

## State Estimation of Direct Field Orientation Control Induction Motor Drive by Using Kalman Filter

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### ABSTRACT

Field oriented control (FOC) method of an induction motor (IM), allows high performance speed and torque response to be achieved from an induction motor. The present work selects the direct field orientation control induction motor (DFOCIM) as an effective method for eliminating the coupling effect between torque and flux. In the present work, two schemes of estimators have been used, standard kalman filter (SKF) and Extended kalman filter (EKF) for estimating stator current, rotor flux and rotor speed. The performance comparison is based on evaluation of estimation error of both estimators. The performances of suggested estimators are assessed in terms of tracking performance and noise rejection capability.

**Keywords:** Direct Field Orientation Control (DFOC); Standard Kalman Filter (SKF); Extended kalman filter (EKF).

تخمين المعطيات للمحرك الحثي ذو سيطرة توجيه المجال المغناطيسي  
المباشر بأستخدام مرشحات الكالمان

### الخلاصة

تحقق سيطرة توجيه المجال المغناطيسي الاداء العالي لاستجابة السرعة والعزم في المحرك الحثي . في العمل الحالي تم اختيار سيطرة توجيه المجال المغناطيسي المباشر كأحد الطرق الفعالة لازالة التأثير المتبادل بين العزم والفيض المغناطيسي للمحرك الحثي وتم استخدام نوعين من التخمين , Standard Kalman Filter (SKF) و Extended Kalman Filter (EKF) لتخمين تيار الجزء الثابت وفيض الجزء الدوار وسرعة الجزء الدوار للمحرك الحثي : وتم مقارنة الاداء بتقدير الخطا للنوعين وتم تقييم الاداء من ناحية تتبع الاداء وكذلك قابلية رفض الضوضاء.

## INTRODUCTION

Field orientation control is already the industrial standard for high performance induction motor drives. In practice, both the indirect field oriented control and the direct field oriented control methods require flux estimation from the motor terminal variables [1]. Induction motor can produce good performance using field oriented control technique. The main idea of the vector control is the control of the torque and the flux separately. In order to decouple the vectors and realize decoupled control most control schemes require accurate flux and motor rotor speed. These formations are provided by a Hall sensors and sensing coils (flux measurement) and incremental encoder (rotor speed measurement). The use of these sensors implies more electronics, high cost, and lower reliability, difficulty in mounting in some cases such as motor drives in harsh environment and high speed drives, increase in weight, increase in size and increase electrical susceptibility [2, 3]. To overcome these problems, in recent years, the elimination of these sensors has been considered as an attractive prospect. The rotor speed and flux are estimated from machine terminals proprieties such as stator currents or voltage as EKF [4, 5]. In this paper, two observes are suggested; SKF and EKF for estimating stator current, rotor flux and rotor speed.

## KALMAN FILTER

Kalman filter directly accounts for the effects of the disturbance noises of a control system and the errors in the parameters will also be handled as noise. The Kalman filter is implemented by the following equations, the system being expressed as a state model

### Kalman filter in Continuous time

Consider the definite stochastic model by the following differential equations [6, 7]:

$$\left. \begin{aligned} \dot{X}(t) &= AX(t) + BU(t) + W(t) \\ Y(t) &= CX(t) + V(t) \end{aligned} \right\} \quad \dots(1)$$

The terms  $W(t)$  and  $V(t)$  are, respectively, the process and measurement noises which are assumed uncorrelated and their covariance matrices are  $Q$  and  $R$ . Their average values are null:

$$\left\{ \begin{aligned} E[W(t)] &= 0 \\ E[V(t)] &= 0 \end{aligned} \right. \quad \dots(2)$$

Their autocorrelations are expressed by:

$$\left\{ \begin{aligned} E[W(t_1)W(t_2)^T] &= Qd(t_2-t_1) \\ E[V(t_1)V(t_2)^T] &= Rd(t_2-t_1) \end{aligned} \right. \quad \dots(3)$$

E: the expectation,  $d(t)$  is function a impulse of Dirac, the matrices  $Q$  and  $R$ , definite nonnegative, is symmetrical and has the spectral concentrations of average power  $W(t)$  and  $V(t)$  [8,9];

They are characterized by the absence of correlation enters  $W(t)$  and  $V(t)$  :

$$E [W(t_1)V(t_2)^T] = 0 \quad \dots(4)$$

And between the noises and the initial state:

$$E [W(t)X_0(t)^T] = E [V(t)X_0(t)^T] \quad \dots(5)$$

One can write in continuous time the system of equations according to

$$\begin{cases} \dot{\hat{X}}(t) = A\hat{X}(t) + BU(t) + K(t)(Y(t) - \hat{Y}(t)) \\ \hat{Y}(t) = C\hat{X}(t) \end{cases} \quad \dots(6)$$

The dynamics error of observation  $e = X(t) - \hat{X}(t)$  is defined, from (1) and (6) according to:

$$\dot{e} = \dot{X}(t) - \dot{\hat{X}}(t) = (A - K(t)C)e + W(t) - K(t)V(t) \quad \dots(7)$$

When the preceding assumptions (2) and (3) are checked, the optimal gain of the kalman filter is given by:

$$K(t) = AP(t)C^T R(t)^{-1} \quad \dots(8)$$

The covariance of the error in estimation  $P(t) = E[ee^T]$  is the solution of the following equation:

$$\dot{P}(t) = AP(t) + P(t)A^T + Q - K(t)RK(t)^T \quad \dots(9)$$

### Kalman filter in discrete time

In this section, we will present two types of kalman filter in discrete time, in first, the discrete kalman filter for the linear systems (Standard Kalman Filter (SKF) and then it's extended for the nonlinear systems, (Extended Kalman Filter (EKF))

2-2-1 standard kalman Filter (SKF)

In our case, the kalman filter is used for the estimate of the vector of state made up  $X_k$  of the stator currents and rotor flux in the model  $(a, b)$ , by introducing the discrete noises  $W_k$  and  $V_k$ , on the state and the exit [9, 10]:

$$\begin{cases} X_{k+1} = A_k X_k + B_k U_k + W_k \\ Y_k = C_k X_k + V_k \end{cases} \dots( 10 )$$

The discrete noises must check the same assumptions as the continuous noises. We will suppose them stationary:

$$\begin{cases} E[W_k] = 0 \\ E[V_k] = 0 \\ E[W_k V_k^T] = 0 \end{cases} \quad \begin{cases} E[W_k W_k^T] = Q d_{k1} \\ E[V_k V_k^T] = R d_{k1} \end{cases} \dots( 11 )$$

$d_{k1}$  : is the Kronecker data function, which is worth 1 if  $K=1$ .  $Q$  and  $R$  are respectively, the covariance's matrices of the noises  $W_k$  and  $V_k$ . Then the procedure of estimate thus breaks up into two stages [3, 5, 11]

**a stage of prediction:**

$$\hat{X}_{k+1/k} = A_{k,k} \hat{X}_{k/k} + B_{k,k} U_k \dots ( 12 )$$

This stage makes it possible to build a first estimate of the vector of state at the moment  $k+1$ . One then seeks to determine his variance:

$$P_{k+1/k} = A_{k,k} P_{k/k} A_{k,k}^T + Q_k \dots( 13 )$$

When:

$Q_k$  : Discrete covariance matrices of the noises of state.

Thus, this measurement of the state makes it possible to predict the exit:

$$\hat{Y}_{k+1/k} = C_{k+1} \hat{X}_{k+1/k} \dots ( 14 )$$

**a stage of correction**

In fact, the stage of prediction makes it possible to have a difference between the measured exit  $Y_{k+1}$  and the predicted exit  $\hat{Y}_{k+1/k}$ . To improve the state, it is thus necessary to hold account of this variation and to correct it by the intermediary of the kalman filter gain  $K_{k+1}$ . By minimizing the variance of the error, one obtains the expression of the new vector of state

$$\hat{X}_{k+1/k+1} = \hat{X}_{k+1/k} + K_{k+1} (Y_{k+1} - \hat{Y}_{k+1/k}) \dots( 15 )$$

The kalman filter gain  $K_{k+1}$  is given starting from the matrix of covariance  $P_{k+1/k}$  and the matrix of covariance of the noises of discrete measurement  $R_k$  :

$$K_{k+1} = P_{k+1/k} C_k^T (C_k P_{k+1/k} C_k^T + R_k)^{-1} \quad \dots(16)$$

And

$$P_{k+1/k+1} = (I_5 - K_{k+1} C_k) P_{k+1/k} \quad \dots(17)$$

In our simulation, the standard kalman filter in discrete time is used for the estimate of the vector of state made up of the stator currents ( $I_{sa}, I_{sb}$ ) and of rotor flux ( $\Phi_{ra}, \Phi_{rb}$ ), the linear model of the IM is discretized by a Taylor series application in order Two:

$$\begin{cases} X_{k+1} = A_k X_k + B_k U_k \\ Y_k = C_k X_k \end{cases} \quad \dots(18)$$

When

$$\begin{cases} A_k = \exp(A(P\Omega)T_e) \approx I_4 + A(P\Omega)T_e + \frac{(A(P\Omega)T_e)^2}{2} \\ B_k = (A(P\Omega))^{-1}(A_k - I_4)B \approx T_e(I_4 + \frac{A(P\Omega)T_e}{2})B \\ C_k = C \end{cases} \quad \dots(19)$$

$$A(P\Omega) = A(w) = \begin{bmatrix} -\frac{1}{t'_s} & 0 & \frac{K_r}{t'_s R_S t_r} & \frac{K_r}{t'_s R_S} P\Omega \\ 0 & -\frac{1}{t'_s} & -\frac{K_r}{t'_s R_S} P\Omega & \frac{K_r}{t'_s R_S t_r} \\ \frac{M}{t_r} & 0 & -\frac{1}{t_r} & -P\Omega \\ 0 & \frac{M}{t_r} & P\Omega & -\frac{1}{t_r} \end{bmatrix};$$

$$B = \begin{bmatrix} \frac{1}{t'_s R_S} & 0 \\ 0 & \frac{1}{t'_s R_S} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$t'_S = s \frac{L_S}{R_S} \quad K_r = \frac{M}{L_r} \quad R_S = R_S + K_r^2 \cdot R_r \quad t_r = \frac{L_r}{R_r} \quad s = 1 - \frac{M^2}{L_S L_r}$$

The structure of discrete standard kalman filter can be put in the form of the figure (1):

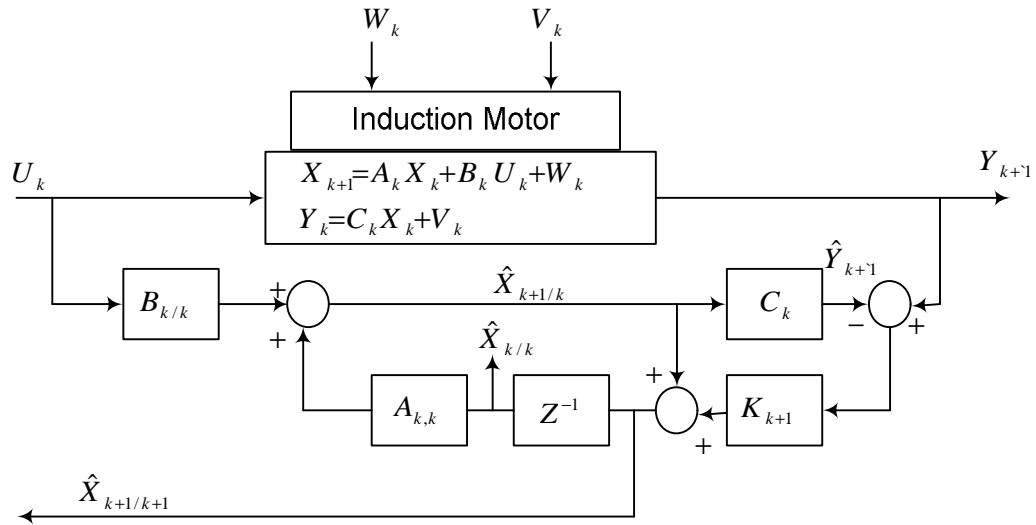


Figure (1) discrete standard kalman filter [8, 11].

**Extended Kalman Filter (EKF)**

The standard kalman filter describes previously, allows the estimate of the state of a linear system. If one wants to estimate the speed in induction motor, a solution consists in extending the vector of state estimated with the speed in IM. The model becomes nonlinear then. The EKF is given by the following equation [5, 12]

$$\begin{cases} X^e_{k+1} = F_k(X^e_k, U^e_k) + W_k = A^e_k X^e_k + B^e_k U^e_k + W_k \\ Y^e_k = H_k(X^e_k) + V_k = C^e_k X^e_k + V_k \end{cases} \dots(20)$$

When:

$$\begin{cases} A^e_k = \frac{dF}{dX^e_k} \Big|_{X^e_k = \hat{X}^e_k} \\ B^e_k = \frac{dF}{dU^e_k} \Big|_{X^e_k = \hat{X}^e_k} \\ C^e_k = \frac{dH}{dX^e_k} \Big|_{X^e_k = \hat{X}^e_k} \end{cases} \dots\dots( 21 )$$

The Extended discrete noises are white, Gaussian and of null average. These noises are defined by their matrices of covariance ( $Q^e_k, R^e_k$ ). The prediction of the state as well as the matrix of covariance of the filter are given by the two following equations

$$\hat{X}^e_{k+1/k} = A^e_{k,k} \hat{X}^e_{k/k} + B^e_{k/k} U^e_k \dots\dots( 22 )$$

$$P^e_{k+1/k} = A^e_{k,k} P^e_{k/k} A^e_{k/k}{}^T + Q^e_k \dots\dots( 23 )$$

When:

$Q^e_k$  : Discrete covariance matrices of the noises of state the index "e" represent the extended vectors.

The kalman filter gain  $K^e_{k+1}$  is calculated by the following equation:

$$K^e_{k+1} = P^e_{k+1/k} C^e_k{}^T (C^e_k P^e_{k+1/k} C^e_k{}^T + R_k)^{-1} \dots\dots(24)$$

$R_k$  : Discrete covariance matrix of the noises of measurement:

Knowing that our filter is recursive, the reactualization of the matrix of covariance of the filter is given by:

$$P^e_{k+1/k+1} = (I_5 - K^e_{k+1} C^e_k) P^e_{k+1/k} \dots\dots( 25 )$$

Finally, the estimate of the state is given by:

$$\hat{X}^e_{k+1/k+1} = \hat{X}^e_{k+1/k} + K^e_{k+1} (Y_{k+1} - C^e_k \hat{X}^e_{k+1/k}) \dots\dots( 26 )$$

**Total Structure of the direct field orientation control induction motor provided with stochastic observer by using Kalman filter**

The basic structure of the direct field orientation control induction motor (DFOCIM) [13] based kalman filters are shown in figure (2) and figure (3). Figure (2) shown the interconnection between SKF and DFOCIM. It's clear that SKF needs actual speed for estimating machine parameter. The block diagram in figure (3) including EKF as estimator. It's clear in the figure that no need to actual speed

and the speed is estimated inside the EKF block. The two types of stochastic observers (SKF and EKF) are simulated using Matlab/Simulink.

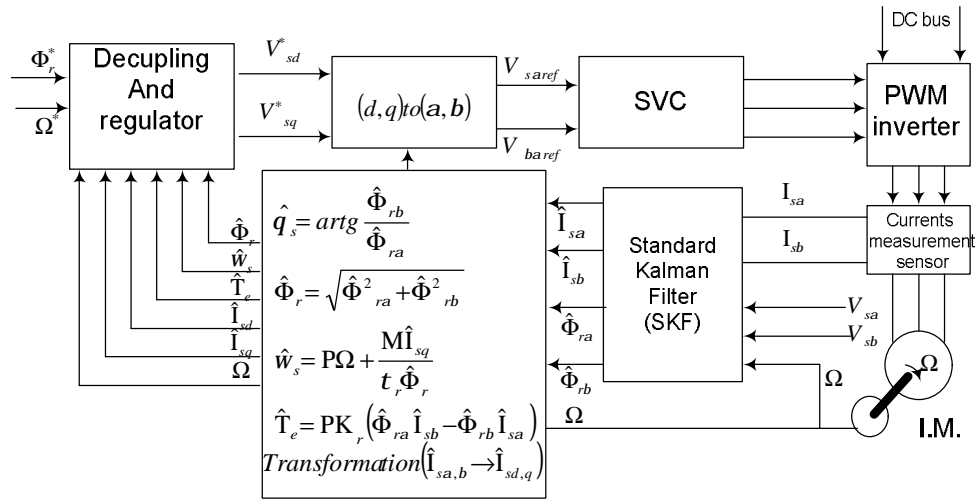


Figure (2) structure of the (DFOCIM) with (SKF)

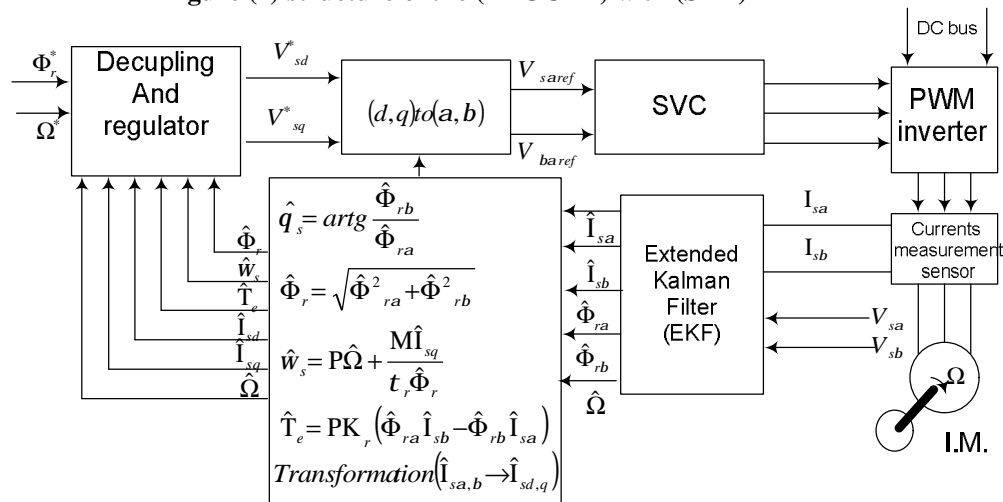


Figure (3) structure of the (DFOCIM) with (EKF).

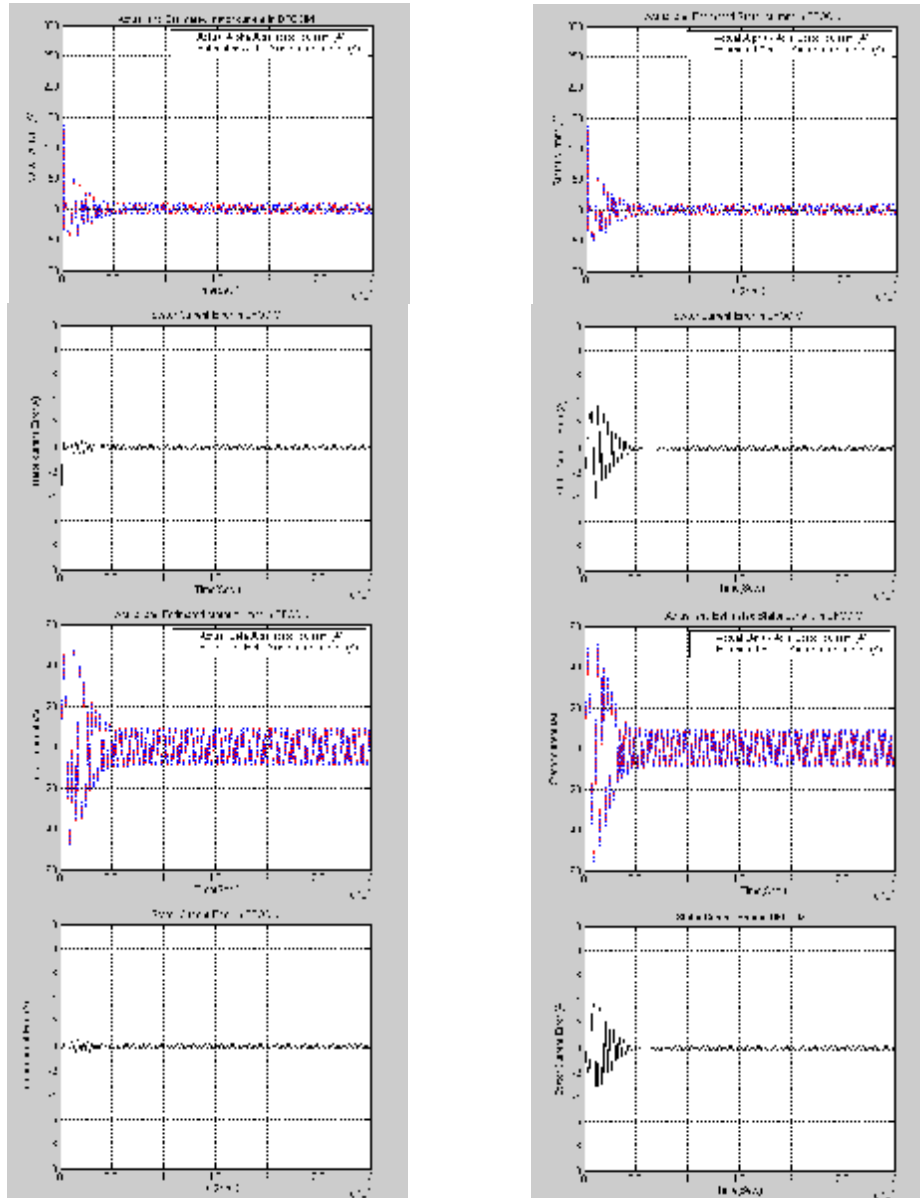
**Simulated Results of the direct field orientation control induction motor provided with stochastic observer by using Kalman filter**

**No load Response**

In figure (4) present the actual, estimated and error in stationary frame (a, b) for stator current and rotor flux at the reference speed 1000tr/min when no load in exerted on rotor shaft. The simulated results show the actual and estimated states



for two types of observers of kalman filter (SKF and EKF), at the no load with a reference speed of 1000 tr/min, are presented on figure. It is evident from figure that EKF give less steady state estimation error. However, SKF shows good transient characters; since it relies on actual speed measurement show in figure (2).



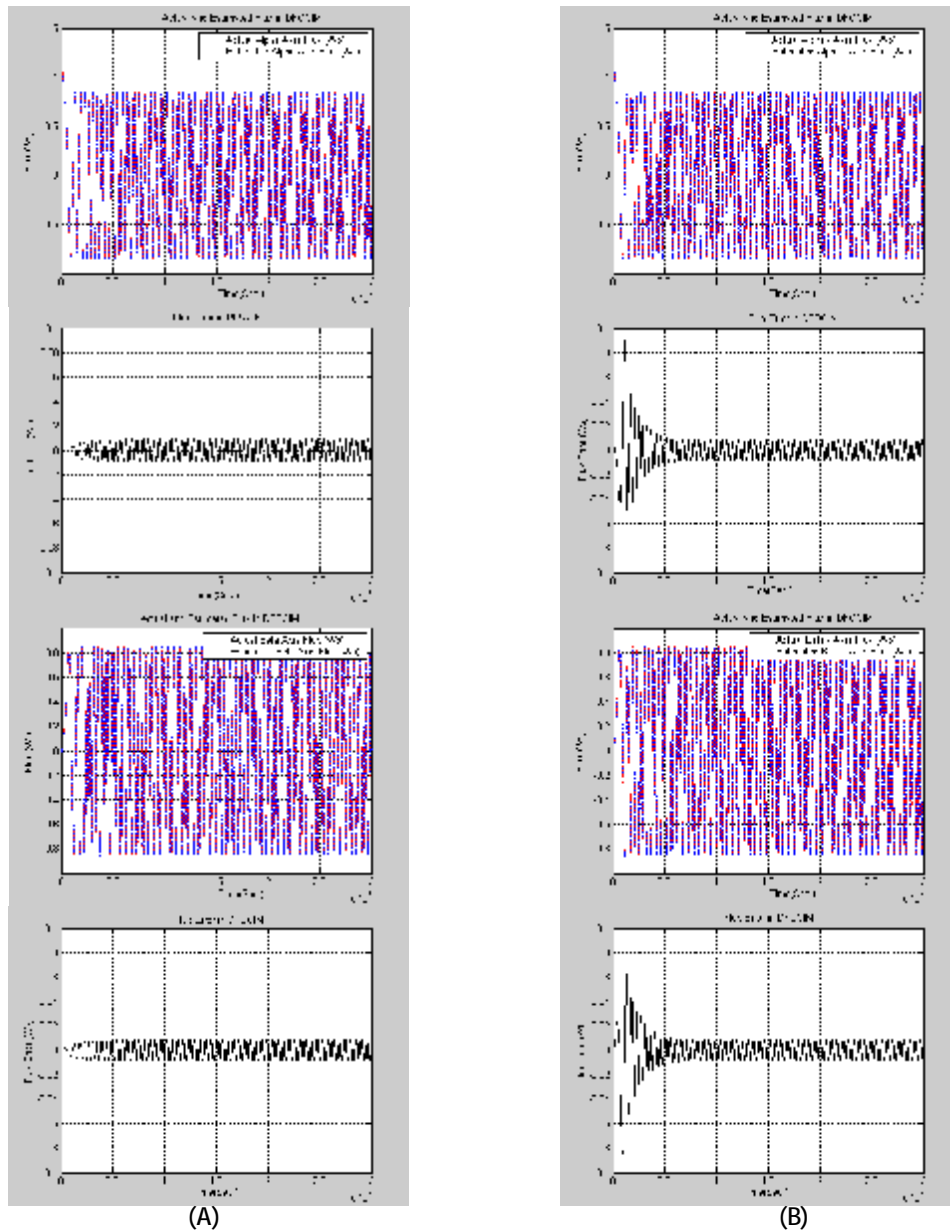
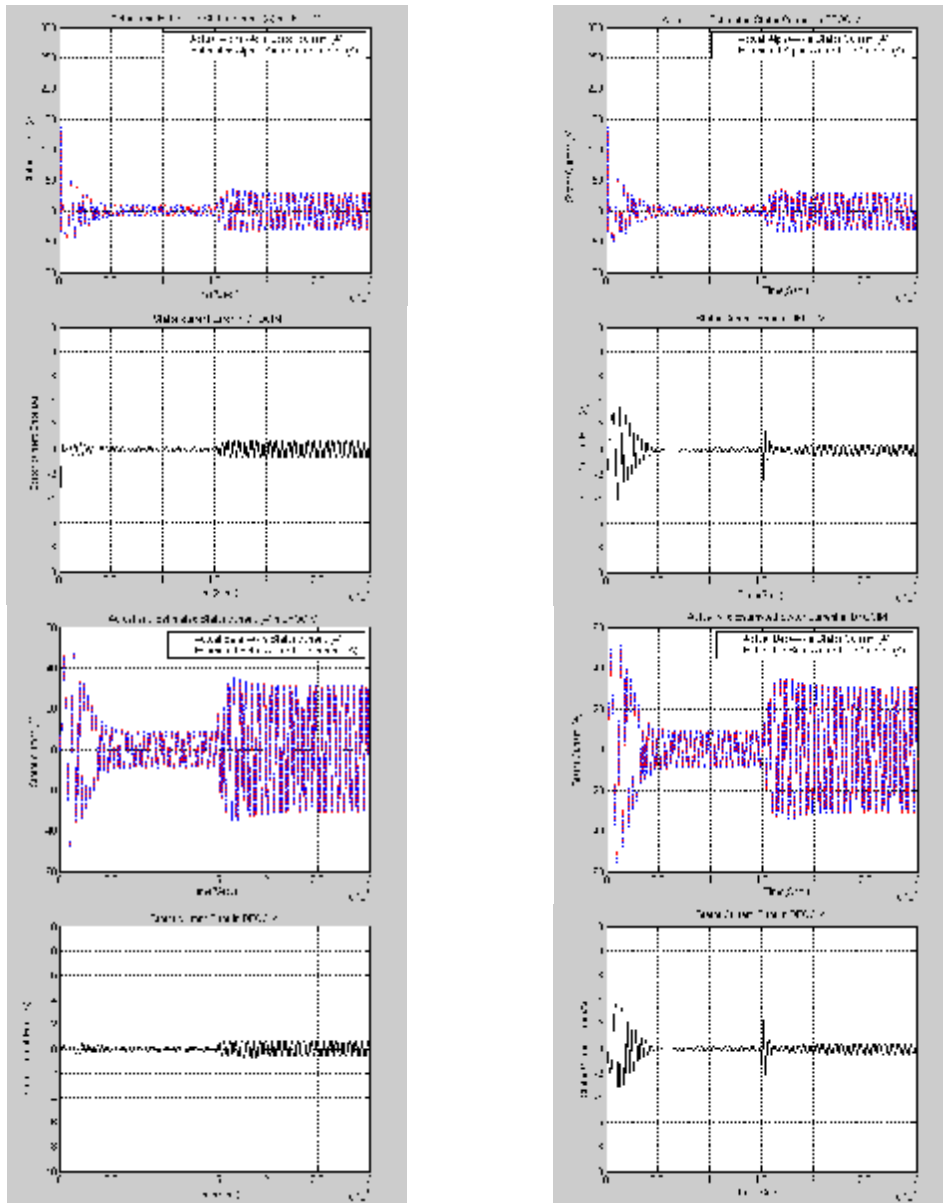


Figure (4) Simulation Results obtained with the two observers of kalman filter and error for a speed of reference 1000 tr/min (No-load)

- A- SKF
- B- EKF

**Load Response**

In figure (5) present the actual, estimated and error in stationary frame ( $a, b$ ) for stator current and rotor flux by two observer of Kalman filter (SKF, EKF) when load Torque of high 50Nm is applied at time 1.5 Sec to 3 Sec. It is clear that estimation error based on EKF shows less sensitive to load changes in steady state that it is counterpart SKF but SKF shows less sensitive to load changes in transient state.



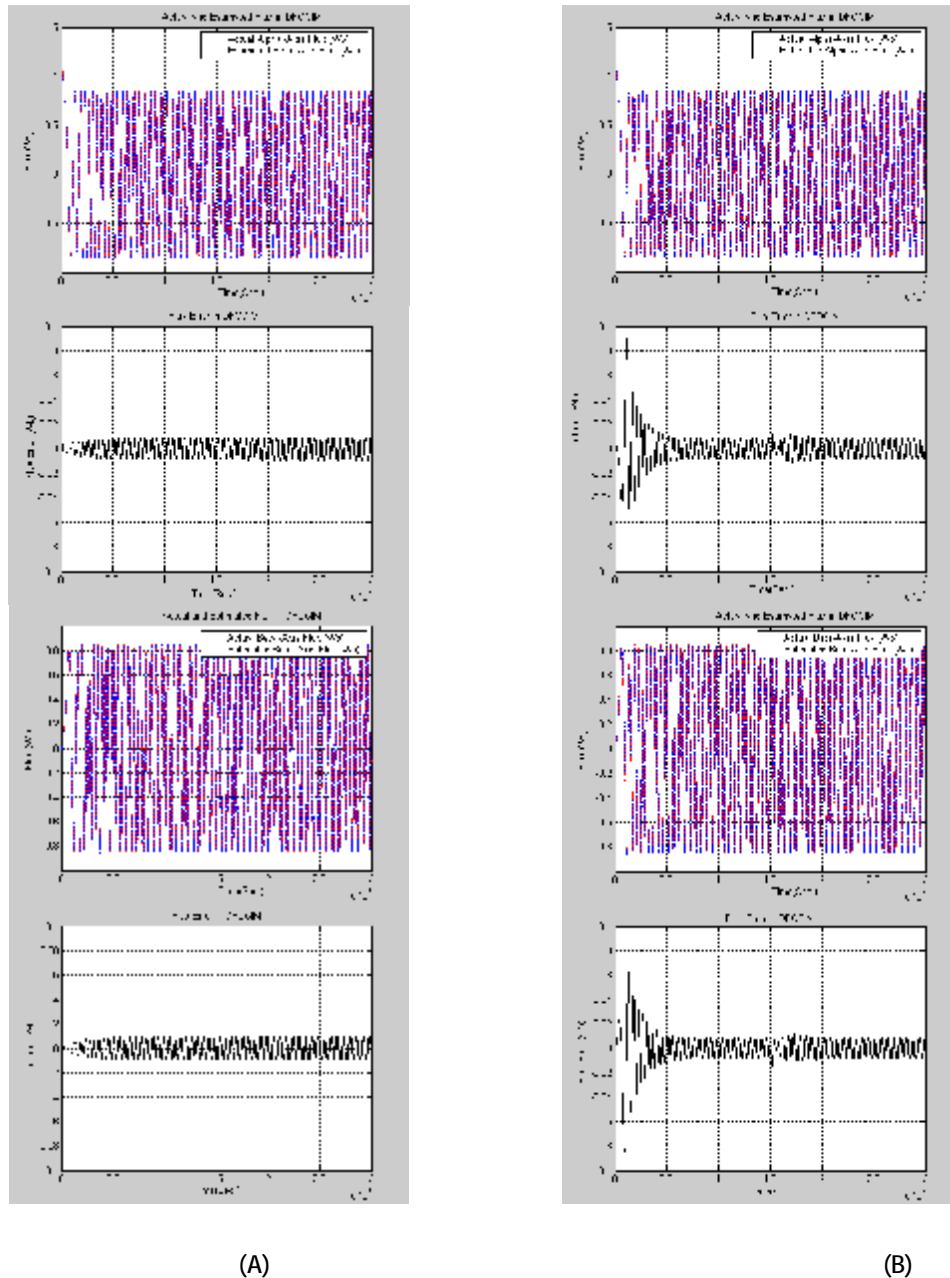
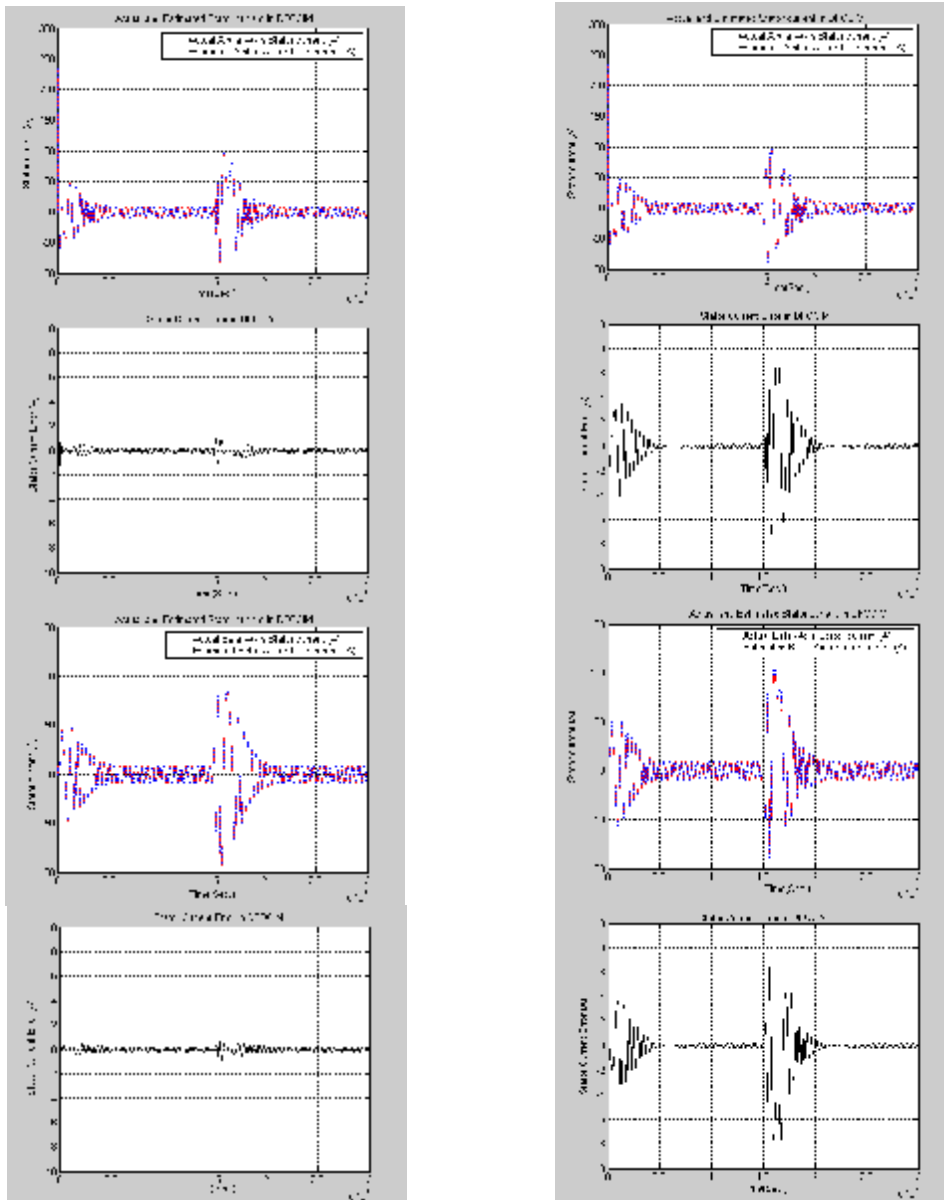


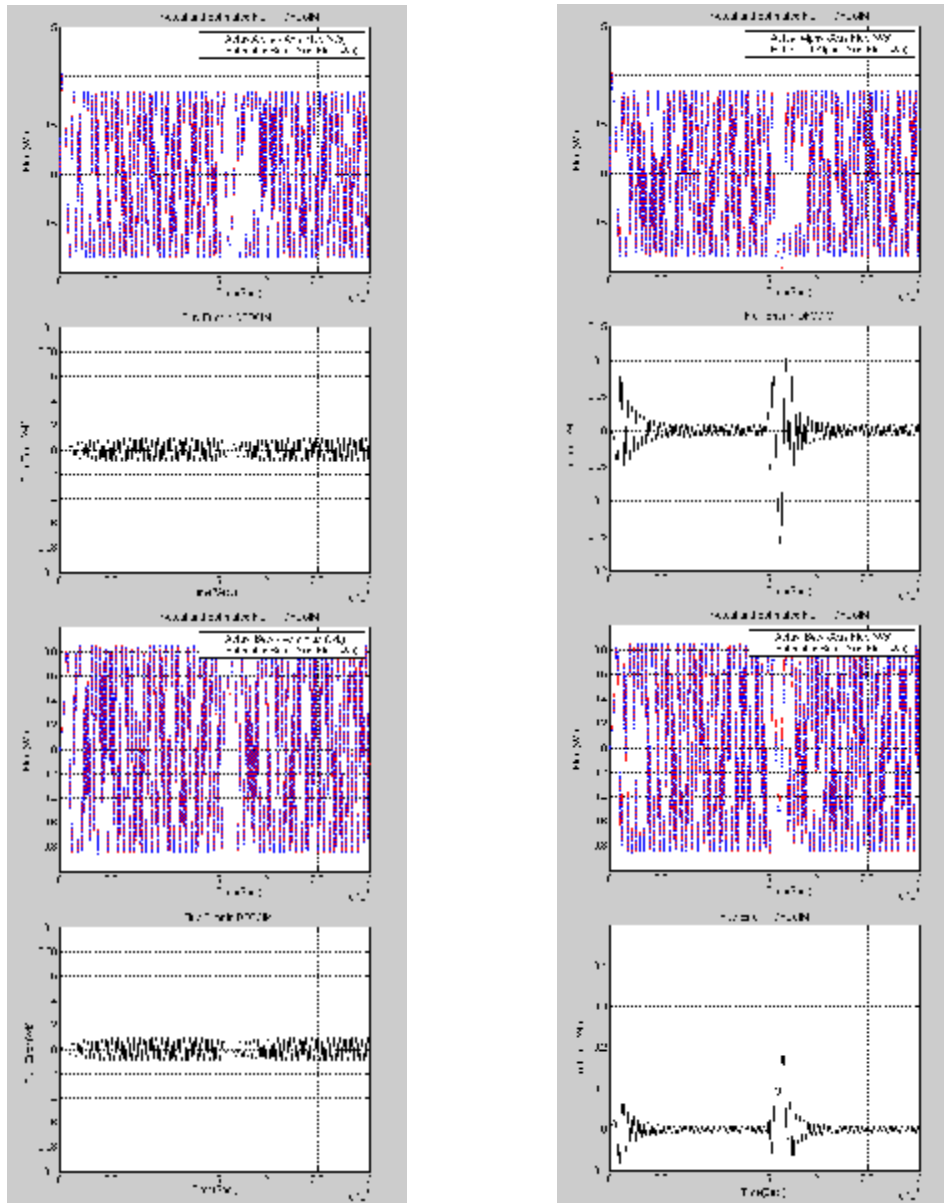
Figure (5) Simulation Results obtained with the two observers of kalman filter and error for a speed of reference 1000 tr/min (load of 50 Nm at t=1.5s to 3s).

A- SKF  
B- EKF

**Tracking Performances**

figure (6) shows the robustness of the algorithm of order associated with the observer with Kalman filter , at the time of the inversion of the instruction speed of 1000 tr/min with -1000 tr/min starting from t=1.5s. It is noted that EKF is robust opposite the abrupt variation the speed of reference in steady state but SKF gives less transient state estimation error.





(A)

(B)

Figure. (6) Simulation Results obtained with the two observers of Kalman filter (inversion of the direction of rotation of 1000 tr/min to -1000 tr/min to  $t=1.5s$ ).

A- SKF  
B- EKF

**Noise Injection**

In order to test the robustness of the observer of Kalman filter (SKF and EKF) to the noises of measurement, In fig (7) at reference flux 0.85 Wb , the noise rejection capabilities of suggested observers have been examined by injecting noise of variance 3A in stator current ( $I_{sa}$  ,  $I_{sb}$  ) . It is clear from the figure that the flux estimate resulting from SKF is more sensitive to such noise then EKF observer.

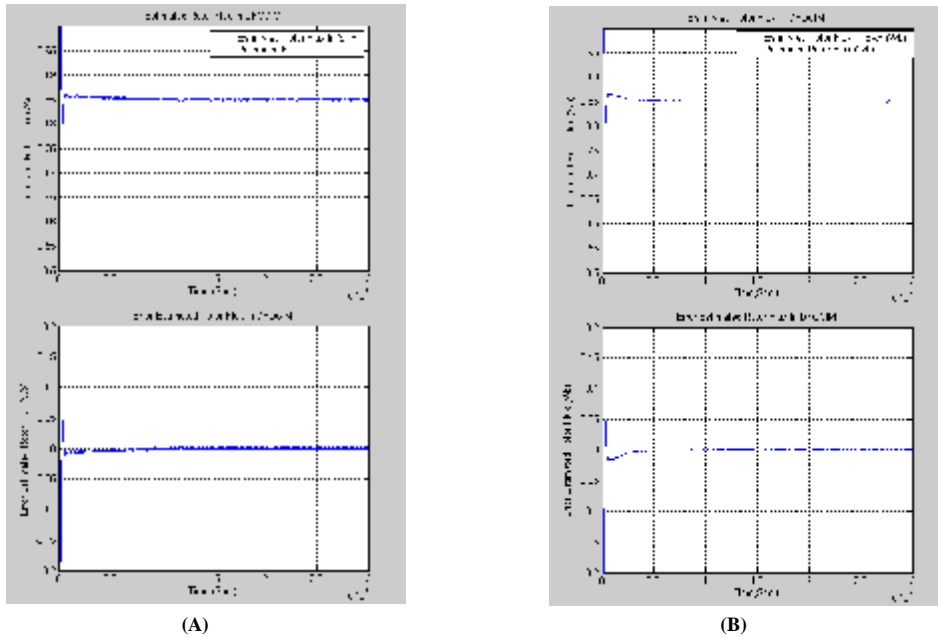
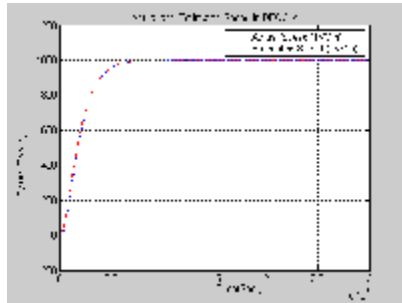


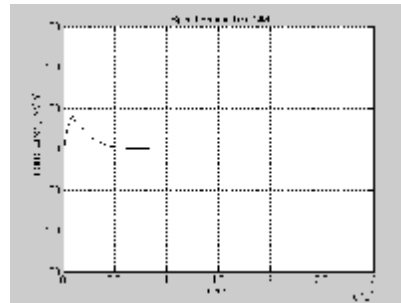
Figure.(7) Comparison of the model of the rotor flux estimated by the two observers of Kalman filter (SKF and EKF) with the injection of a noise measurement of variance of 3A  
 A- SKF  
 B- EKF

**Speed Estimation**

At no load in figure (8-A) shown the actual and estimation speed and figure (8-B) shown the estimation error speed in EKF observer. To check the robustness of the EKF, The actual and estimation speed are identical and estimation error reach zero value in steady state.



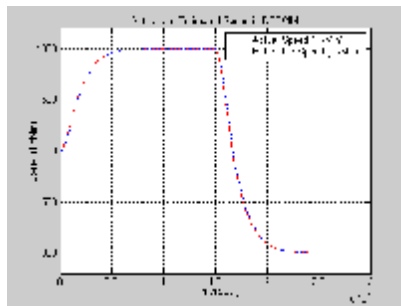
(8- A)



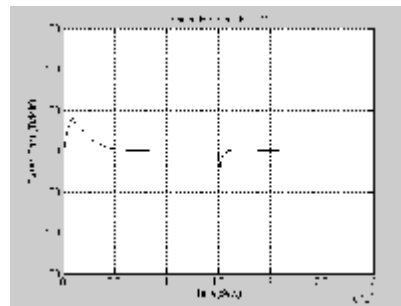
(8- B)

Figure (8) Simulation Result Obtained with EKF.

The figure (9-A) the speed estimated by extended kalman filter (EKF) and the figure (9-B) the estimation error, at the time of application of a reference speed of  $\pm 1000(Tr / Min)$ . According to these results, one notes that the observer by EKF expresses well still robustness with respect to the abrupt variation the speed of reference, since the estimate speed is always made satisfactory way



(9-A)

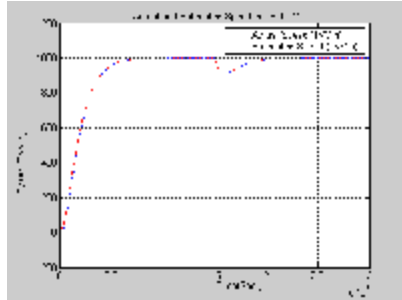


(9-B)

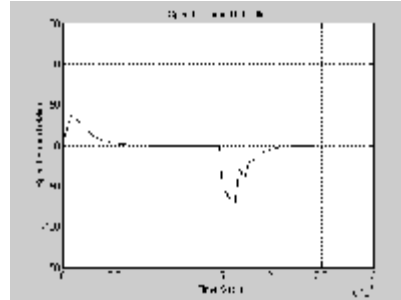
Figure (9) Simulation Result Obtained with EKF.

To check the robustness of the EKF opposite the load torque. One applied a load (50 Nm) at the time  $t=1.5s$  to  $3s$  (fig. 10). According to this result one notes that the algorithm of estimate is robust opposite the variation of the load (50Nm), owing to the fact that the estimate speed is always made satisfactory way in transient and steady state





(10-A)



(10-B)

Figure (10) Simulation Result Obtained with EKF.

**CONCLUSIONS**

In this paper, we propose SKF observer to estimate rotor flux and stator current and EKF observer to estimate rotor flux, Stator current and rotor speed applied in DFOCIM. The simulated results show the good estimation for rotor flux and stator current in EKF applied in DFOCIM but SKF is more advantage in transient state than EKF due to estimate of first values of the components of the vector of state. EKF showed high performance in speed estimated and showed high performance in terms of noise rejection with compare with SKF in DFOCIM.

**Appendix I**

The parameters IM are listed in Table (1)

Table (1): Parameters of IM

Nominal power	$P_n$	7,5	Kw
Nominal speed	$\Omega_n$	1450	tr/min
Nominal torque	T	50	Nm
Number of Paris pole	P	2	p.u
Stator resistance	$R_s$	0,63	$\Omega$
Rotor resistance	$R_r$	0,4	$\Omega$
Stator inductance	$L_s$	0,097	H
Rotor inductance	$L_r$	0,091	H
Mutual inductance	M	0.091	H
moment of inertia	J	0,22	Kg.m <sup>2</sup>

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*List of Symbols*

Symbol	Description
$A$	The state matrix.
$B$	The input matrix.
$C$	The output matrix.
$E$	Expected value.
$e$	Error.
$I$	Identity matrix.
$i_{sa} (I_{sb})$	$a$ -axis ( $b$ -axis) of Stator current.
$i_{dr} (i_{qr})$	D-axis (q-axis) of rotor current.
$i_{sd} (i_{sq})$	D-axis (q-axis) of stator current.
$M$	Magnetizing inductance.
$L_r$	Rotor self inductance.
$P$	Number of motor pole pairs.
$P_k$	Error covariance matrix.
$Q_k$	Input noise covariance matrix.
$R_r$	Rotor resistance.
$R_s$	Stator resistance.
$t_k$	Time at index k.
$U$	Input vector.
$V_t$	Measurement Noise
$V_{sd} (V_{sq})$	D-axis (q-axis) of stator voltage.
$V_{sa} (V_{sb})$	$a$ -axis ( $b$ -axis) of stator voltage.
$w$	Electrical speed.
$W_k$	Process noise
$K$	Kalman filter Gain
$R_K$	Measurement
$x$	State vector.
$x_k$	State vector at index k.
$y$	Output.
$q$	Angle (rad).
$q_s$	Stator angle.

$\Phi_r$	Flux in rotor
$\Phi_{ra} (\Phi_{rb})$	$a$ -axis ( $b$ -axis) of rotor flux.
$\Omega$	Mechanical speed
$0$	Initial value.
'	A Superscript indicates to Rotor quantities referred to the Stator.
T	A Superscript indicates to Transpose.
$d$	Derivative.
$k$	A subscript indicates to the index of sampling time.
$e$	A Superscript indicates to extended kalman filter.
$\wedge$	A Superscript indicates to estimation.