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Estimation of Flexural Creep Modulus of Short Fiber Reinforced Polymeric Composite Materials Using (FEM).

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ABSTRACT

The present work is a theoretical macro mechanics study, which concentrates on linear viscoelastic creep behavior of short fiber reinforced polymeric composite materials to estimate their flexural creep modulus at constant room temperature and at a time range of **1 to 10^9** seconds. The loading cases are concentrated and the deformations have been limited with small strains without permanent deformation. A three-parameter model is used as a mathematical viscoelastic model. A finite element program in Quick Basic had been designed to obtain the flexural creep modulus.

Halpin-Tsai equation shows an improvement in the modulus values for composite materials reinforced with short fibers in both aligned and random reinforcement through flexural creep estimations. On the other hand, the modified rule of mixtures shows a slight effect of reinforcement on this property.

الخلاصة:

البحث الحالي دراسة نظرية ماكروميكانيكية تركز على السلوك الخطي اللزج-المرن للزحف للمواد البوليمرية المقواة بالياف قصيرة لتحديد معامل مرونة الانحناء في درجة حرارة ثابتة وفترة زمنية تتراوح بين ($1-10^9$) ثانية.حالات التحميل هي أحمال متمركزة أما التشوهات فهي محددة بانفعالات صغيرة دون التشوه الدائمي . استخدم الموديل ذو العناصر الثلاثة كموديل رياضي لتمثيل السلوك اللزج- المرن. استخدمت طريقة العناصر المحددة كطريقة عددية للحصول على معامل المرونة وقد تم تصميم برنامج بلغة البيسك السريع لهذا الغرض. معادلة هالبن-تساي أظهرت تحسن في معامل المرونة للمادة المركبة ذات الأرضية البوليمرية المقواة بألياف قصيرة من كلا النوعين اتجاهية وعشوائية من خلال تنبؤات الانحناء الزحفي. من ناحية أخرى أظهرت قاعدة المخاليط المعدلة تأثيرا قليلا للتقوية على خواص المادة المركبة ذات الأرضية البوليمرية من تنبؤات الانحناء الزحفي.

modified the finite element solution for elasticity problems in two-dimensions to linear and nonlinear viscoelastic creep behavior of solid glassy polymers through changing material properties in each time step.

The present work is aimed to investigate numerically the time dependent flexural creep modulus of polymer reinforced with short aligned and randomly oriented fibers. The material systems chosen are polypropylene and short E-glass fiber reinforced polypropylene composite. Both polymeric and polymeric matrix composite (PMC) specimens have been assumed to be fabricated by injection molding. The general assumptions of the present work can be stated as follows:

1-The polymeric specimens are considered to be isotropic.
2- The recovery behavior has not been considered.
3-Stress concentrations at fiber ends have been ignored.
4-The present work has been concerned with the linear viscoelastic behavior in polymer and PMC and does not reach to failure.
5-The bonding between fiber and matrix is perfect.

In this work 3-parameters model will be used to represent the viscoelastic behavior of polypropylene polymer. The response of this model contains both initial elastic and decreasing strain rate which represents creep response fully [Resen, A. S. and Alhadithi]. The time dependent modulus of this model has three constants (E_1, E_2, μ_2). These constants have been determined according to experimental curve of polypropylene at constant stress (8.3 MPa) and at (20^oC) from [Rollason, E. C.] by curve fitting procedure, so: E_1=1.7GPa, E_2=0.23GPa, and μ_2=2.9 x 10^{16} (Poise).

$$J(t)=\frac{\varepsilon(t)}{\sigma_o}=\frac{1}{E_1}+\frac{1}{E_2}(1-e^{-t/t_r}) \tag{1}$$

$$E(t)=\frac{1}{J(t)}=\frac{E_1E_2}{E_1+E_2-E_1e^{-t/t_r}} \tag{2}$$

Where $t_r = \mu_2/E_2$

Flexural Creep Estimation of Polymer

The viscoelastic characteristics of composite materials are usually due to a viscoelastic matrix material [Jones], so it is essential to estimate the flexural creep modulus of polymer- the matrix. In this work three-point flexural creep estimation for simply supported beams was conducted due to the ASTM D790 on specimens with a length, L = 130 mm; depth, h = 8.125 mm; width, b = 6.4 mm; and L/h = 16. All the assumptions of simple bending theory (elementary beam theory) are consistent with the flexure specimen used. The beam cross section is uniform and symmetrical about the plane of loading - (x-y) plane. A simplifying assumption will be used that the shear forces do not contribute significantly to the overall deformation [Lardner].

The time dependent deflection of a linear viscoelastic beam under sustained loading can be determined as [Popov]:

$$\delta(x,t)=\delta_{el}(x)E_bJ(t) \tag{3}$$

$$J(t)=\frac{\delta(x,t)}{\delta_{el}(x)E_b} \tag{4}$$

So

$$\varepsilon_p(x,t)=-\frac{M(x)y}{I}\cdot\frac{\delta(x,t)}{\delta_{el}(x)E_b} \tag{5}$$

Where I is moment of inertia (= $bh^3/12$) , E_b bending modulus which is constant material property, M(x)= PX/2 and the elastic deflection in terms of x is [Timoshinko & Young]:

$$\delta_{el}(x) = \frac{Px}{12E_bI}(x^2 - \frac{3}{4}L^2) \qquad 0 \le x \le \frac{L}{2} \tag{6}$$

δ (x, t) is the viscoelastic deflection of the beam that can be determined from the finite element analysis. Then the time dependent flexural creep modulus of polymer can be determined as [Resen, & Alhadithi, Ward]:

$$E_p(t) = \frac{1}{J_p(t)} \tag{7}$$

Each strain value **ε (x, t)** at position x and time t must be checked due to the limit of linear viscoelastic behavior where the strain must be less than **0.5%.**

Flexural Creep Estimation of PMC:

1- Aligned Short Fiber PMC

For flexural creep estimation of aligned short fiber composite, the specimens were stressed and strained in the fiber dominated longitudinal direction and the effect of time dependence has been admitted to the modulus of polymer $\mathbf{E_p}$ **(t).** The mechanical response can be predicted according to the two following approaches:

-Modified Rule of Mixtures

By assuming perfect alignment, the time dependent longitudinal modulus of unidirectional short fiber reinforced composites $\mathbf{E_{CL}}$ **(t)** can be obtained as:

$$E_{CL}(t) = \lambda_1(t)V_fE_f + V_mE_p(t) \tag{8}$$

Where :V_f and V_m are volume fractions of fiber and matrix respectively; $\mathbf{E_f}$ modulus of elasticity of E-glass fiber wherein $E_{Lf} = E_{Tf} = E_f$, E_p (t) is to be obtained from finite element computer program; $\boldsymbol{\lambda_1}$ is the factor which corrects the modulus for the shortness of the fibers with a modification represented by time dependence according to the presence of time dependent shear modulus G_p (t) as follows:

$$\lambda_1(t) = \left\{1 - \left[\frac{\tanh(n(t)a)}{n(t)a}\right]\right\} \tag{9}$$

Where: a is the fiber aspect ratio and equal to l/d; n is a dimensionless group of constants and can be estimated as:

$$n(t) = \sqrt{\frac{2G_p(t)}{E_fLn(2R/d)}} \tag{10}$$

where:

$$G_p(t) = \frac{E_p(t)}{2(1+\nu_p(t))} \tag{11}$$

For viscoelastic materials $\nu_p(t)$ is specialized to [Lakes]:

$$\nu_p(t) = 0.5 - \frac{E_p(t)}{6k_p} \tag{12}$$

When the bulk modulus of polymer, k_p, is approximated as [Jones]:

$$k_p = \frac{E_p}{3(1-2\nu_p)} \tag{13}$$

Where E_p and ν_p are modulus of elasticity and Poisson's ratio for initially elastic matrix respectively. The time dependent longitudinal strains for PMC will be:

$$\varepsilon_{CL}(t) = \frac{\sigma}{E_{CL}(t)} \tag{14}$$

- Halpin-Tsai Equations

The longitudinal modulus and transverse modulus of aligned short fiber PMC can be written by modifying the equation quoted in [Agarwal, & Broutman] as follows:

$$E_{CL}(t) = E_p(t)\frac{1+\frac{2L_f}{d}\lambda_L(t)V_f}{1-\lambda_L(t)V_f} \tag{15}$$

Where

$$\lambda_L(t) = \frac{(E_f/E_p(t))-1}{(E_f/E_p(t))+\frac{2L_f}{d}} \tag{16}$$

and

$$E_{CT}(t) = E_p(t)\frac{1+2\lambda_T(t)V_f}{1-\lambda_T(t)V_f} \tag{17}$$

Where

$$\lambda_T(t) = \frac{(E_f/E_p(t))-1}{(E_f/E_p(t))+2} \tag{18}$$

2- Random Short Fiber Composite

Applying the approach of Halpin-Tsai quoted in [Agarwal, & Broutman]with modification due to time dependent the modulus of random short fiber PMC can be determined as follows:

$$E_{Cr}(t) = \frac{3}{8}E_{CL}(t) + \frac{5}{8}E_{CT}(t) \tag{19}$$

Finite Element Modeling for Flexural Specimen

The simply supported beam is descretized by using one-dimensional linear elements with two nodes for each and one degree of freedom for each node. Due to symmetry a **half of beam length (65 mm)** was analyzed. The finite element mesh of the beam is shown in Fig. (1). **The boundary conditions are:**

a. At x = 0، and at x =L, δ_{el} = 0, δ(x, t) = 0, M = 0.

b. At x = L/2, δ_{el} (x) = max., δ(x, t) = max., and M = maximum.

The one-dimensional linear differential equation for deflection of a simply supported beam subjected to concentrated load is [Segerlind]:
For a sequence of nodes : r, s, and m as shown in fig.(2) the nodal residual equation of node s in terms of r& m after evaluation using Galerkin's formulation is :

$$EI\frac{d^2\phi}{dx^2} - \frac{Px}{2} = 0 \tag{20}$$

$$R_s = -\left(\frac{EI}{L_e}\right)^{(e)} \Phi_r + \left[\left(\frac{EI}{L_e}\right)^{(e)} + \left(\frac{EI}{L_e}\right)^{(e+1)}\right]\Phi_s - \left(\frac{EI}{L_e}\right)^{(e+1)} \Phi_m \tag{21}$$

$$+\frac{P}{12L_e}(-X_s^2(2L_e + X_m) + X_m^3 + X_s^2(2L_e - X_r) + X_r^3)$$

RESULTS AND DISCUSSION

A special quasi-static finite element computer program was designed to solve flexural beam problem of polymer and its composite reinforced with short fibers through the linear viscoelastic region. A period of time between ($1\text{-}10^9$) seconds has been considered. Maximum and minimum values of the property understudy have been obtained at t = 10^9 seconds. The values of the mechanical property that exceeds over the linear viscoelastic limits (ε > 0.5%) will be decreased to zero due to the present study that is limited with linear viscoelastic behavior. The values out of the linear viscoelastic limits have been found at node 14. The following data have been considered in the present analysis [Peng, X. & Cao, Eichhorns, and Sanadi et.al.]:
V_f= 0.2 ; a = 40 ; d = 14 μm ; E_f = 70 GPa ; E_b = 1.4 GPa ; ν_p = 0.3 and P = 250 N.

Flexural Creep of Polypropylene

Fig.(3) refers to the time dependence of flexural strain of polypropylene specimen according to Eq. (5). Strain increases with time starting from node 2 to node 14. Maximum strain of node 13 is **$4.5\text{x}10^{-3}$**.

Fig.(4) refers to the effect of time on flexural modulus of polypropylene polymer due to Eq. (7). Flexural modulus has been decreased with time beginning from node 2 to node 14. Minimum modulus for node 13 is **25** GPa.

Flexural Creep of PMC (according to Halpin-Tsai Equation):

- Longitudinal modulus of composite with aligned short fibers:

Fig.(5) denote the time dependence of longitudinal flexural modulus for aligned short E-glass fiber reinforced polypropylene due to Halpin-Tsai equation

(Eq.15). Flexural modulus curves are decreasing with time.Minimum value of longitudinal flexural modulus is 500 GPa and it is obtained at node 13.

-Transverse modulus of composite with aligned short fibers:

Fig.(6) represents the time dependence of transverse flexural modulus of aligned short E-glass fibers reinforcing polypropylene obtained from Halpin-Tsai equation (Eq.17). Maximum transverse modulus is equal to **50** GPa. In general, the lateral mechanical properties are very close to the results of PP polymer alone because the lateral properties of aligned composite are matrix dominated.

-Modulus of composite with random short fibers:

Fig.(**7)** refers to the effect of time on flexural creep modulus of random short E-glass reinforced polypropylene obtained from Halpin-Tsai equation (Eq.19). The creep modulus is equal to **25** GPa for. It can be concluded that the effect of reinforcement in random case is small with respect to polymer alone. The stiffness values are the same without improvement as the random composite material is isotropic and its mechanical properties are equal in all directions.

Flexural Creep of PMC (Modified Rule of Mixtures):

- Longitudinal modulus of composite with aligned short fibers:

Fig.(8) shows the effect of time on longitudinal flexural modulus of aligned short E-glass fiber reinforced polypropylene by using the modified rule of mixtures (Eq.8).Maximum longitudinal modulus **is 31**GPa. It is clear that there is little improvement in stiffness.

The rule of mixtures gives accurate results of composite reinforced with aligned continuous fibers. According to the present study, the modified rule of mixtures is less accurate in predicting of the mechanical properties of composite reinforced with aligned short fibers.

CONCLUSIONS

1- There is constancy in flexural curves for modulus during the periods **$1\text{-}10^6$** seconds, so the region of glassy behavior of polypropylene polymer alone and polypropylene reinforced with short E-glass fibers in the forms of aligned and random reinforcement is limited by this period.

2- It can be observed that using a wider range of time may give three regions in the curves of creep modulus. These regions are glassy, viscoelastic, and rubbery.

3-. The mechanical property that have been obtained from flexural creep estimations are identical to the time dependent typical behavior of polymer and **PMC**. This behavior reflects the success of the theoretical analysis applied in the present work.

4- The reinforcement of polypropylene polymer by short E-glass fiber in its two forms, aligned and random, improves modulus according to Halpin-Tsai equation.

5- Halpin-Tsai equation is more accurate than modified rule of mixtures in the theoretical prediction of the mechanical properties of the understudy composite material reinforced with short fibers.

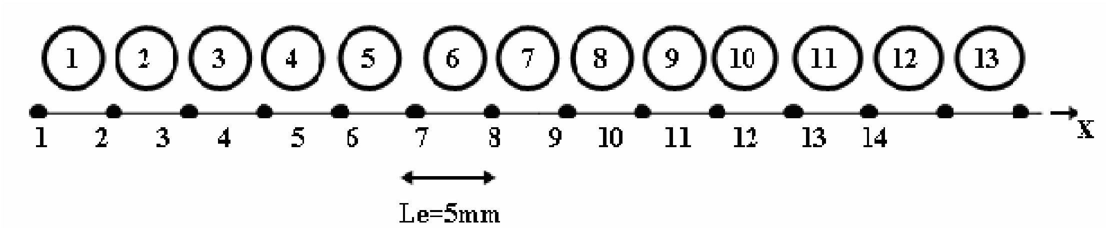


Fig.(1): Finite element mesh for flexural specimen

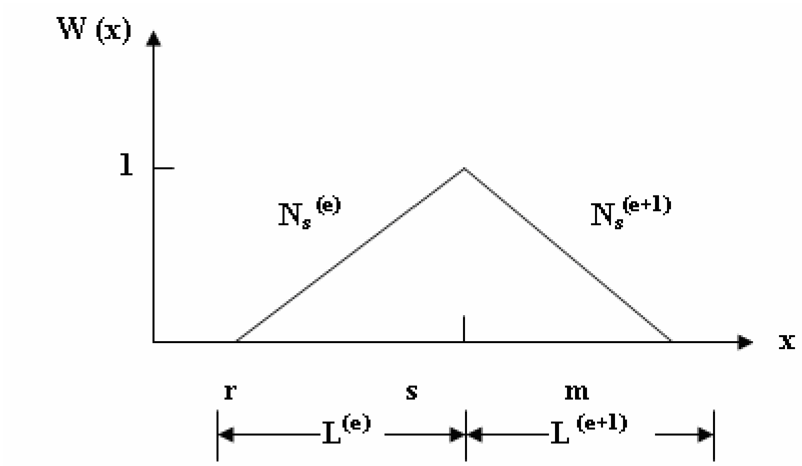


Fig.(2): The weighting function for an interior node [Segerlind].

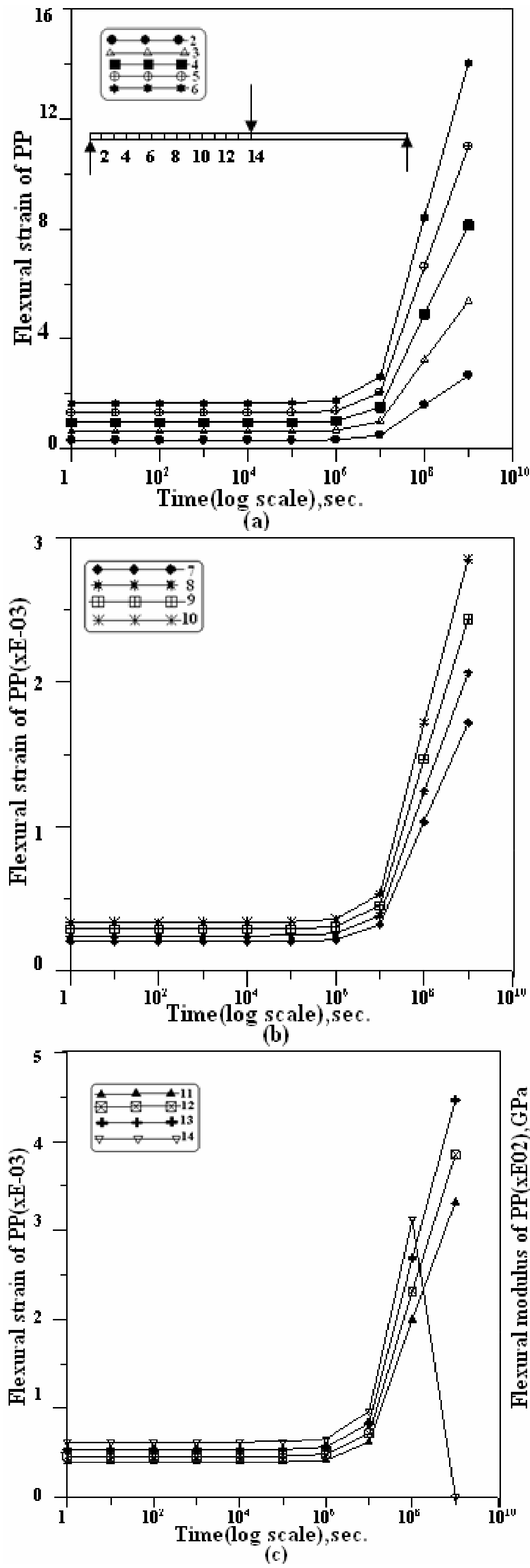


Fig.(3): Time dependent flexural strain for PP, (a) nodes(2-6); (b) nodes(7-10); (c) nodes(11-14)

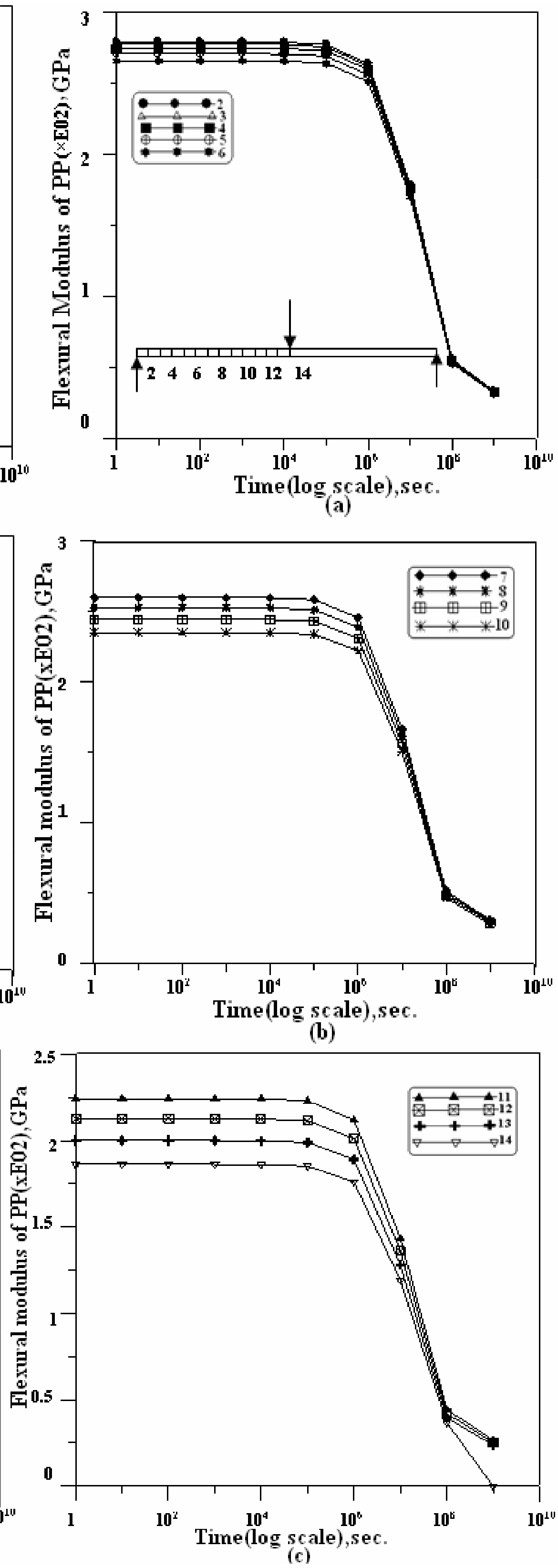


Fig.(4): Time dependent flexural modulus for PP, (a) nodes(2-6); (b) nodes(7-10); (c) nodes(11-14)

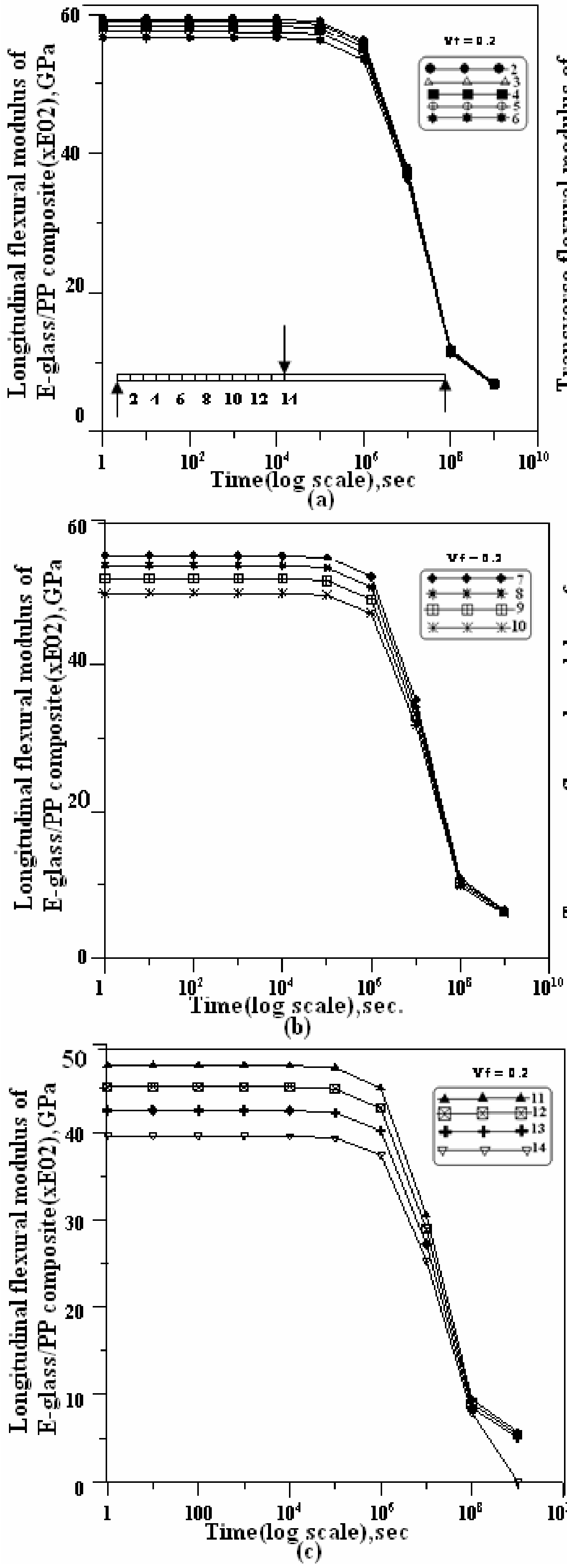


Fig.(5): Time dependent longitudinal flexural modulus for aligned short E-glass/PP composite(Halpin-Tsai eq.) for nodes: a-(2-6);b-(7-10);c- (11-14)

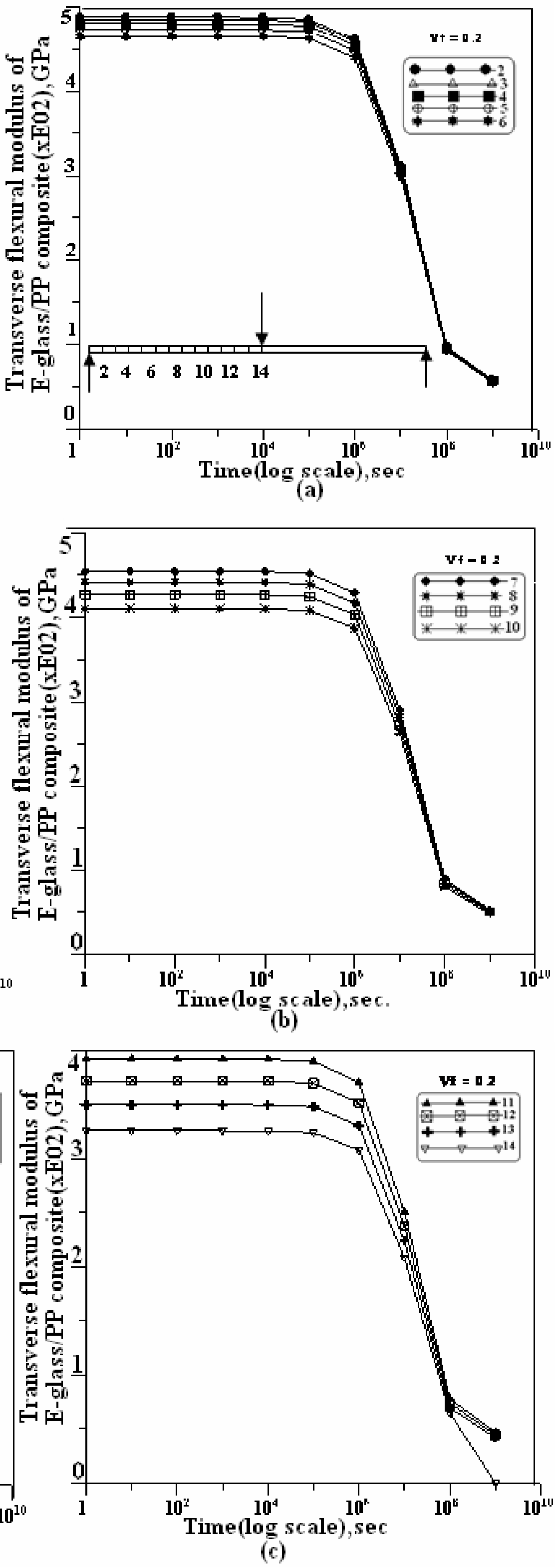
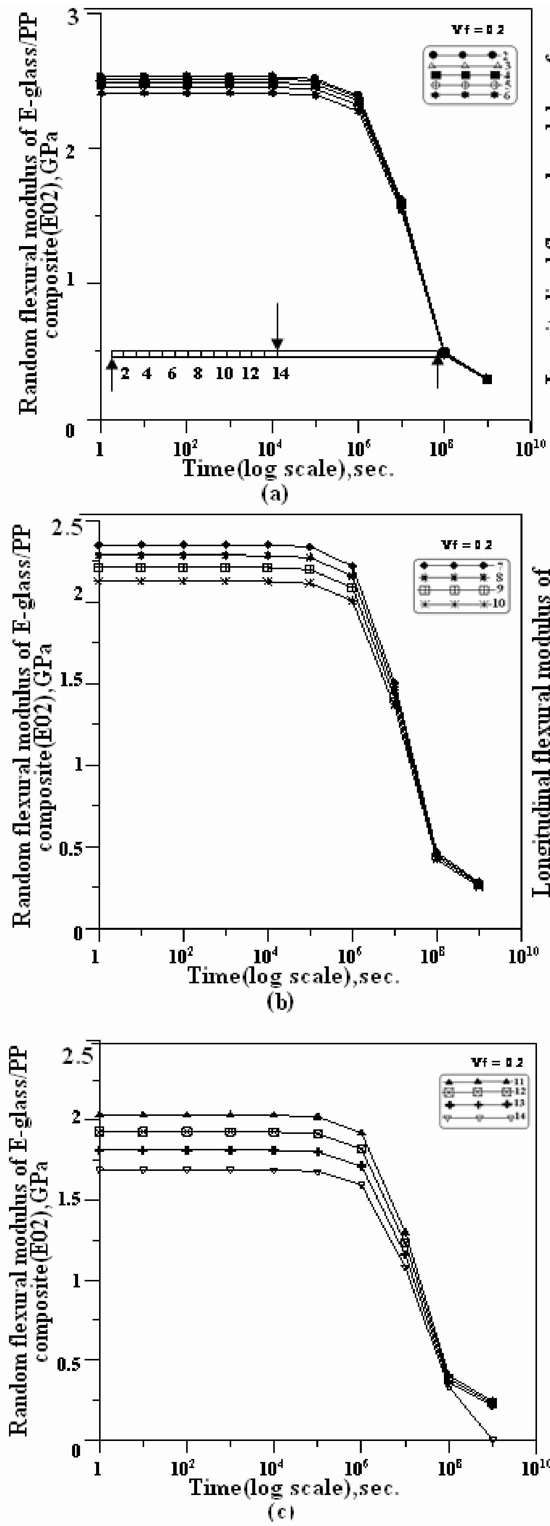


Fig.(6): Time dependent transverse flexural modulus for aligned short E-glass/PP composite(Halpin-Tsai eq.) for nodes: a-(2-6);b-(7-10);c- (11-14).



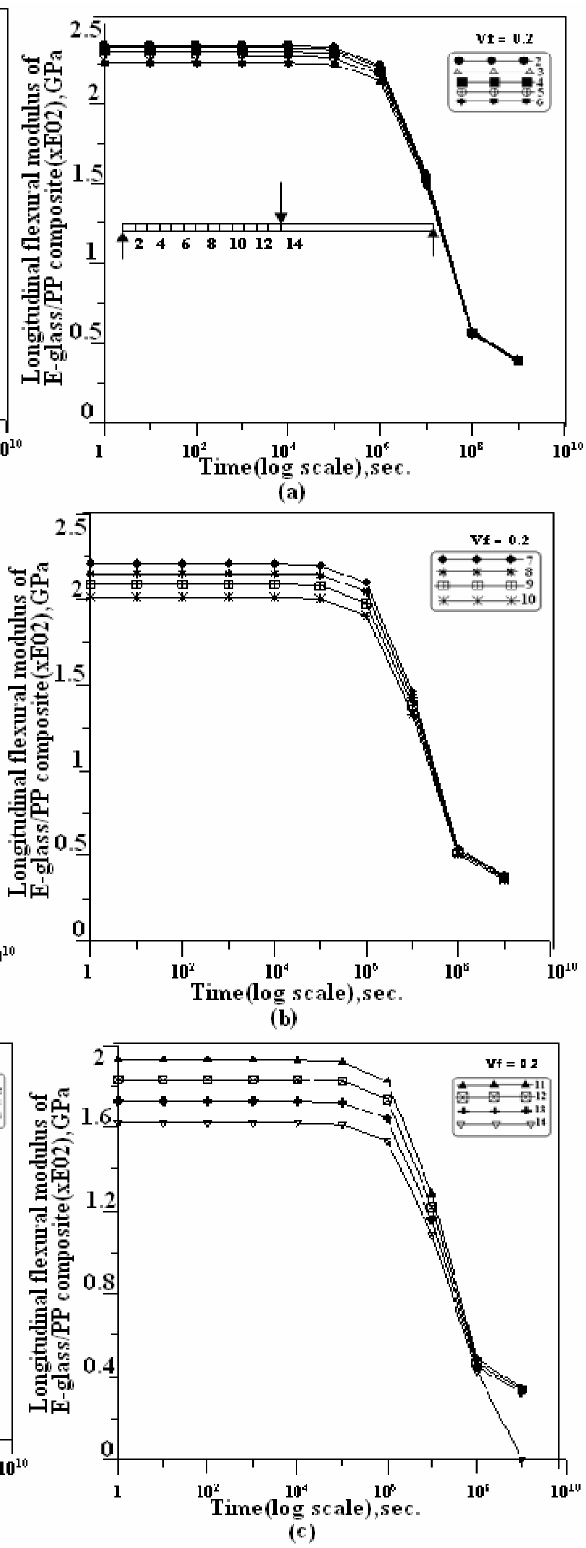


Fig.(7): Time dependent flexural modulus For random short E-glass/PP composite (Halpin-Tsai eq.) for nodes: a-(2-6);b-(7-10);c-(11-14)

Fig.(8): Time dependent longitudinal flexural modulus for aligned short E-glass/PP composite (modified rule of mixture) for nodes:a-(2-6);b-(7-10); c-(11-14)

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