 is very important one because with that accepted approximation [3], it provides an analytical form to determine the effect of C_{Auger} on t_d in the presence of flexible

choices of laser cavity parameters and at reasonable range of temperature for many types of laser sources. *Note:* we used N_{th} rather I_{th} in order to give more physical form and that will be useful to complete the analysis further to include the next two stages (not indicated here).

However, one simply can substitute $N_{th}(T) = (I_{th}(T) T_e(T)) / qV$, where $T_e(T)$ is the TD of carrier life time, in order to calculate t_d in term of $I_{th}(T)$.

Results of Discussion:

In this section, the mathematical calculations have been performed by using MathCAD software and the related programs of numerical calculation and the calculations corresponding to our model (i.e. eqn. 11) are shown in appendix C and D respectively. The values of laser parameters at $T_o = 25^\circ C$ are provided in

table I, In fig.1, the accuracy of t_d calculation result by the proposed model (see eqn. 11) was compared with the one calculated by numerical method for different values of u at $T=50^\circ C$ (selected arbitrary). For instance, at $u = 2$ and $\sigma = 0.5$, t_d equals 0.833 and 0.823 nsec when it is calculated by our model and numerical

method respectively. While for $u = 3$, t_d equals 0.466 and 0.452 nsec respectively for the same value of σ . These results show high accuracy of calculation by proposed model. In fig. 2, a comparison between t_d calculation by numerical method and approximated expression forms at $T = 25^\circ\text{C}$ and $u = 2$ are shown. Due to approximations, there are expected differences among t_d curve values drawn. For instance, at $\sigma = 0.5$, $t_d = 0.506, 0.552, 0.544$ and 0.666 nsec corresponding to the calculations by eqns. 12, 13, 14 and numerical method respectively. The TD of t_d for different values of I and σ , as T changed through an accepted range from 25°C to 100°C [7,8], are depicted through figs. 3 and 4 respectively. Fig.3 shows the effect of T on t_d for different values of σ . It is noted that t_d increases with increasing of T due to increasing of threshold carrier density. For instance, t_d is about 0.77 nsec at $T=25^\circ\text{C}$ and $\sigma = 0.4$ while it increases to 1.924 nsec as T rises to 100°C for the same value of σ . The effect of T is reduced for higher values of σ because $t_d \rightarrow \text{zero}$ as $\sigma \rightarrow 1$, i.e. $N_i \rightarrow N_{th}$, therefore, the time delay may be eliminated if the laser source is turned-on from a value closed to the threshold current. Thus, all the shown curves meet at zero value of t_d when σ set to 1, i.e. $N_i = N_{th}$. Also from the results shown in fig. 3, the effect of residual carrier density, which is considered here as a variable value of initial carrier density, can be observed. It is significant in direct digital modulation of

semiconductor lasers especially at high data rates where the initial carrier density is varied depending on the time intervals among the digital current pulses. Actually, after the pervious pulse is turned-off, the carrier density diminishes relatively slowly according to the carrier lifetime. When the next current pulse is applied, t_d depends on the residual carrier density which acts as initial one to this pulse. Thus, if the time interval is relatively short, residual carrier density is significant and t_d is reduced.

The TD of t_d for different values of I is depicted at fig. 4. As I increases, t_d decreases at specified values of σ . This behavior of t_d can be readily concluded by noting the mathematical expression of t_d . It should be mentioned that we assumed I in all results corresponding to $I_{th}(T_0)$, i.e. I is temperature independent. This assumption is made according to the normal operation of laser source. In other words, the current source supplies a fixed value of injection current. Finally the effect of Auger coefficient, C_{Auger} , is shown in fig.5 where t_d is calculated corresponding to eqn. 14. The general effect of C_{Auger} is to increase t_d as it increases, where it is known that C_{Auger} increases with temperature, therefore, a reasonable expectation that t_d will increase further if the TD of C_{Auger} is taken into account and that can be done by considering C_{Auger} as $C_{Auger}(T)$ at eqn. 14. In fig. 5, we represented a general effect of increasing of C_{Auger} and not especially by temperature effect.

Parametr	Description	Value	Unit
d	Active region thickness	0.04	μm
W	Active region width	1	μm
L	Cavity length	400	μm
A	Nonradiative recombination coefficient	1.10^{-8}	sec^{-1}
B	radiative recombination coefficient	8.10^{-17}	m^3/sec
C_{Auger}	Auger recombination coefficient	10^{-40}	m^6/sec
g_0	Differential gain	$7.6.10^{-20}$	m^2
Γ	Confinement factor	0.08	
α_{int}	Internal loss	15	cm^{-1}
v_g	Group velocity	$7.5.10^9$	cm/sec
N_0	Carrier density at transparency	1.10^{-8}	cm^{-3}
g_0	Gain constant	$7.6.10^{-16}$	cm^2

Table. I: Parameter values for QWDFB at room temperature.

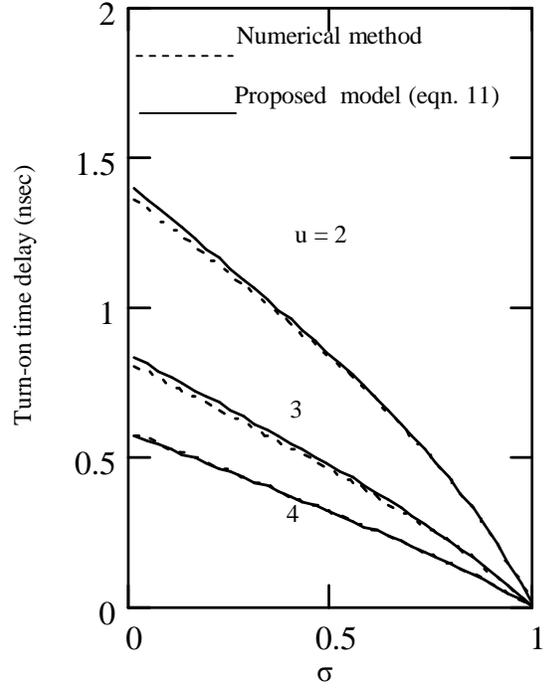


Fig.1: Turn-on time delay vs. σ for different values of u at $T=50^\circ\text{C}$. The calculated curve by proposed model is compared with those calculated by numerical method.

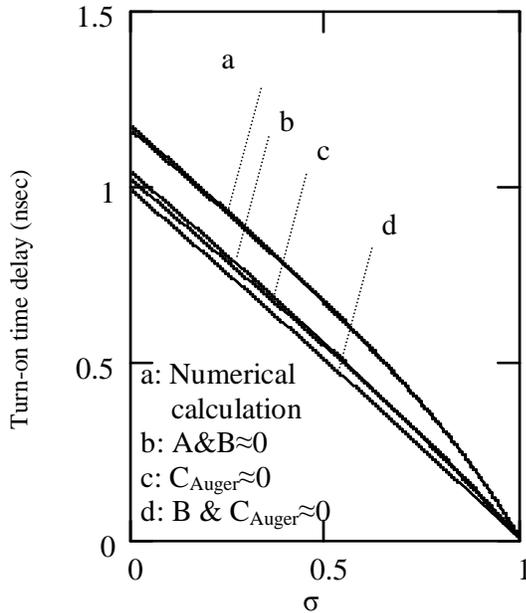


Fig.2: A comparison among various expressions of t_d with α at $T_0=25^\circ\text{C}$ and $I=2 I_{\text{th}}(T_0)$.

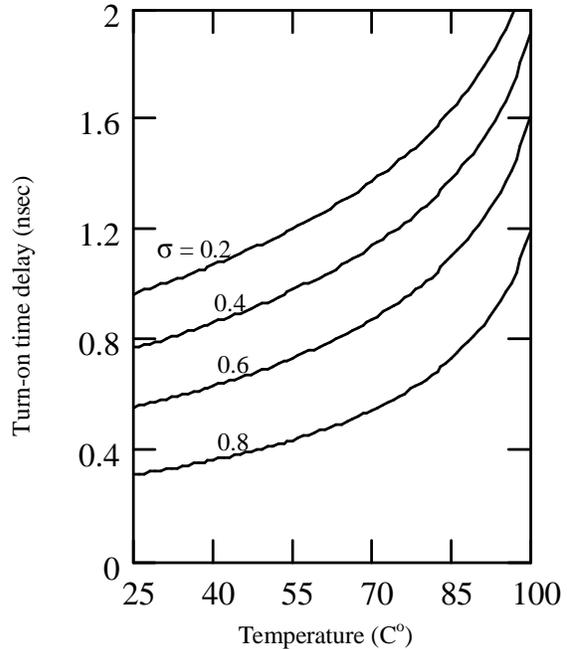


Fig.3: The temperature dependence of t_d for different values of σ at $I=2 I_{\text{th}}(T_0)$.

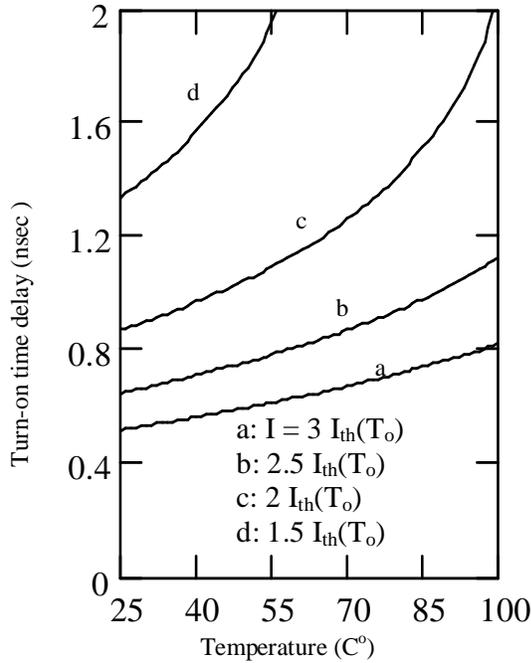


Fig.4: The temperature dependence of t_d for different values of bias current, I at $\sigma=0.3$.

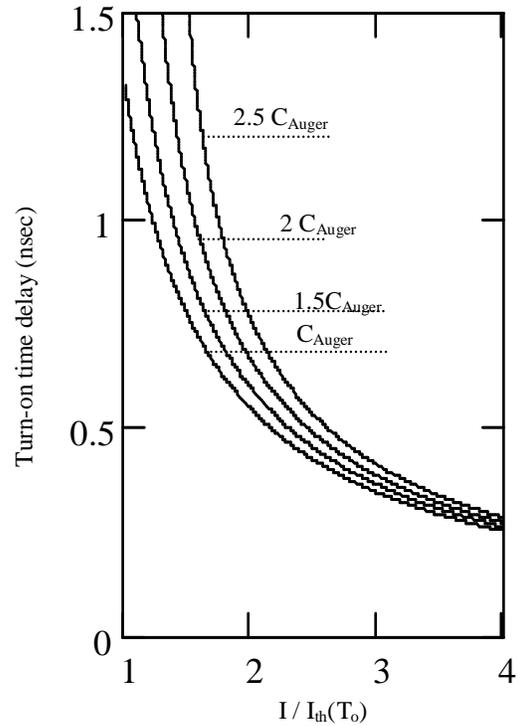


Fig.5: Effect of Auger coefficient on t_d at $\sigma = 0.5$ and $T_0=25^\circ\text{C}$.

Conclusions:

The presented study provides:

- 1-Very accurate expression form for turn-on time delay of semiconductor lasers in term of TD of threshold carrier density.
- 2- flexibility to study theoretically the influence of device current, initial carrier density and Auger effect on turn-on time delay of uncooled QWDFB lasers at wide range of temperature where the results shows that t_d increases as both temperature and Auger coefficient increases and it is decreases as both device current initial carrier density increased further .
- 3- Theoretical calculation of the TD of threshold carrier density where it is determined exactly according to the TD laser cavity parameters and in turn, threshold carrier density can be determined

- while many studies depend on the well know exponential form that describe the TD of threshold current depending on characteristic temperature and current.
- 4- A knowledge about the effect of residual carrier density or nonconstant initial carrier density (as functions of T) which has significance at direct digital modulation of semiconductor lasers.
- 5- A mathematical software program to determine turn-on time delay of laser sources at any values of laser parameters and any degree of temperature within a specified range.
- 6- Theoretical background to complete the analysis further to include the temperature effect on transient response and relaxation phenomena.

Appendix A
Some details about eqn .9

In order to explain the evaluation of eqn.9, we assume the following integral:

$$F(X) = \int_{x_1}^{x_2} G(X)dX \quad \dots(A1)$$

where x_1 and x_2 are arbitrary positive real numbers and $G(X)$ is

$$G(X) = \frac{K}{i - K_1X + K_2X^2 + K_3X^3} \quad \dots(A2)$$

where K is constant and the dominator of (A2) is a polynomial of 3 degree and i, K_1, K_2 and K_3 are coefficients with known values.

After substitution the values of K and coefficients in (A2), the solution of (A1) can be written as

$$F(X) = (F_1(X) + jF_2(X)) \Big|_{x=x_1}^{x=x_2} \quad \dots(A3)$$

$$= F_1(X) \Big|_{x=x_1}^{x=x_2} + jF_2(X) \Big|_{x=x_1}^{x=x_2}$$

where $F_1(X)$ and $F_2(X)$ are specified functions related to the result of integration process and j is the imaginary unit.

Assuming a specified values of x_1 and x_2 , (A3) can be re-written as

$$F(X) = F_1(X) \Big|_{x_2=1} - F_1(X) \Big|_{0 \leq x_1 \leq 1}$$

$$+ j \left(F_2(X) \Big|_{x_2=1} - F_2(X) \Big|_{0 \leq x_1 \leq 1} \right) \quad \dots(A4)$$

$$= F_1(X) \Big|_{x_2=1} - F_1(X) \Big|_{0 \leq x_1 \leq 1}$$

where the imaginary part of (A4) are cancelled through the subtraction process because always $F_2(X) \Big|_{x_2=1} = F_2(X) \Big|_{0 \leq x_1 \leq 1}$ for the indicated values of x_1 and x_2 . The function $F_1(X)$ then can be written after arrangement some terms, i.e. the commons terms are placed out, as

$$t(N) = N_1 \left[\ln |N - N_2| - 0.5 \ln \left[(N + 0.66 N_2)^2 + (N_2)^2 \right] - 1.6 \operatorname{atan2} (N + 0.6 N_2, -N_2) \right] \quad \dots(A5)$$

where $N_1 = -4.951 \cdot 10^{-10}$ and $N_2 =$

$2.277 \cdot 10^{18}$ are numbers related to the integration result.

Appendix B
Derivation of $N_1(u)$ and $N_2(u)$

We re-integrated the rate equation for different values of I , i.e different values of

u , then obtained the following data as shown in the tables below :

	u	N_1
	0	1
0	1	7.96
1	1.5	6.03
2	2	4.95
3	2.5	4.25
4	3	3.76
5	3.5	3.38
6	4	3.09
7	4.5	2.85
8	5	2.66

$\times 10^{-10}$

Table II

	u	N_2
	0	1
0	1	1.72
1	1.5	2.03
2	2	2.28
3	2.5	2.49
4	3	2.67
5	3.5	2.83
6	4	2.98
7	4.5	3.11
8	5	3.24

$\times 10^{18}$

Table III

To obtain $N_1(u)$ by MathCAD software

Let:

$$X_1 = \text{data}^{(0)} \quad \dots (B1)$$

$$Y_1 = \text{data}^{(1)} \quad \dots (B2)$$

where X_1 and Y_1 are the given data in table II for column 0 and 1 respectively. A simpler way to perform a customized log fit is by using the **Infit** function, which fits data to the model function

$$y(x) = a \cdot \ln(x) + b \quad \dots (B3)$$

where a, b are arbitrary numbers.

Then define

$$K_1 = \text{Infit}(X_1, Y_1) \quad \dots (B4)$$

Following the same procedure, $N_2(u)$ can be written as

$$N_2(u) = [K_{20} \ln(u) + K_{21}] \cdot 0.997 \quad \dots (B7)$$

According to the given data

$$K_1 = \begin{pmatrix} -2.993 \\ 7.343 \end{pmatrix}$$

and from the model function we write

$$N_1(u) = K_{10} \ln(u) + K_{11} \quad \dots (B5)$$

then for more accurate results, $N_1(u)$ may be written as the following after multiplying by a correction factor 0.984

$$N_1(u) = [K_{10} \ln(u) + K_{11}] \cdot 0.984 \quad \dots (B6)$$

Fig.6 depicted a comparison between the given data and the resulting function from the curve fitting method.

where $K_2 = \begin{pmatrix} 0.969 \\ 1.647 \end{pmatrix}$ and fig. 7 shows the

resulting function comparing of the data of table III.

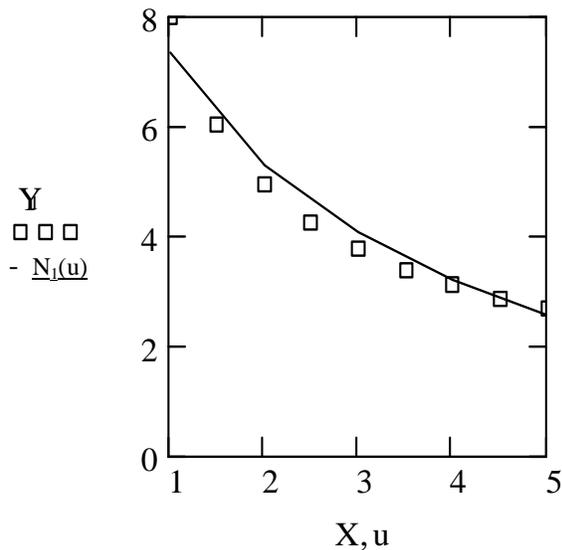


Fig.6. A comparison between the data and the resulting function by curve fitting method

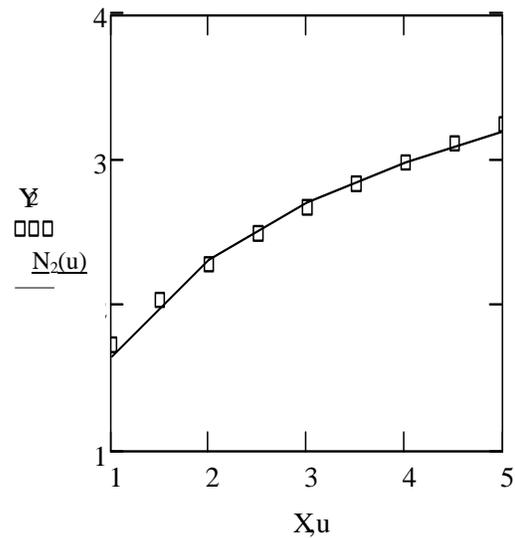


Fig.7. A comparison between the data and the resulting function by curve fitting method

Appendix C
Mathcad software program to determine t_d numerically

$$t_d := \begin{cases} c \leftarrow 3 \cdot 10^{10} \\ n \leftarrow 3.5 \\ \Gamma \leftarrow 0.08 \\ A \leftarrow 1 \cdot 10^8 \\ B \leftarrow 8 \cdot 10^{-11} \\ \text{Cauger} \leftarrow 10 \cdot 10^{-29} \\ q \leftarrow 1.6 \cdot 10^{-19} \\ w \leftarrow 1 \cdot 10^{-4} \\ d \leftarrow 4 \cdot 10^{-6} \\ L \leftarrow 0.04 \\ V \leftarrow w \cdot L \cdot d \\ T_o \leftarrow 25 \\ N_{oo} \leftarrow 10^{18} \\ go \leftarrow 7.6 \cdot 10^{-16} \\ \sigma_{io} \leftarrow 15 \\ N_{th} \leftarrow \frac{T + 273}{T_o + 273} \cdot N_{oo} + \frac{1}{c \cdot \left[\left[\frac{(T_o + 273)}{T + 273} \right] \cdot go \right] \cdot \frac{1}{\left(\frac{c}{n} \right) \cdot \left[\left[\left(\frac{T + 273}{T_o + 273} \right) \cdot \sigma_{io} + \left(\frac{1}{L} \right) \cdot \ln \left(\frac{1}{0.32} \right) \right] \right]}} \\ N_{tho} \leftarrow N_{oo} + \frac{1}{c \cdot go \cdot \frac{1}{\left(\frac{c}{n} \right) \cdot \left[\left[\sigma_{io} + \left(\frac{1}{L} \right) \cdot \ln \left(\frac{1}{0.32} \right) \right] \right]}} \\ \int_{\sigma \cdot N_{th}}^{N_{th}} \left[\left[u \cdot \left[w \cdot L \cdot \left[(q \cdot d) \cdot \frac{N_{tho}}{1} \right] \right] \right] - q \cdot V \cdot \left(A + B \cdot N + \text{Cauger} \cdot N^2 \right) \cdot N \right] dN \end{cases}$$

Appendix D
Our model Mathcad software program

$$\begin{aligned}
 t_d := & \left[\begin{array}{l}
 c \leftarrow 3 \cdot 10^{10} \\
 n \leftarrow 3.5 \\
 \Gamma \leftarrow 0.08 \\
 T_o \leftarrow 25 \\
 N_{oo} \leftarrow 10^{18} \\
 g_o \leftarrow 7.6 \cdot 10^{-16} \\
 \sigma_{io} \leftarrow 15 \\
 N_1 \leftarrow -(-2.993 \ln(u) + 7.343) \cdot 0.98410^{-10} \\
 N_2 \leftarrow (0.969 \ln(u) + 1.647) \cdot 10^{18} \cdot 0.997 \\
 N \leftarrow \left[\frac{T + 273}{T_o + 273} \cdot N_{oo} + \frac{1}{c \cdot \left[\frac{(T_o + 273)}{T + 273} \right] \cdot g_o \cdot \left(\frac{c}{n} \right) \cdot \left[\left[\frac{T + 273}{T_o + 273} \right] \cdot \sigma_{io} + \left(\frac{1}{0.04} \right) \cdot \ln \left(\frac{1}{0.32} \right) \right]} \Gamma \cdot n} \right] \\
 N_x \leftarrow x \cdot \left[\frac{T + 273}{T_o + 273} \cdot N_{oo} + \frac{1}{c \cdot \left[\frac{(T_o + 273)}{T + 273} \right] \cdot g_o \cdot \left(\frac{c}{n} \right) \cdot \left[\left[\frac{T + 273}{T_o + 273} \right] \cdot \sigma_{io} + \left(\frac{1}{0.04} \right) \cdot \ln \left(\frac{1}{0.32} \right) \right]} \Gamma \cdot n} \right] \\
 td1 \leftarrow N_1 \cdot \ln(1 - |N - N_2|) - 0.5 N_1 \cdot \ln \left[(N + 0.66 N_2)^2 + (1 \cdot N_2)^2 \right] - 2 \cdot 0.8 N_1 \cdot \text{atan2}(N + 0.66 N_2, -1 \cdot N_2) \\
 tdx \leftarrow N_1 \cdot \ln(1 - |N_x - N_2|) - 0.5 N_1 \cdot \ln \left[(N_x + 0.66 N_2)^2 + (1 \cdot N_2)^2 \right] - 2 \cdot 0.8 N_1 \cdot \text{atan2}(N_x + 0.66 N_2, -1 \cdot N_2) \\
 (td1 - tdx) \cdot 10^9
 \end{array} \right.
 \end{aligned}$$

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الخلاصة:

في هذا البحث, تأثير درجة حرارة العمل على التأخير الزمني لبدء التشغيل لأشباه الموصلات الليزرية, قد اعيد دراسته نظريا. اقترحنا صيغه لحساب الاعتماديه الحرارية لزمان بدء التشغيل بموجب الاعتمادية الحراريه لبارمترات الفجوة الليزرية. تم اشتقاق صيغه تحليلية دقيقه لحساب التأخير الزمني لبدء التشغيل كداله لدرجة الحرارة من خلال الاعتماديه الحراريه لكثافة الحاملات عند مستوى العتبة والتي تم حسابها اعتمادا على الاعتماديه الحراريه على بارمترات الليزر وليس اعتمادا على علاقه بانكوف الاسيه المعروفه لدرجة الحرارة المميزة والتيار المميز. نتائج المحاكاة للبحث اظهرت توافق كبير بين قيم زمن بدء التشغيل المحسوبه بواسطه الصيغه المقترحه وتلك المحسوبه عن طريق الطريقه العدديه. ايضا اعدنا اشتقاق علاقات تقريبية للتأخير الزمني لبدء التشغيل حسب الصيغه المقترحه.