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The Distribution of the Outflow for Linear Reservoirs Using Fast Fourier Transformation Method

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الملخص

تعد نظرية الخزانات العشوائية من أشهر التطبيقات للعمليات العشوائية وسلاسل ماركوف، التي تؤدي دوراً مهماً في الإحصاء الرياضي وتطبيقاته,الهدف الرئيسي لهذا لبحث إيجاد توزيع السحب لخزانات خطية باستخدام طريقة محولات فورير السريعة.

ABSTRACT

One of the most important problems in linear reservoir is the distribution of the storage.The main goal of this paper is to find the distribution of the outflow of linear reservoir and then the storage distribution.This is done by applying a simple technique which is the Fast Fourier Transformation (FFT) Method.

1. INTRODUCTION

Stochastic reservoir theory was initiated by Moran (1954), who has assumed that the inflow X_t into the reservoir during the time-interval (t, t+1) forms a sequence of independent and identically distributed random variables. The main interest is then the storage process $\{Z_t\}$ which is a Markov chain and defined by the following stochastic difference equation (Moran, 1954)

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$$Z_{t+1} = \min(Z_t + X_t, C + W) - \min(Z_t + X_t, W) \tag{1}$$

where:

a) X_t is the inflow during the time - interval (t , t+1) .

b) Z_t is the content of the reservoir before the inflow X_t .

c) W is the release at the end of each time-interval.

d) C represents the reservoir capacity .

Equation (1) can be re- written as:

$$Z_{t+1} = \begin{cases} 0 & , Z_t + X_t \le W \\ Z_t + X_t - W & , W \le Z_t + X_t < C + W \\ C & , Z_t + X_t \ge C + W \end{cases} \tag{2}$$

Many research articles had been published in this case , among them, Kelmes(1974),Phatarford (1988),Saleem(1979),Saleem (2006) and Saleem and Melkonian(2006).

2. The Fast Fourier Transformation Method

By a linear reservoir , we mean a reservoir of unbounded capacity (topless) , in which the outflow rate is directly proportional to the temporary content.

In the non-seasonal linear reservoir, the relation is written as (Klemes 1974):

$$Y(t) = \beta Z(t) \tag{3}$$

where Y(t) is the amount of water released from the reservoir ,
Z(t) is the content of the reservoir ; $0 < \beta < 1$ is a constant.

Now we give a review of some results **:**

In the discrete time the appropriate inflow, outflow, and the storage variables are:

X_t = inflow quantity during time (t,t+l) .
Y_t = outflow quantity during time (t,t+l) .
Z_t = quantity of water stored at time t.

Equation (3) becomes:

$$Y_t = (1-b) Z_t , \quad 0 < b < 1 , \quad t = 0,1,\dots \tag{4}$$

The continuing equation for the reservoir (Eq.2) with W= Y_t becomes :

$$Z_{t+1} - Z_t = X_t - Y_t \tag{5}$$

From equations (4) and (5} we obtain

$$Y_{t+1} = b\,Y_t + c\,X_t \tag{6}$$

where $0 < b < 1$, $0 < c < 1$, $b+c = 1$, $t = 0,1,\ldots$

The difference equation (6) has the unique solution :

$$Y_{t+1} = b^{t+1} Y_0 + c \sum_{r=0}^{t} b^{t-r} X_r \qquad t = 0,1,\ldots$$

Equivalently for arbitrary m (m < t)

$$Y_{t+1} = b^{t-m} Y_{m+1} + c \sum_{m+1}^{t} b^{t-r} X_r \ ; \quad t = m, m+1, \ldots \tag{7}$$

Letting $m \to \infty$,we obtain the asymptotic equilibrium solution :

$$Y_{t+1} = c \sum_{r=0}^{\infty} b^{r} X_{t-r} \tag{2.6}$$

that is the weighted sum of Makovian variables.
Now suppose that $\{X_t\}$ is the gamma distributed Markov chain , it follows that the Laplace Transform of the outflow can be written as (saleem (1979)):

$$L(Y,\theta) = \prod_{r=1}^{\infty} H(\theta_r)$$

$$\theta_1 = c\theta, \qquad \theta_{r+1} = b^r \theta_1 + G(\theta_r) \qquad r = 1,2,3,\ldots$$

$$H(\theta) = \{\ 1 + \alpha(1-\rho)\theta\ \}^{-p} \ , \qquad G(\theta) = \frac{\rho\,\theta}{1 + \alpha(1-\rho)\theta} \ ,$$

where ρ is the correlation between X_t and X_{t+1} .

the problem is to find the inverse of the Laplace transform which is given by (9). This inversion will give us the outflow distribution .

3. Inversion of Laplace Transform

Laplace transform of a function f(t) can be expressed as :

$$F(s) = \int_0^{\infty} e^{-st} f(t)\,dt \qquad (10)$$

There are two restrictions on f(t) that are necessary for the integral to exist:

1. f(t) must be piecewise continuous.
2. f(t) must be of exponential order:

i. e $|f(t)| < Me^{\sigma t}$,where M is constant

The inversion integral is defined as:

$$f(t) = \frac{1}{2\pi i} \int_{\sigma - i\omega}^{\sigma + i\omega} e^{st} F(s)\,ds \qquad (11)$$

The Fast Fourier Transform method:

There are several methods that give a solution to (11) . We apply the method of the Fast Fourier Series approximation (FFT), (Hassanzadele and pooladi 2007)

$$f(t) = \frac{e^{\sigma t}}{2\pi i} \int_{-\omega_f}^{\omega_f} F(\sigma + i\omega) e^{i\omega t} d\omega \qquad (12)$$

4. THE RESULTS

The DISTRIBUTION OF THE OUTFLOW

Table (1) gives the Laplace Transform (L.T.) of the outflow distribution as given by (9) for chosen values of the parameters.Table (2) gives a numerical inversion of this L.T. obtained for the following chosen parameters :

$\theta = 0, 2, (0.1)$, p= 2,3,4,…

$\alpha = 1$; $b = c = \frac{1}{2}$, $\rho = 0, 6, (.1)$

Graphs of Distribution of the Outflow Given by FFT Method

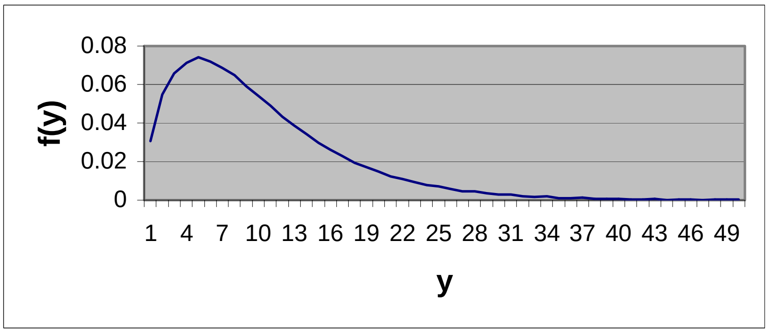

Fig .1

The probability density function of the outflow with parameters
$p = 2 \quad , \quad \alpha = 1 \quad , \rho = 0$

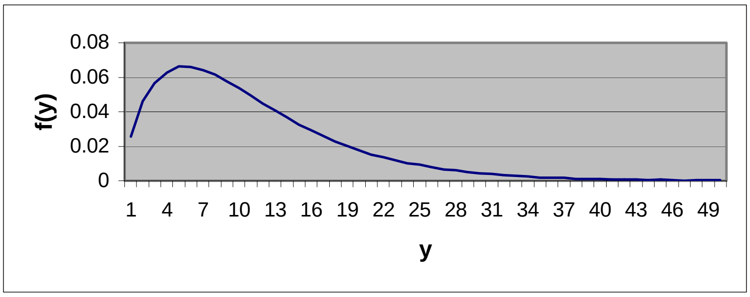

Fig .2

The probability density function of the outflow with parameters
$p = 2 \quad , \quad \alpha = 1 \quad , \rho = 0.1$

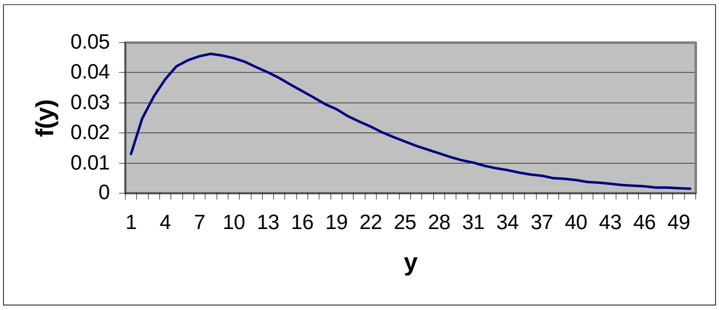

Fig.3

The probability density function of the outflow with parameters
$p = 2 \quad , \quad \alpha = 1 \quad , \rho = 0.6$

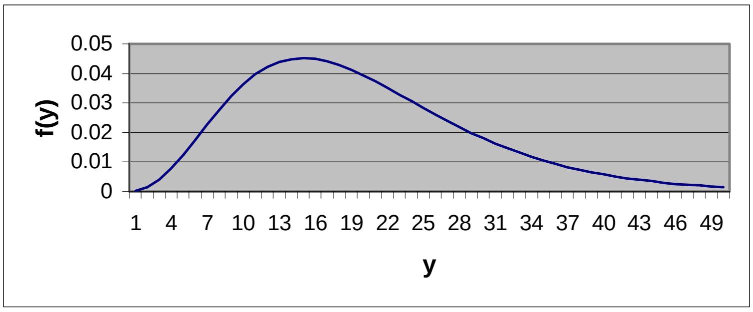

Fig. 4

The probability density function of the outflow with parameters
$p = 4 \quad , \quad \alpha = 1 \quad , \rho = 0$

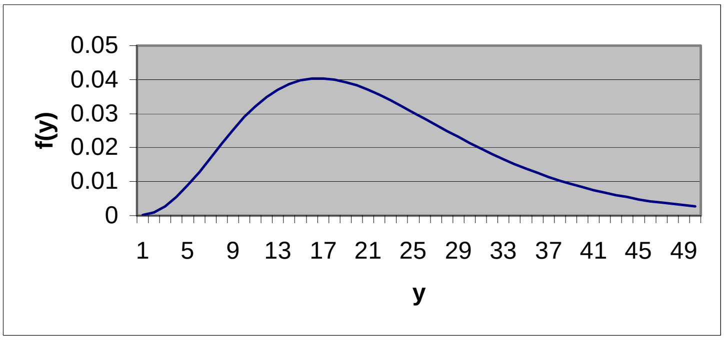

Fig. 5

The probability density function of the outflow with parameters
$p = 4 \quad , \quad \alpha = 1 \quad , \rho = 0.1$

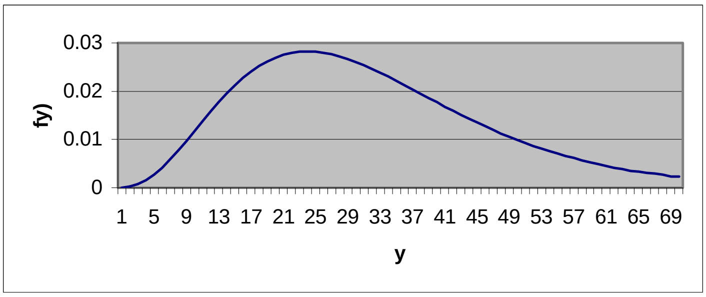

Fig. 6

The probability density function of the outflow with parameters
$p = 4 \quad , \quad \alpha = 1 \quad , \rho = 0.6$

It is seen that these distributions qualitatively resemble the (gamma) form of the inflow distribution.

CONCLUSIONS

1- FFT method can be calculated by computer program, so it takes a minimal time which is less than 10^{-5} second.
2- FFT method used to evaluate the time domain solution for a variety of problems that arise in applications of stochastic reservoirs theory. In most cases the results of the inversions compared well with the analytical inversion.
3- FFT method is both accurate and fast , accuracy can be improved by increasing the value of m.

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APPENDIXES

Table (1)

Laplace Transforms of the Outflow Distributions

p	ρ	θ	L(Y, θ)	P	ρ	θ	L(Y, θ)	p	ρ	θ	L(Y, θ)
2	0	0	1	2	0.1	0	1	2	0.6	0	1
2	0	0.1	0.821549255	2	0.1	0.1	0.806	2	0.6	0.1	0.732
2	0	0.2	0.679098943	2	0.1	0.2	0.654	2	0.6	0.2	0.544
2	0	0.3	0.564540331	2	0.1	0.3	0.534	2	0.6	0.3	0.41
2	0	0.4	0.471780724	2	0.1	0.4	0.44	2	0.6	0.4	0.312
2	0	0.5	0.396195855	2	0.1	0.5	0.364	2	0.6	0.5	0.241
2	0	0.6	0.334243293	2	0.1	0.6	0.302	2	0.6	0.6	0.187
2	0	0.7	0.283186031	2	0.1	0.7	0.253	2	0.6	0.7	0.147
2	0	0.8	0.240892538	2	0.1	0.8	0.212	2	0.6	0.8	0.116
2	0	0.9	0.205690544	2	0.1	0.9	0.179	2	0.6	0.9	0.093
2	0	1	0.176259049	2	0.1	1	0.151	2	0.6	1	0.074
2	0	1.1	0.151547815	2	0.1	1.1	0.129	2	0.6	1.1	0.06
2	0	1.2	0.130716835	2	0.1	1.2	0.11	2	0.6	1.2	0.049
2	0	1.3	0.113090443	2	0.1	1.3	0.094	2	0.6	1.3	0.04
2	0	1.4	0.09812226	2	0.1	1.4	0.081	2	0.6	1.4	0.033
2	0	1.5	0.085368235	2	0.1	1.5	0.07	2	0.6	1.5	0.027
2	0	1.6	0.074465765	2	0.1	1.6	0.06	2	0.6	1.6	0.022
2	0	1.7	0.065117421	2	0.1	1.7	0.052	2	0.6	1.7	0.019
2	0	1.8	0.057078191	2	0.1	1.8	0.045	2	0.6	1.8	0.016
2	0	1.9	0.050145426	2	0.1	1.9	0.039	2	0.6	1.9	0.013
2	0	2	0.044150868	2	0.1	2	0.034	2	0.6	2	0.011
2	0	2.1	0.038954298	2	0.1	2.1	0.03	2	0.6	2.1	0.00311
2	0	2.2	0.034438454	2	0.1	2.2	0.026	2	0.6	2.2	0.007896
2	0	2.3	0.030504944	2	0.1	2.3	0.023	2	0.6	2.3	0.006718
2	0	2.4	0.027070947	2	0.1	2.4	0.02	2	0.6	2.4	0.005732
2	0	2.5	0.024066529	2	0.1	2.5	0.018	2	0.6	2.5	0.004906

2	0	2.6	0.021432471	2	0.1	2.6	0.016	2	0.6	2.6	0.00421
2	0	2.7	0.019118478	2	0.1	2.7	0.014	2	0.6	2.7	0.003622
2	0	2.8	0.017081727	2	0.1	2.8	0.013	2	0.6	2.8	0.003125
2	0	2.9	0.015285653	2	0.1	2.9	0.011	2	0.6	2.9	0.002702
2	0	3	0.013698963	2	0.1	3	0.009913	2	0.6	3	0.002342
2	0	3.1	0.012294808	2	0.1	3.1	0.008841	2	0.6	3.1	0.002034
2	0	3.2	0.011050097	2	0.1	3.2	0.007896	2	0.6	3.2	0.001771
2	0	3.3	0.009944929	2	0.1	3.3	0.007063	2	0.6	3.3	1.55E-03
2	0	3.4	0.008962114	2	0.1	3.4	0.006328	2	0.6	3.4	1.35E-03
2	0	3.5	0.008086773	2	0.1	3.5	0.005676	2	0.6	3.5	1.18E-03
2	0	3.6	0.007305999	2	0.1	3.6	0.5099	2	0.6	3.6	1.04E-03
2	0	3.7	0.006608578	2	0.1	3.7	0.000459	2	0.6	3.7	9.13E-03
2	0	3.8	0.005984743	2	0.1	3.8	0.004131	2	0.6	3.8	8.04E-04
2	0	3.9	0.005425976	2	0.1	3.9	0.003725	2	0.6	3.9	7.09E-04
2	0	4	0.004924833	2	0.1	4	0.003363	2	0.6	4	6.26E-04
2	0	4.1	0.004474799	2	0.1	4.1	0.00304	2	0.6	4.1	5.54E-04
2	0	4.2	0.00407016	2	0.1	4.2	0.002751	2	0.6	4.2	4.91E-04
2	0	4.3	0.003705897	2	0.1	4.3	0.002492	2	0.6	4.3	4.36E-04
2	0	4.4	0.003377597	2	0.1	4.4	0.00226	2	0.6	4.4	3.88E-04
2	0	4.5	0.00308137	2	0.1	4.5	0.002052	2	0.6	4.5	3.45E-04
2	0	4.6	0.002813786	2	0.1	4.6	0.001865	2	0.6	4.6	3.08E-04
2	0	4.7	0.002571813	2	0.1	4.7	0.001697	2	0.6	4.7	2.75E-04
2	0	4.8	0.002352767	2	0.1	4.8	0.001545	2	0.6	4.8	2.46E-04
2	0	4.9	0.002154273	2	0.1	4.9	0.001408	2	0.6	4.9	2.20E-04
2	0	5	0.001974219	2	0.1	5	0.001285	2	0.6	5	1.97E-04
p	ρ	θ	$L(Y,\theta)$	P	ρ	θ	$L(Y,\theta)$	p	ρ	θ	$L(Y,\theta)$
4	0	0	1	4	0.1	0	1	4	0.6	0	1
4	0	0.1	0.674943178	4	0.1	0.1	0.649	4	0.6	0.1	0.536
4	0	0.2	0.461175375	4	0.1	0.2	0.0428	4	0.6	0.2	0.296
4	0	0.3	0.318705785	4	0.1	0.3	0.286	4	0.6	0.3	0.168
4	0	0.4	0.222577051	4	0.1	0.4	0.193	4	0.6	0.4	0.098
4	0	0.5	0.156971156	4	0.1	0.5	0.132	4	0.6	0.5	0.058
4	0	0.6	0.111718579	4	0.1	0.6	0.091	4	0.6	0.6	0.035
4	0	0.7	0.080194328	4	0.1	0.7	0.064	4	0.6	0.7	0.022

4	0	0.8	0.058029215	4	0.1	0.8	0.045	4	0.6	0.8	0.014
4	0	0.9	0.0423086	4	0.1	0.9	0.032	4	0.6	0.9	0.008583
4	0	1	0.031067252	4	0.1	1	0.023	4	0.6	1	0.005528
4	0	1.1	0.02296674	4	0.1	1.1	0.017	4	0.6	1.1	0.003606
4	0	1.2	0.017086891	4	0.1	1.2	0.012	4	0.6	1.2	0.00238
4	0	1.3	0.012789448	4	0.1	1.3	0.008827	4	0.6	1.3	0.001071
4	0	1.4	0.009627978	4	0.1	1.4	0.00651	4	0.6	1.4	0.0007294
4	0	1.5	0.007287736	4	0.1	1.5	0.00483	4	0.6	1.5	0.0005013
4	0	1.6	0.00554515	4	0.1	1.6	0.003606	4	0.6	1.6	0.0003475
4	0	1.7	0.004240278	4	0.1	1.7	0.002707	4	0.6	1.7	0.0003475
4	0	1.8	0.00325792	4	0.1	1.8	0.002043	4	0.6	1.8	0.0002428
4	0	1.9	0.002514564	4	0.1	1.9	0.001549	4	0.6	1.9	0.000171
4	0	2	0.001949299	4	0.1	2	0.001151	4	0.6	2	0.0001213
4	0	2.1	0.001517437	4	0.1	2.1	0.000904	4	0.6	2.1	0.0008669
4	0	2.2	0.001186007	4	0.1	2.2	0.000696	4	0.6	2.2	0.0006235
4	0	2.3	0.000930552	4	0.1	2.3	0.000537	4	0.6	2.3	0.00004513
4	0	2.4	0.000732836	4	0.1	2.4	0.000417	4	0.6	2.4	0.00003286
4	0	2.5	0.000579198	4	0.1	2.5	0.000325	4	0.6	2.5	0.00002407
4	0	2.6	0.000459351	4	0.1	2.6	0.000254	4	0.6	2.6	0.00001772
4	0	2.7	0.000365516	4	0.1	2.7	0.000199	4	0.6	2.7	0.00001312
4	0	2.8	0.000291785	4	0.1	2.8	0.000157	4	0.6	2.8	0.00009763
4	0	2.9	0.000233651	4	0.1	2.9	0.000124	4	0.6	2.9	0.000073
4	0	3	0.000187662	4	0.1	3	9.83E-05	4	0.6	3	5.484E-06
4	0	3.1	0.000151162	4	0.1	3.1	7.82E-05	4	0.6	3.1	4.138E-06
4	0	3.2	0.000122105	4	0.1	3.2	6.24E-05	4	0.6	3.2	3.136E-06
4	0	3.3	9.89E-05	4	0.1	3.3	4.99E-05	4	0.6	3.3	2.39E-06
4	0	3.4	8.03E-05	4	0.1	3.4	4.00E-05	4	0.6	3.4	1.82E-06
4	0	3.5	6.54E-05	4	0.1	3.5	3.22E-05	4	0.6	3.5	1.40E-06
4	0	3.6	5.34E-05	4	0.1	3.6	0.000026	4	0.6	3.6	1.08E-06
4	0	3.7	4.37E-05	4	0.1	3.7	2.10E-05	4	0.6	3.7	8.33E-07
4	0	3.8	3.58E-05	4	0.1	3.8	1.71E-05	4	0.6	3.8	6.46E-07
4	0	3.9	2.94E-05	4	0.1	3.9	1.39E-05	4	0.6	3.9	5.03E-07
4	0	4	2.43E-05	4	0.1	4	1.13E-05	4	0.6	4	3.92E-07
4	0	4.1	2.00E-05	4	0.1	4.1	9.24E-06	4	0.6	4.1	3.07E-07

4	0	4.2	1.66E-05	4	0.1	4.2	7.57E-06	4	0.6	4.2	2.42E-07
4	0	4.3	1.37E-05	4	0.1	4.3	6.21E-06	4	0.6	4.3	1.90E-07
4	0	4.4	1.14E-05	4	0.1	4.4	5.11E-06	4	0.6	4.4	1.50E-07
4	0	4.5	9.49E-06	4	0.1	4.5	4.21E-06	4	0.6	4.5	1.19E-07
4	0	4.6	7.92E-06	4	0.1	4.6	3.48E-06	4	0.6	4.6	9.48E-08
4	0	4.7	6.61E-06	4	0.1	4.7	2.88E-06	4	0.6	4.7	7.55E-08
4	0	4.8	5.54E-06	4	0.1	4.8	2.39E-06	4	0.6	4.8	6.03E-08
4	0	4.9	4.64E-06	4	0.1	4.9	1.98E-06	4	0.6	4.9	4.83E-08
4	0	5	3.90E-06	4	0.1	5	1.65E-06	4	0.6	5	3.88E-08

Table (2)
Outflow Distribution by FFT Method

Y	p	ρ	f(y)	Y	p	ρ	f(y)	Y	p	ρ	f(y)
0.1	2	0	0.307150077	0.1	2	0.1	0.254199	0.1	2	0.6	0.129986176
0.2	2	0	0.547512519	0.2	2	0.1	0.461216	0.2	2	0.6	0.247610999
0.3	2	0	0.657467124	0.3	2	0.1	0.565592	0.3	2	0.6	0.32200665
0.4	2	0	0.714099427	0.4	2	0.1	0.626992	0.4	2	0.6	0.377222013
0.5	2	0	0.742416965	0.5	2	0.1	0.663958	0.5	2	0.6	0.420791601
0.6	2	0	0.720482865	0.6	2	0.1	0.658294	0.6	2	0.6	0.442208412
0.7	2	0	0.688878633	0.7	2	0.1	0.641988	0.7	2	0.6	0.455242968
0.8	2	0	0.650694021	0.8	2	0.1	0.617959	0.8	2	0.6	0.461885045
0.9	2	0	0.592467283	0.9	2	0.1	0.574965	0.9	2	0.6	0.455727235
1	2	0	0.541001822	1	2	0.1	0.535336	1	2	0.6	0.447624046
1.1	2	0	0.490942847	1.1	2	0.1	0.494934	1.1	2	0.6	0.436002327
1.2	2	0	0.432696564	1.2	2	0.1	0.446317	1.2	2	0.6	0.417373415
1.3	2	0	0.386764384	1.3	2	0.1	0.406166	1.3	2	0.6	0.400118463
1.4	2	0	0.343237272	1.4	2	0.1	0.367251	1.4	2	0.6	0.381345745
1.5	2	0	0.296082953	1.5	2	0.1	0.324474	1.5	2	0.6	0.358544288
1.6	2	0	0.261903614	1.6	2	0.1	0.291875	1.6	2	0.6	0.338934088
1.7	2	0	0.228658154	1.7	2	0.1	0.259707	1.7	2	0.6	0.318032153
1.8	2	0	0.193887935	1.8	2	0.1	0.226353	1.8	2	0.6	0.295491225
1.9	2	0	0.171582608	1.9	2	0.1	0.202723	1.9	2	0.6	0.279950571
2	2	0	0.147746683	2	2	0.1	0.17807	2	2	0.6	0.257105035
2.1	2	0	0.123313102	2.1	2	0.1	0.153055	2.1	2	0.6	0.23693493
2.2	2	0	0.110088149	2.2	2	0.1	0.137437	2.2	2	0.6	0.220522488
2.3	2	0	0.09288462	2.3	2	0.1	0.119064	2.3	2	0.6	0.202983379
2.4	2	0	0.076531788	2.4	2	0.1	0.101423	2.4	2	0.6	0.185756768
2.5	2	0	0.069660055	2.5	2	0.1	0.091982	2.5	2	0.6	0.172418257
2.6	2	0	0.057411425	2.6	2	0.1	0.078143	2.6	2	0.6	0.157482727
2.7	2	0	0.04667076	2.7	2	0.1	0.066082	2.7	2	0.6	0.143729419
2.8	2	0	0.044069107	2.8	2	0.1	0.060755	2.8	2	0.6	0.133063031
2.9	2	0	0.034571818	2.9	2	0.1	0.050085	2.9	2	0.6	0.120558135
3	2	0	0.028008384	3	2	0.1	0.042701	3	2	0.6	0.109737766
3.1	2	0	0.027587783	3.1	2	0.1	0.039955	3.1	2	0.6	0.101789406

3.2	2	0	0.020202414	3.2	2	0.1	0.031535	3.2	2	0.6	0.091157232
3.3	2	0	0.016844725	3.3	2	0.1	0.027054	3.3	2	0.6	0.082692289
3.4	2	0	0.017978751	3.4	2	0.1	0.026439	3.4	2	0.6	0.076674084
3.5	2	0	0.010955727	3.5	2	0.1	0.019173	3.5	2	0.6	0.068473291
3.6	2	0	0.010214434	3.6	2	0.1	0.017592	3.6	2	0.6	0.062421539
3.7	2	0	0.011757159	3.7	2	0.1	0.017636	3.7	2	0.6	0.057610077
3.8	2	0	0.005385818	3.8	2	0.1	0.011571	3.8	2	0.6	0.050545334
3.9	2	0	0.006308608	3.9	2	0.1	0.011671	3.9	2	0.6	0.046683702
4	2	0	0.007842335	4	2	0.1	0.011764	4	2	0.6	0.043132845
4.1	2	0	0.001917968	4.1	2	0.1	0.006201	4.1	2	0.6	0.037235522
4.2	2	0	0.004414902	4.2	2	0.1	0.007715	4.2	2	0.6	0.034365403
4.3	2	0	0.005090824	4.3	2	0.1	0.007992	4.3	2	0.6	0.031967497
4.4	2	0	0.000193729	4.4	2	0.1	0.002879	4.4	2	0.6	0.027094993
4.5	2	0	0.003507007	4.5	2	0.1	0.005431	4.5	2	0.6	0.025729484
4.6	2	0	0.003313975	4.6	2	0.1	0.004999	4.6	2	0.6	0.023423173
4.7	2	0	0.001427438	4.7	2	0.1	0.000802	4.7	2	0.6	0.019244479
4.8	2	0	0.003220955	4.8	2	0.1	0.004275	4.8	2	0.6	0.019069162
4.9	2	0	0.00195927	4.9	2	0.1	0.003098	4.9	2	0.6	0.017060554
5	2	0	0.002068735	5	2	0.1	0.004293	5	2	0.6	0.0139528
0.1	4	0	0.0023094	0.1	4	0.1	0.001544	0.1	4	0.6	0.000378247
0.2	4	0	0.0141834	0.2	4	0.1	0.00974039	0.2	4	0.6	0.002563261
0.3	4	0	0.0394764	0.3	4	0.1	0.02761574	0.3	4	0.6	0.007612032
0.4	4	0	0.0768381	0.4	4	0.1	0.0546486	0.4	4	0.6	0.015771071
0.5	4	0	0.122723	0.5	4	0.1	0.08890766	0.5	4	0.6	0.02716623
0.6	4	0	0.1735771	0.6	4	0.1	0.12851006	0.6	4	0.6	0.041524688
0.7	4	0	0.2256847	0.7	4	0.1	0.17023192	0.7	4	0.6	0.058306981
0.8	4	0	0.2758216	0.8	4	0.1	0.2122517	0.8	4	0.6	0.076664047
0.9	4	0	0.3216126	0.9	4	0.1	0.25247683	0.9	4	0.6	0.096264994
1	4	0	0.3610456	1	4	0.1	0.28942578	1	4	0.6	0.11672222
1.1	4	0	0.3935056	1.1	4	0.1	0.32166031	1.1	4	0.6	0.13711651
1.2	4	0	0.4183463	1.2	4	0.1	0.34878408	1.2	4	0.6	0.156879867
1.3	4	0	0.4353934	1.3	4	0.1	0.37038674	1.3	4	0.6	0.176122674
1.4	4	0	0.4452949	1.4	4	0.1	0.38656608	1.4	4	0.6	0.194196601
1.5	4	0	0.4483156	1.5	4	0.1	0.39694043	1.5	4	0.6	0.21063801

1.6	4	0	0.4453762	1.6	4	0.1	0.40251924	1.6	4	0.6	0.225769318
1.7	4	0	0.4375844	1.7	4	0.1	0.4033437	1.7	4	0.6	0.23881429
1.8	4	0	0.4250504	1.8	4	0.1	0.39979993	1.8	4	0.6	0.250100406
1.9	4	0	0.4094939	1.9	4	0.1	0.39267816	1.9	4	0.6	0.259709428
2	4	0	0.3909803	2	4	0.1	0.38259294	2	4	0.6	0.267427947
2.1	4	0	0.3706077	2.1	4	0.1	0.36993931	2.1	4	0.6	0.273026569
2.2	4	0	0.3486125	2.2	4	0.1	0.35519007	2.2	4	0.6	0.276951319
2.3	4	0	0.3263514	2.3	4	0.1	0.33926197	2.3	4	0.6	0.279371117
2.4	4	0	0.3035266	2.4	4	0.1	0.32173072	2.4	4	0.6	0.280121206
2.5	4	0	0.2809494	2.5	4	0.1	0.30365626	2.5	4	0.6	0.279409943
2.6	4	0	0.2587501	2.6	4	0.1	0.28546236	2.6	4	0.6	0.277488552
2.7	4	0	0.2370709	2.7	4	0.1	0.26722108	2.7	4	0.6	0.27424234
2.8	4	0	0.2164948	2.8	4	0.1	0.24858399	2.8	4	0.6	0.26997676
2.9	4	0	0.1967934	2.9	4	0.1	0.23092202	2.9	4	0.6	0.264607377
3	4	0	0.1786108	3	4	0.1	0.21350653	3	4	0.6	0.258503607
3.1	4	0	0.1612458	3.1	4	0.1	0.19691969	3.1	4	0.6	0.251619607
3.2	4	0	0.1453588	3.2	4	0.1	0.18083624	3.2	4	0.6	0.244399931
3.3	4	0	0.1306742	3.3	4	0.1	0.16538458	3.3	4	0.6	0.236336603
3.4	4	0	0.1168619	3.4	4	0.1	0.15123302	3.4	4	0.6	0.228435891
3.5	4	0	0.1040794	3.5	4	0.1	0.13804216	3.5	4	0.6	0.219662318
3.6	4	0	0.0930648	3.6	4	0.1	0.12541055	3.6	4	0.6	0.211098294
3.7	4	0	0.0823001	3.7	4	0.1	0.11345658	3.7	4	0.6	0.202223127
3.8	4	0	0.0730776	3.8	4	0.1	0.10291761	3.8	4	0.6	0.193046635
3.9	4	0	0.0649787	3.9	4	0.1	0.09273654	3.9	4	0.6	0.184211365
4	4	0	0.0575105	4	4	0.1	0.08365158	4	4	0.6	0.175799018
4.1	4	0	0.0507757	4.1	4	0.1	0.07514805	4.1	4	0.6	0.166544336
4.2	4	0	0.0441841	4.2	4	0.1	0.06732814	4.2	4	0.6	0.158501652
4.3	4	0	0.0392328	4.3	4	0.1	0.06030232	4.3	4	0.6	0.149666007
4.4	4	0	0.0346214	4.4	4	0.1	0.05418999	4.4	4	0.6	0.141496076
4.5	4	0	0.0296279	4.5	4	0.1	0.04834024	4.5	4	0.6	0.134105191
4.6	4	0	0.0258463	4.6	4	0.1	0.04280787	4.6	4	0.6	0.126000519
4.7	4	0	0.0225913	4.7	4	0.1	0.03848896	4.7	4	0.6	0.118813742
4.8	4	0	0.0199359	4.8	4	0.1	0.03380442	4.8	4	0.6	0.111814631