Performance Optimization of Multi-Quantum Wells Laser Used in Optical Communications

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In this work, an analytical treatment of some design parameters of multi-quantum well semiconductor laser aiming to optimize the performance of such systems is presented. The treatment concentrates on three main parameters; the reflectivity of the front mirror, the effective lifetime of the photon inside laser cavity, and the number of quantum wells in the laser structure. These three parameters relate to several other parameters which impose to achieve design compensation. Such compensation can be simulated by analytical treatments because the characteristics of these lasers have been established well in form of small and large signal solutions. These lasers are increasingly studied due to their characteristic features in optical communications architectures, low-dimensioned photonics and optoelectronics devices.

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1. Introduction

Semiconductor lasers are excellent optical sources for several applications including: high-speed optical time-division multiplexed (OTDM) communication systems, optical clocking and sampling, etc [1]. When a voltage is applied to a degenerated *pn* junction device, considerable electrons and holes are injected into the transition region. If the bias is large enough, the transition region contains a high concentration of conduction band electrons and a high concentration of valence band holes. In other words, a population inversion exists around the junction. This population inversion region is also called the active region.

Another condition to sustain continuous laser operation from the device is the optical cavity. Its main function is to implement a laser oscillator, or to build up the intensity of stimulated emissions by means of an optical resonator. Since only multiples of the half-wavelength can exist in an optical cavity, the radiation wavelength that can build up in the cavity is determined by the cavity length *L*,

$$L = M\frac{\lambda_0}{2n} \quad (1)$$

where M is an integer, λ_0 is the free space wavelength, and n is the refractive index of the semiconductor. The resonant frequency of the cavity, i.e., a mode of cavity, satisfies the above relationship.

The device described thus far is a homo-junction laser, since both *p* and *n* regions are fabricated with the same semiconductor. The multiple quantum well structure schematically sketched in Fig. (1) extends the advantages of the single quantum well laser [1].

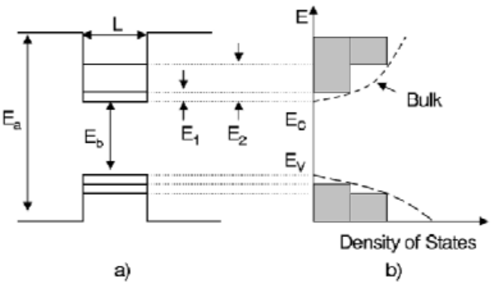

Fig. (1) (a) Schematic energy diagram of a semiconductor QW structure. E_a and E_b are the energy gaps that make up the barriers and well. E_1 and E_2 are the first two energy states confined in the well of width L. (b) Density of States in a quantum well and bulk material. The dashed curve represents the 3D density of states for bulk material while the solid line is the 2D density states for a particle confined to a quantum well [1]

The introduction of compressive strain into the multiple quantum well lasers leads to low internal loss, high quantum efficiency, low threshold current operation, and small line-width enhancement factors [2-4]. This configuration forms a rectangular quantum potential well from the energy bands. When the thickness of the narrow bandgap material, L, is on the order of the de Broglie wavelength of a thermalized electron ($\lambda=h/p$, where h is Planck's constant and p is the momentum of the electron), quantum size effects occur forming quantized energy states within the well [5]. These states represent the probability of having an electron or hole in any respective position within the well and are found from the solution to the time independent Schrodinger wave equation in two dimensions.

2. The Model

Laser action and optical gain occur in a semiconductor because of the non-equilibrium of carriers that populate the bands. This state is attained as described above.

The absorption of incident photons is given by the absorption rate as

$$r_{abs} = P[1 - f_n(E_2)] f_p(E_1) N_p(E) \quad (2)$$

While the stimulated emission rate is given as

$$r_{st} = P f_n(E_2)[1 - f_p(E_1)] N_p(E) \quad (3)$$

where P is the transition probability, N_p is the density of photons of energy E, and f_n and f_p are the Fermi functions. For lasing to occur, there must be more stimulated radiative processes than photon absorption processes, thus $r_{st}>r_{abs}$. This condition leads to $f_n(E_n)>f_p(E_p)$ where E_n-E_p is the transition energy from the conduction band to the valence band. This condition causes population inversion: where the concentration of electrons in the conduction band is greater than the concentration of holes in the valence band. The gain, represented by stimulated downward transitions of carriers, should be at least equal to the losses, stimulated upward transitions, for lasing to occur [2].

Longitudinal modes are formed along the length of the active region parallel to the semiconductor layers. These modes define laser resonance. This is the process whereby photons generated in the active region oscillate along the length perpendicular to the cleaved "mirror" facets producing gain and thus laser emission. The cavity is resonant because only certain frequencies (wavelengths) of light are allowed within it. These resonant frequencies are governed by the cavity length d. The appropriate choice will stimulate a certain wavelength while damping all others. This condition is

$$d = \frac{q\lambda}{2n} \qquad q = 1,2,3,... \quad (4)$$

where λ is the wavelength, and n is the effective index of refraction within the active region. The resonant frequency follows from this expression as

$$v_q = \frac{qc}{2nd} \qquad q = 1,2,3,... \quad (5)$$

where c is the speed of light. These frequencies are the longitudinal modes which are solutions to the Helmholtz equation with appropriate boundary conditions.

Rate equations for each longitudinal mode, λ, with photon density (S_λ) and carrier density (N_λ), which couple into this mode, are:

$$\frac{\partial N_\lambda}{\partial t} = \frac{J_\lambda}{q} - B_\lambda N_\lambda^2 - \frac{N_\lambda}{2\tau_0} + \sum_k \frac{N_\lambda}{\tau_{k,\lambda}} - \sum_k \frac{N_\lambda}{\tau_{k\lambda,\lambda}} - \frac{\partial S_\lambda}{\partial x}\frac{\partial x}{\partial t} \quad (6)$$

$$\frac{\partial S_\lambda}{\partial t} = \beta_\lambda B_\lambda N_\lambda^2 - \frac{S_\lambda}{\tau_{ph,\lambda}} + \frac{\partial S_\lambda}{\partial x}\frac{\partial x}{\partial t}, \qquad \lambda = 1,2,...,\lambda_{\max} \quad (7)$$

We will consider one mode with photon density (S), whose photon energy is closest to the peak gain, rather than using set of differentia; equations for all waveguide modes. The intensity of this mode will grow faster than all others and eventually dominate [8]. This simplification avoids the problem of finding the parameters and coefficients for every single mode. On the other hand, it does no enable to calculate the emission spectrum of the laser diode. For a single longitudinal mode, the rate equations reduce to:

$$\frac{\partial N}{\partial t} = \frac{J}{q} - BN^2 - \frac{N}{2\tau_0} - v_{gr}\Gamma l(N - N_{tr})S \quad (8)$$

$$\frac{\partial S}{\partial t} = \beta BN^2 - \frac{S}{\tau_{ph}} - v_{gr}\Gamma l(N - N_{tr})S \quad (9)$$

$$P_L = -v_{gr} SW \ln\frac{1}{\sqrt{R_1}} \quad (10)$$

Here, the photon lifetime (τ_{ph}) is related to the quality factor (Q) of the laser cavity as [6]:

$$\tau_{ph} = \frac{\lambda Q}{2\pi c} \quad (11)$$

Assuming a time harmonic solution and ignoring higher order terms, the rate equations become [7]

$$j\omega n_L = \frac{j_L}{q} - \frac{n_L}{\tau_{eff}} - v_{gr}\Gamma l(N - N_{tr})s_L - v_{gr}\Gamma n_L l S_0 \quad (12)$$

$$j\omega s_L = \frac{s_L}{\tau_{ph}} + v_{gr}\Gamma l(N - N_{tr})s_L + v_{gr}\Gamma n_L l S_0 \quad (13)$$

where τ_{eff} is the same as for LED. Using the following

$$\frac{1}{\tau_{ph}} = v_{gr}\Gamma l(N - N_{tr}) \quad (14)$$

These equations can be solved yielding

$$j_L = j\omega q n_L + q n_L\left(\frac{1}{\tau_{ph}} + v_{gr}\Gamma l S_0\right) + q n_L \frac{v_{gr}\Gamma l S_0}{j\omega \tau_{ph}} \quad (15)$$

Replacing n_1 by relating it to the small signal voltage v_1

$$v_L = \frac{m V_t n_L}{N_0} \quad (16)$$

The equation for the small signal current, i_1, can be written as

$$i_L = \left(j\omega C + \frac{1}{R} + \frac{1}{j\omega L}\right) v_L \quad (17)$$

where $C = \dfrac{qN_0 A}{mV_t}$, A is the area of the laser diode and

$$m = \frac{N_0 e^{N/N_c}}{N_c(e^{N/N_c} - 1)} + \frac{N_0 e^{N/N_v}}{N_v(e^{N/N_v} - 1)} \quad (18)$$

$$\frac{1}{R} = C(\frac{1}{\tau_{eff}} + \Gamma l S_0 v_{gr}) \qquad (19)$$

and

$$L = \frac{1}{C}\left(\frac{\tau_{ph}}{\Gamma l S_0 v_{gr}}\right) \qquad (20)$$

Adding parasitic elements and the circuit described by the Eq. (16) we obtain the following equivalent circuit, where L_B is a series inductance, primarily due to the bond wire, R_S is the series resistance in the device and C_p is the parallel capacitance due to the laser contact and bonding pad [8].

The resistor R_d, in series with the inductor, L, is due to gain saturation [5] and can be obtained by adding a gain saturation term to Eq. (16). The optical output power is proportional to the current through inductor L, which is given by:

$$i_L = \frac{qAs}{\tau_{ph}} = qAsv_{gr}(\alpha + \frac{1}{L_C}\ln\frac{1}{\sqrt{R_1 + R_2}}) \qquad (21)$$

and the corresponding power emitted from mirror R_1

$$P_L = sh\nu W v_{gr} \ln\frac{1}{\sqrt{R_1 + R_2}} \qquad (22)$$

Ignoring the parasitic elements and the gain saturation resistance, R_d, we can find the AC responsivity P_L/i_L as:

$$\frac{P_L}{i_L} = \frac{h\nu}{q}\frac{\ln\frac{1}{\sqrt{R_1}}}{\ln\frac{1}{\sqrt{R_1+R_2}} + \alpha L_C}\frac{1}{1 + j\omega\frac{L}{R} + (j\omega)^2 LC} \qquad (23)$$

From which we find the relaxation frequency of the laser

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{\Gamma S_0 l v_{gr}}{\tau_{ph}}} = \sqrt{\frac{\Gamma l P_0}{h\nu\tau_{ph} W \ln\frac{1}{\sqrt{R_1}}}} \qquad (24)$$

or the relaxation frequency is proportional to the square root of the DC output power. The amplitude at the relaxation frequency relative to that at zero frequency equals [8]

$$\frac{P_L|_{\omega=\omega_0}}{P_L|_{\omega=0}} = \frac{R}{L\omega_0} = \frac{1}{\frac{1}{\omega_0\tau_{eff}} + \omega_0\tau_{ph}} \qquad (25)$$

Comparing threshold currents of laser diodes with identical dimensions and material parameters but with a different number of quantum wells (m), we can find that the threshold currents are not simple multiples of that of a single quantum well laser.

Assuming that the modal gain (g) is linearly proportional to the carrier concentration in the wells and that the carriers are equally distributed between the m wells. For m quantum wells, the modal gain can be expressed as:

$$g = lm(N - N_{tr}) = lm\Delta N \qquad (26)$$

where l is the differential gain coefficient and N_{tr} is the transparency carrier density.

Since the total modal gain is independent of the number of quantum wells, we can express the carrier density as a function of modal gain at lasing [6]:

$$N = \frac{g}{lm} + N_0 = \frac{\Delta N}{m} \qquad (27)$$

Then the radiative recombination current at threshold is [8]

$$J_{th} = qB_1 m(N_{tr} + \frac{\Delta N}{m})^2 = qB_1(N_{tr}^2 m + 2N_{tr}\Delta N + \frac{\Delta N^2}{m}) \qquad (28)$$

where B_1 is the quantum-well radiative recombination rate coefficient

This means that the threshold current density is a constant plus a component which is proportional to the number of quantum wells. The last terms can be ignored for m>>1 and ΔN<<N_{tr}.

3. Results and Discussion

Figure (2) shows the variation of the quantum efficiency with the reflectivity of the front mirror of laser resonator at different cavity lengths (L). It is clear that the efficiency decreases with increasing reflectivity as well as increasing cavity length.

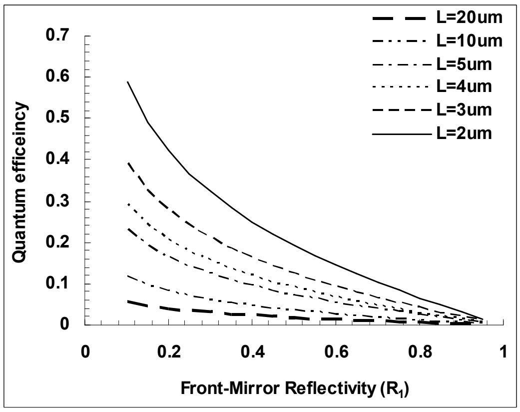

Fig. (2) Variation of quantum efficiency with the reflectivity of output mirror for different lengths of laser cavity (*L*)

Therefore, efficient lasers are obtained by reducing the waveguide losses, increasing the reflectivity of the back mirror, decreasing the reflectivity of the front mirror and decreasing the length of the cavity. Decreasing the reflectivity of the mirror also increases the threshold current and is therefore less desirable. Decreasing the cavity length at first decreases the threshold current but then rapidly increases the threshold current.

Figure (3) shows the variation of amplitude of laser power at relaxation frequency to that at zero frequency with the relaxation frequency for two different cases. The difference between values for two cases is too small except the deviation toward larger difference at the lower relaxation frequencies (ω_0), when we suppose that the effective lifetime (τ_{eff}) coincides the photon lifetime (τ_{ph}), which is in turn an ideal case. As the effective lifetime being longer, the photon inside laser cavity has much more chances to stimulate the laser active medium to produce much more photons, hence much more power. This can be achieved by maximizing the photon lifetime throughout two main parameters; the first is minimizing the losses inside cavity, and the second is optimizing both cavity length and front mirror reflectivity.

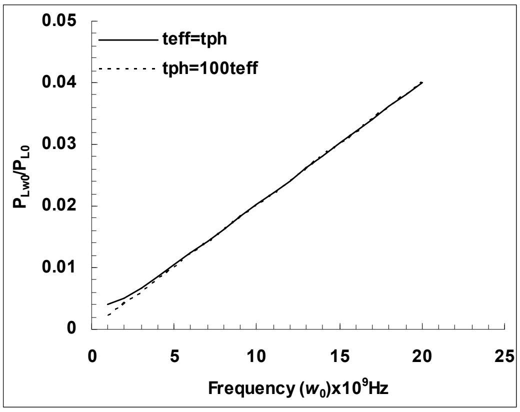

Fig. (3) Variation of amplitude of laser power at relaxation frequency to that at zero frequency with the relaxation frequency for two different cases: equal photon and effective lifetimes (continuous line) and effective lifetime shorter by two orders of magnitudes than the photon lifetime (dashed line)

Figure (4) shows the variation of threshold current density (J_{th}) with the number of quantum wells in the laser structure (m). According to Eq. (28) and Fig. (4), the threshold current density is a constant plus a component which is proportional to the number of quantum wells. Threshold current density increases fast as the number of quantum wells does within hundreds range, whilst it keeps increasing slowly as the number m keeps increasing through thousands. Due to structural reasons, there is a limit for the number of quantum wells inside laser structure as the latter can terminate over such limits. According to Eqs (18) and (26), we would work to increase the gain by increasing the carrier density but this procedure imposes much more wells (higher m), which increases the threshold current density required for the optimum performance of laser device. On the other side, decreasing carrier density imposes decreasing gain (g) as well as decreasing threshold current density as the number of quantum wells (m) will decrease too. Therefore, compensation in all parameters mentioned above should be taken into account to optimize the performance of the supposed laser design.

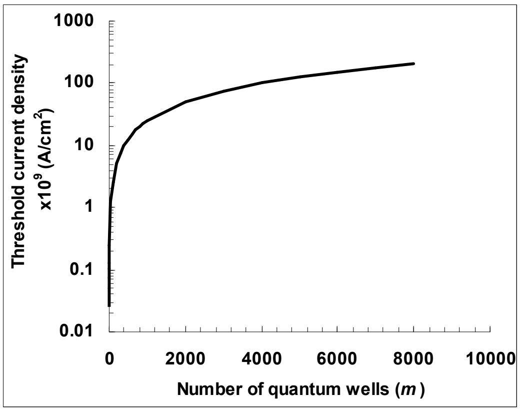

Fig. (4) Variation of threshold current density (J_{th}) with the number of quantum wells in the laser structure (m)

4. Conclusions

From the results of analytical treatments presented in this study, the optimum performance of multi-quantum wells lasers is determined by several physical and engineering considerations. These considerations can be reduced to some parameters those are controlled by earlier design steps. Both cavity length and front mirror reflectivity are controlled to determine the quantum efficiency of laser system, hence, the effective lifetime is controlled to maximize the laser power at the relaxation frequency. As well, the number of quantum wells in the laser structure is a parameter to control threshold current density.

References

[1] J. Mulet, M. Kroh and J. Mørk, *Opt. Express*, 14(3) (2006) 1119-1124.
[2] U. Koren et al., *Appl. Phys. Lett.*, 51 (1987) 1744-1746.
[3] A. Kasukawa, I. Imajo and T. Makino, *Electron. Lett.*, 25 (1989) 104-105.
[4] T.L. Koch et al., *Electron. Lett.*, 24 (1988) 1431-1432.
[5] N.J. Holonyak, R.M. Koblas and R.D. Dupuis, *IEEE J. Quantum Electron.*, QE-16(2) (1980) 170-185.
[6] J. Kaiser et al., *IEEE J. Selected Topics in Quantum Electronics*, 10(5) (2004) 968-973
[7] K.A. Williams et al., *IEEE Journal of Selected Topics in Quantum Electronics*, 5 (1999) 822-31.
[8] S. Sujecki et al., *IEE Proc. Optoelectronics*, 150(3) (2003) 246-252.

