

## Torsional Behavior of High-Strength Reinforced Concrete Beams

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### ABSTRACT:

Recent methods for torsional design of reinforced concrete beams tend to the use of space truss analogy, instead of the earlier skew bending theory. A total of (43) rectangular beams made of high strength concrete (HSC) that failed under pure torsion are considered in this work. These have been taken from the literature.

Regression analysis was performed on the results to obtain two representative equations to predict: cracking torsional moment  $T_{cr}$  and torsional resistance moment  $T_r$ . The first equation is based on (3) major parameters that include concrete compressive strength  $f'_c$  and sectional dimensions, while the second one is based on (7) major parameters which include the quantification of the influence of both transverse and longitudinal reinforcement.

When the ACI 318M-05 Code design equation was applied, it gave a coefficient of variation (COV) of (2.8) percent for the ratio of tested / calculated torsional strength ( $T_{u-test} / T_{r-calc.}$ ), however, the proposed equation has led to a COV of (1.2) percent.

**Keywords:** beams; cracking torsional moment; high strength concrete; longitudinal reinforcement; torsional resistance moment; transverse reinforcement.

### سلوك اللي في العتبات الخرسانية المسلحة المصنوعة من خرسانة عالية المقاومة

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### الخلاصة:

تعتمد الطرق الحديثة لتصميم اللي في العتبات الخرسانية المسلحة على نظرية المسنم الفضائي (Space Truss) بدلا من الإنشاء المائل (Skew Bending). تمت دراسة (43) عتبة خرسانية مسلحة مستطيلة المقطع مصنوعة من خرسانة عالية المقاومة فشلت تحت تأثير اللي الخالص مأخوذة من بحوث سابقة.

حلت النتائج بطريقة التحليل الإرتدادي للحصول على معادلتين لحساب عزم اللي الذي يسبب التشقق ( $T_{cr}$ ) ومقاومة اللي التصميمية ( $T_r$ ). تعتمد المعادلة الأولى على ثلاثة معاملات رئيسية هي مقاومة إنضغاط الخرسانة وأبعاد المقطع، بينما تعتمد المعادلة الثانية على سبعة معاملات رئيسية تشمل على التقييم الكمي لتأثير حديد التسليح الطولي والعرضي للعتبات.

عندما طبقت طريقة التصميم للمدونة (ACI ٣١٨M-٠٥)، تم الحصول على معامل تغاير مقداره (٤٢,٨%) لنسبة مقاومة اللي العملية / المقاومة التصميمية ( $T_{u-test} / T_{r-calc}$ ). وبالمقارنة فإن تطبيق المعادلة المقترحة نتج عنه معامل تغاير مقداره (١١,٢%)

## INTRODUCTION:

Use of HSC leads to more economical building structures resulting from small sections of structural members and large usable floor areas. It also leads to a reduction in overall building height and dead loads resulting from the use of thinner slabs and shallower beams [1, 2, 3, 4].

Pure torsion only occurs infrequently in practice. Normally, it arises as a combined action with bending and/or shear. It can become a predominant action in structures such as eccentrically loaded box beams, curved girders, spandrel beams, structures of irregular shapes, and spiral staircases [5, 6]. However, in bridges, torsion constitutes a significant design action because of eccentric forces. Since large bridge construction is an obvious application of HSC, an investigation of reinforced HSC beams subjected to pure torsion is of interest. In ١٩٦٩ the then ACI Committee ٤٣٨ published its report recommending torsional design based on the skew bending theory [7]. The ACI ٣١٨ Committee used this theory starting from the ١٩٧١ Code [8] which continued up to the ١٩٨٩ Code [9]. BS-٨٥ [10] and BS-٩٧ [11] Code versions also used the same approach.

The most recognized theoretical model of pure torsion in reinforced concrete is the space truss model. Based on post-doctoral research published by MacGregor and Ghoneim [12], the ACI Code in ١٩٩٥ [13] accepted this model. This is now included in the latest ACI ٣١٨-٠٨ Code [14]. The Canadian [15], AASHTO-LRFD [16], and European [17] Codes also use space truss analogy for torsional design.

There are a number of more accurate but more complex design procedures in the literature, [18, 19] but they are not considered in this work.

## RESEARCH SIGNIFICANCE:

This paper provides an evaluation of the design provisions for pure torsion based on (٧) different code approaches: ٢ using skew bending theory (ACI ٣١٨M-٨٩ [9] and BS-٩٧ [11]); plus ٥ using space truss analogy (ACI ٣١٨M-٩٩ [10], ACI ٣١٨M-٠٥ [12], Canadian [15], AASHTO-LRFD [16], and EURO [17]). In addition, a number of equations adopted by some researchers to predict  $T_{cr}$  value are included. A total of (٤٣) tests of torsional failure of tested beams is used to evaluate the previous (٧) methods. Two proposed equations which are based on regression analysis are also introduced. The first one estimates the cracking torsional

moment  $T_{cr}$  of HSC beams, while the second one predicts the torsional resistance moment of such beams.

## EXPERIMENTAL RESULTS:

All available experimental results from test series on pure torsion are obtained from the literature. The ranges of the variables of these ( $\epsilon^3$ ) rectangular solid section beams are listed in Table (1). The main significant parameters are concrete compressive strength  $f'_c$ , aspect ratio ( $\frac{y}{x}$ ), sectional area  $A_{cp}$ , nominal stirrup strength  $\rho_v.f_{yt}$ , and nominal longitudinal steel strength  $\rho_\ell.f_{y\ell}$ . These beams include 0, 10, 12, 14, and 16 specimens from the references 22, 23, 24, 0 and 20 respectively.

**Table 1- Ranges of the variables for the  $\epsilon^3$  tested beams.**

Detail	$f'_c$ (MPa)	$\frac{y}{x}$	$A_{cp}$ (mm <sup>2</sup> )	$\rho_v.f_{yt}$ (MPa)	$\rho_\ell.f_{y\ell}$ (MPa)
Low	41,700	1,430	40000	1,700	2,880
High	109,800	4,000	170000	7,200	24,700
High/Low	2,633	2,797	4,370	4,260	8,076

Where:

$f'_c$  = cylinder compressive strength of concrete, MPa

$x$  = the shorter side of the cross section, mm

$y$  = the longer side of the cross section, mm

$A_{cp}$  = area enclosed by outside perimeter of concrete cross section, mm<sup>2</sup>

$$\rho_v = \text{stirrups ratio} = \frac{2A_t}{b.s}$$

$f_{yt}$  = specified yield strength of transverse reinforcement, MPa

$$\rho_\ell = \text{longitudinal steel ratio} = \frac{A_\ell}{b.h}$$

$f_{y\ell}$  = specified yield strength of longitudinal reinforcement, MPa

## EVALUATION OF EXPERIMENTAL RESULTS:

### Cracking Torsional Moment Equations:

Following are the methods considered in this work to estimate the cracking torsional moment of the beams:

#### 1. ACI 318M-89 Code [4] method:

$$T_{cr} = \frac{1}{6} \sqrt{f'_c} \cdot \sum x^2 \cdot y \quad (1)$$

Where:

$T_{cr}$  = cracking torsional moment, N.mm

#### 2. ACI 318M-08 Code [11] method:

$$T_{cr} = 0.33 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \quad (2)$$

Where:

$P_{cp}$  = outside perimeter of concrete cross section, mm.

#### 3. Canadian-94 Code [12] method:

$$T_{cr} = 0.4 \phi_c \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \quad (3)$$

Where  $\phi_c = 0.7$

#### 4. Hsu and Mo's [13] method:

$$T_{cr} = 0.5 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \quad (4)$$

#### 5. Koutchoukali and Belarbi's [14] method:

$$T_{cr} = 0.46 \sqrt{f'_c} \left( \frac{A_{cp}^2}{P_{cp}} \right) \quad (5)$$

#### 6. Fang and Shiau's [15] method:

$$T_{cr} = 0.095 \sqrt{f'_c} x^2 y \quad (6)$$

### Torsion Design Equations:

(V) Methods of existing design codes are included in this study to predict the torsional resistance moment of the beams. To make comparison between design methods, torsional

resistance  $T_{r-calc}$ . is used instead of nominal  $T_{n-calc}$ . throughout (e.g.  $T_{r-calc} = \alpha \cdot T_{n-calc}$ . per ACI 318M-09 Code [4] method).

The design code methods are based on two approaches:

**a. Skew Bending Theory:**

Torsional strength of beams is composed of two parts: the concrete contribution  $T_c$  and the reinforcement contribution  $T_s$ .

**1. ACI 318M-09 Code [4] method:**

$$T_{r-calc} \leq T_{rACI-t-89} = 0.85 \left[ \frac{\sqrt{f'_c}}{15} \sum x^2 \cdot y + \alpha_t \cdot \frac{A_t \cdot x_1 \cdot y_1 \cdot f_{yt}}{S} \right] \quad (7,1)$$

$$T_{r-calc} \leq T_{rACI-l-89} = 0.85 \left[ \frac{\sqrt{f'_c}}{15} \sum x^2 \cdot y + \alpha_l \cdot \frac{A_l \cdot x_1 \cdot y_1 \cdot f_{yl}}{P_h} \right] \quad (7,2)$$

Where:

$T_{rACI-t-89}$  = torsional resistance moment provided by concrete and stirrups, calculated by ACI-09 method, N.mm.

$\alpha_t$  =  $0.16 + 0.33(y/x) \leq 1.0$

$A_t$  = area of one leg of closed stirrup resisting torsion within spacing  $S$ , mm<sup>2</sup>.

$x$  = shorter centre-to-centre dimension of closed rectangular stirrup, mm.

$y$  = longer centre-to-centre dimension of closed rectangular stirrup, mm.

$S$  = spacing of transverse torsional reinforcement in direction parallel to longitudinal reinforcement, mm.

$T_{rACI-l-89}$  = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by ACI-09 method, N.mm.

$A_l$  = area of longitudinal reinforcement required for torsion, mm<sup>2</sup>.

$P_h$  = perimeter of centerline of outermost closed transverse torsional reinforcement, mm.

**2. BS 8110-97 Code [5] method:**

$$T_{r-calc} \leq T_{rBS-t} = 0.0375 x^2 \left( y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6 A_t \cdot x_1 \cdot y_1 (0.95 f_{yt})}{S} \quad (8,1)$$

$$T_{r-calc} \leq T_{rBS-l} = 0.0375 x^2 \left( y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6 A_l \cdot x_1 \cdot y_1 (0.95 f_{yl})}{P_h} \quad (8,2)$$

Where it is assumed that  $f'_c = 0.85 f_{cu}$ ,

$T_{rBS-t}$  = torsional resistance moment provided by concrete and stirrups, calculated by BS-81 method, N.mm.

$T_{rBS-l}$  = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by BS-81 method, N.mm.

### b. Space Truss Analogy:

This new method is considerably simpler to understand and apply than the previous one. It can also be used for prestressed concrete loaded in torsion, a case not covered by the ACI 318M-89 [1] Code. It assumes that the concrete contribution  $T_c = 0$ . In this method, the beam cross section is idealized as a tube. After cracking, the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners, and concrete compression diagonals approximately centered on the stirrups. The diagonals are at an angle  $\theta$  to the member longitudinal axis.

The most significant difference between the torsion provisions of the ACI Codes and the AASHTO-LRFD [11] specifications is the specified value of  $\theta$ . For non prestressed sections, the ACI Code recommends  $(\theta = 30^\circ)$  degrees, while the AASHTO [11] provisions permit a value of about  $(\theta = 45^\circ)$  degrees (based on the longitudinal strain at mid-span of the section) [16, 17]. The methods adopted this analogy are:

#### 1. ACI 318M-89 Code [1] method:

$$T_{r-calc} \leq T_{rACI-t-89} = 0.85 \left[ \frac{1.7 A_{oh} A_t f_{yt}}{S} \right] \quad (9,1)$$

$$T_{r-calc} \leq T_{rACI-l-89} = 0.85 \left[ \frac{1.7 A_{oh} A_l f_{yl}}{P_h} \right] \quad (9,2)$$

Where:

$T_{rACI-t-89}$  = torsional resistance moment provided by stirrups, calculated by ACI-89 method, N.mm.

$T_{rACI-l-89}$  = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-89 method, N.mm.

$A_o$  = area enclosed by centerline of outermost closed transverse torsional reinforcement, mm<sup>2</sup>.

#### 2. ACI 318M-08 Code [11] method:

$$T_{r-calc} \leq T_{rACI-t-05} = 0.75 \left[ \frac{1.7 A_{oh} \cdot A_t \cdot f_{yt}}{S} \right] \quad (11,1)$$

$$T_{r-calc} \leq T_{rACI-l-05} = 0.75 \left[ \frac{1.7 A_{oh} \cdot A_\ell \cdot f_{y\ell}}{P_h} \right] \quad (11,2)$$

Where:

$T_{rACI-t-05}$  = torsional resistance moment provided by stirrups, calculated by ACI-05 method, N.mm.

$T_{rACI-l-05}$  = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-05 method, N.mm.

**12. Canadian-94 Code [12] method:**

$$T_{r-calc} \leq T_{rCan-t} = 0.85 \left[ \frac{1.7 A_{oh} \cdot A_t \cdot f_{yt}}{S} \right] \quad (12,1)$$

$$T_{r-calc} \leq T_{rCan-l} = 0.85 \left[ \frac{1.7 A_{oh} \cdot A_\ell \cdot f_{y\ell}}{P_h} \right] \quad (12,2)$$

Where:

$T_{rCan-t}$  = torsional resistance moment provided by stirrups, calculated by Canadian Code method, N.mm.

$T_{rCan-l}$  = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by Canadian Code method, N.mm.

It can be seen that the Canadian Code [12] method is symmetric with the ACI 318M-99 Code [11] method.

**13. AASHTO-LRFD-98 Bridge Design Specifications [13] method:**

$$T_{r-calc} \leq T_{rAASHTO-t} = 0.85 \left[ \frac{1.7 A_{oh} \cdot A_t \cdot f_{yt}}{S} \cdot Cot \theta \right] \quad (13,1)$$

$$T_{r-calc} \leq T_{rAASHTO-l} = 0.85 \left[ \frac{1.7 A_{oh} \cdot A_\ell \cdot f_{y\ell}}{0.9 P_h} \cdot tan \theta \right] \quad (13,2)$$

Where:

$T_{rAASHTO-t}$  = torsional resistance moment provided by stirrups, calculated by AASHTO method, N.mm.

$T_{r \text{ AASHTO-}\ell}$  = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by AASHTO method, N.mm.

$\theta$  = angle of inclination of compression diagonals to the member longitudinal axis, equal to 36 degrees.

o. **EURO-89 Code<sup>[13]</sup> method:**

$$T_{rEU} = 1.7 A_{oh} \sqrt{\frac{A_t}{S} \cdot f_{yt} \cdot \frac{A_\ell}{P_h} \cdot f_{y\ell}} \quad (13)$$

Where:

$T_{rEU}$  = torsional resistance moment calculated by EURO method, N.mm.

**Statistical Evaluation of Existing Methods:**

Table (5) shows the results of the cracking torsional moment of (26) specimens (out of 33 tested beams- not all the values of  $T_{cr}$  are included in the references). The comparison between these results and predicted values ( $T_{cr-test} / T_{cr-calc.}$ ) leads to a range of (0.846-1.931) for the mean of this ratio. It can be seen that the ACI 318M-89 Code<sup>[4]</sup> method is the one with the greatest amount (all the 26 specimens) of unacceptable predictions-based on the value of ( $T_{cr-test} / T_{cr-calc.}$ ) < 1. The lowest ratio for this code is (0.717).

In contrast, the ACI 318M-01<sup>[11]</sup>, Canadian<sup>[10]</sup>, and Fang and Shiau<sup>[12]</sup> methods lead to good predictions with no results of the previous ratio < 1, but the ACI 318M-00 Code<sup>[11]</sup> method seems to be the best due to the lowest values of low and high of the ratio ( $T_{cr-test} / T_{cr-calc.}$ ) among the other two methods. The coefficient of variation (COV) gives a good indication as a measure of the relevance of the prediction method for the ratio ( $T_{cr-test} / T_{cr-calc.}$ ). It can be seen that the difference in COV values of all methods is very small (ranging between 9.38-9.48 percent), therefore this coefficient does not indicate which method is the best.

**Table 5- Statistical analysis of the ratio ( $T_{cr-test} / T_{cr-calc.}$ ) for 26 tests.**

Detail	ACI-89 <sup>[4]</sup>	ACI-01 <sup>[11]</sup>	Canadian <sup>[10]</sup>	Reference (18)	Reference (20)	Reference (20)	Proposed Eq. (14)
$\bar{x}$	0.846	1.400	1.931	0.927	1.008	1.480	1.220
S.D.	0.080	0.132	0.181	0.087	0.090	0.141	0.109
COV, %	9.479	9.379	9.379	9.379	9.379	9.479	8.926
Low	0.717	1.100	1.088	0.762	0.829	1.207	1.028
High	0.973	1.638	2.202	1.081	1.170	1.708	1.419
High/Low	1.308	1.418	1.418	1.418	1.418	1.308	1.381
Number < 1	26	0	0	22	9	0	0



Table (3) shows the values of the results of the (43) tested beams, compared with the predicted strength ( $T_{u-test} / T_{r-calc.}$ ). The range of the mean of this ratio is (1,141-1,809). Based on the value of ( $T_{u-test} / T_{r-calc.}$ ) < 1, the EURO [17] method leads to unsafe predictions (14 specimens). The lowest ratio for this code is (0,394). On the other hand, the ACI 318M-08 Code [11] method is the most conservative of the existing methods with only (0) results with the previous ratio < 1. From table (3) it can be seen that the ACI 318M-99[12], Canadian [10], and ACI 318M-08[11] Code methods lead to the least relevant prediction with a high COV of (42,821) percent for each one of them. From this point of view, the best COV is (24,192) percent for the ACI 318M-89 Code [9] method. The COV values are (33,872, 36,138, and 41,300) percent for BS [11], AASHTO [13], and EURO [17] methods, respectively.

**Table 3- Statistical analysis of the ratio ( $T_{u-test} / T_{r-calc.}$ ) for 43 tests.**

Detail	ACI-89[9]	BS[11]	ACI-99[12] and Canadian[10]	ACI-08[11]	AASHTO[13]	EURO[17]	Proposed Eq. (10)
$\bar{x}$	1,388	1,264	1,741	1,809	1,472	1,141	1,200
S.D.	0,336	0,428	0,702	0,796	0,532	0,472	0,140
COV, %	24,192	33,872	42,821	42,821	36,138	41,300	11,172
Low	0,801	0,633	0,703	0,797	0,511	0,394	0,999
High	2,112	2,173	3,030	3,433	2,304	2,033	1,489
High/Low	2,630	3,430	4,308	4,308	4,608	5,108	1,490
Number< 1	0	12	8	0	10	14	1

### Regression Analysis of Test Results:

Using regression analysis, the (26) and (43) test results of cracking and resistance moment, respectively were analyzed by computer. The aim is to obtain simple and conservative equations to predict cracking torsional moment and torsional resistance moment

of HSC rectangular section beams under pure torsion, that give the lowest possible COV values of the ratios ( $T_{cr-test} / T_{cr-calc.}$ ) and ( $T_{u-test} / T_{r-calc.}$ ). This has led to the following prediction equations:

$$T_{cr-Proposed} = 0.115(f'_c)^{0.6} \cdot x^{1.92} \cdot y \quad (14)$$

Where:

$T_{cr-proposed}$  = cracking torsional moment calculated by proposed method, N.mm

$$T_{r-proposed} = 6.2 \frac{A_{oh}^{1.2}}{S^{0.55}} \left( \frac{A_t \cdot A_\ell}{P_h} \right)^{0.26} (f_{yt} \cdot f_{y\ell})^{0.026} \quad (15)$$

Where:

$T_{r-proposed}$  = torsional resistance moment provided by stirrups and longitudinal torsion reinforcement, calculated by proposed method, N.mm.

Equation (14) is based on the 3 main parameters  $f'_c$ ,  $x$ , and  $y$ , while equation (15) is based on the 7 main parameters  $A_{oh}$ ,  $S$ ,  $A_t$ ,  $A_\ell$ ,  $P_h$ ,  $f_{yt}$ , and  $f_{y\ell}$ . Tables 4 and 5 had show the summary of statistical evaluation of the proposed methods. The proposed equation (14) which estimates  $T_{cr}$  gives the best COV value of (8.926) percent among all other methods with no result having the ratio of ( $T_{cr-test} / T_{cr-calc.}$ ) < 1 (Table 4).

As shown in Table (5), when the proposed equation (15) [that predicts  $T_r$ ] was applied, it led to much safer prediction with only one specimen (out of 13) having the ratio of ( $T_{u-test} / T_{r-calc.}$ ) < 1 essentially 0.999  $\cong$  1. It can be seen that there is a great reduction in the COV value that was obtained by applying the proposed equation (15) [COV = 11.14 percent]. In addition, the value of high/low of the previous ratio was (1.49) for this equation, while the range of this ratio was (2.630-0.108) for all other methods.

To illustrate the relevance of the proposed method – equation (15), the ratio of ( $T_{u-test} / T_{r-calc.}$ ) has been compared by this method with that of the latest available ACI 318M-08 Code [11] procedure (which is the same as the procedure of the ACI 318-08 Code [14]). These are shown in Figs. 14, 15, 16, 17, and 18.

The comparison in Fig. 1 between the ACI 318M-08 [11] method and the proposed equation (15) shows clearly that for the range of  $f'_c$  (17, 21 – 29, 34) MPa, the proposed method shows much less scatter in the results. In addition, the number of unsafe results ( $T_{u-test} / T_{r-calc.}$ ) < 1 is greater for the ACI 318M-08 Code [11] method, despite the fact that this ratio is high in several cases (up to 3.433). It is to be noted that there is a tendency toward greater safety with rising  $f'_c$  values for both methods which is an important advantage since much fewer tests are made on HSC beams in torsion.

Similar conclusions regarding the much greater scatter and the number of unsafe results by the ACI 318M-08 Code [11] method can be seen in Fig. 2 (influence of the aspect

ratio  $\frac{y}{x}$ ,  $\gamma$  (influence of sectional area  $A_{cp}$ ),  $\xi$  (influence of the nominal stirrup strength  $\rho_v f_{yt}$ ), and  $\rho$  (influence of the nominal longitudinal steel strength  $\rho_l f_{yl}$ ). For the ACI 318M-08 Code [11] method, there is a significant rise in the factor of safety with rising value of  $\frac{y}{x}$ , while the safety factor of the proposed method – equation (10) is not influenced by variation of  $\frac{y}{x}$  value- Fig. 2. The influence of  $A_{cp}$  is indicated in Fig. 3 which shows that for ACI 318M-08 [11] method, the factor of safety decreases with increasing  $A_{cp}$  value. In contrast, the safety factor of the proposed method is approximately constant with variation of  $A_{cp}$  value.

Figs. 4 and 5 show clear trends for the overestimation of the influence of the nominal steel strength ( $\rho_v f_{yt}$  and  $\rho_l f_{yl}$ ) by the ACI 318M-08 [11] method. On the other hand, the proposed method shows no variation in the safety factor with rising value of  $\rho_v f_{yt}$  (ranging between 1,7-2,2) MPa, and  $\rho_l f_{yl}$  (ranging between 2,88-29,5) MPa.

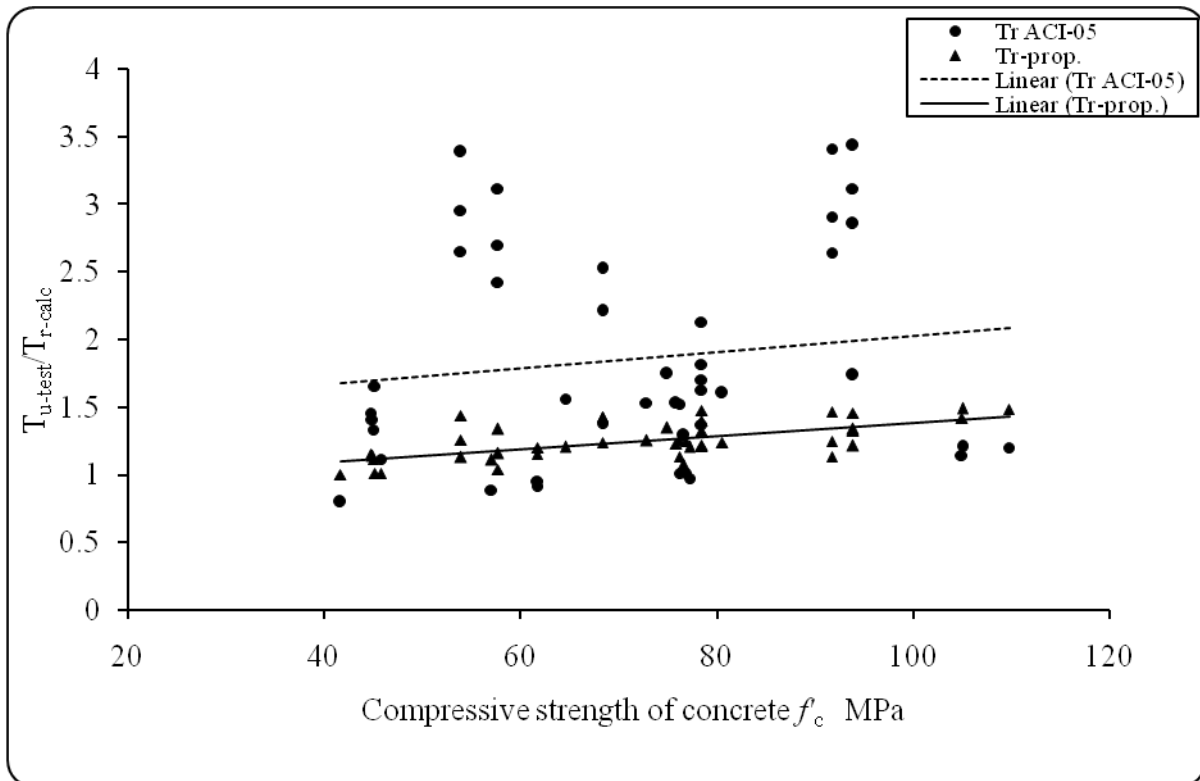


Fig. 1 - Influence of compressive strength of concrete  $f'_c$  on test results

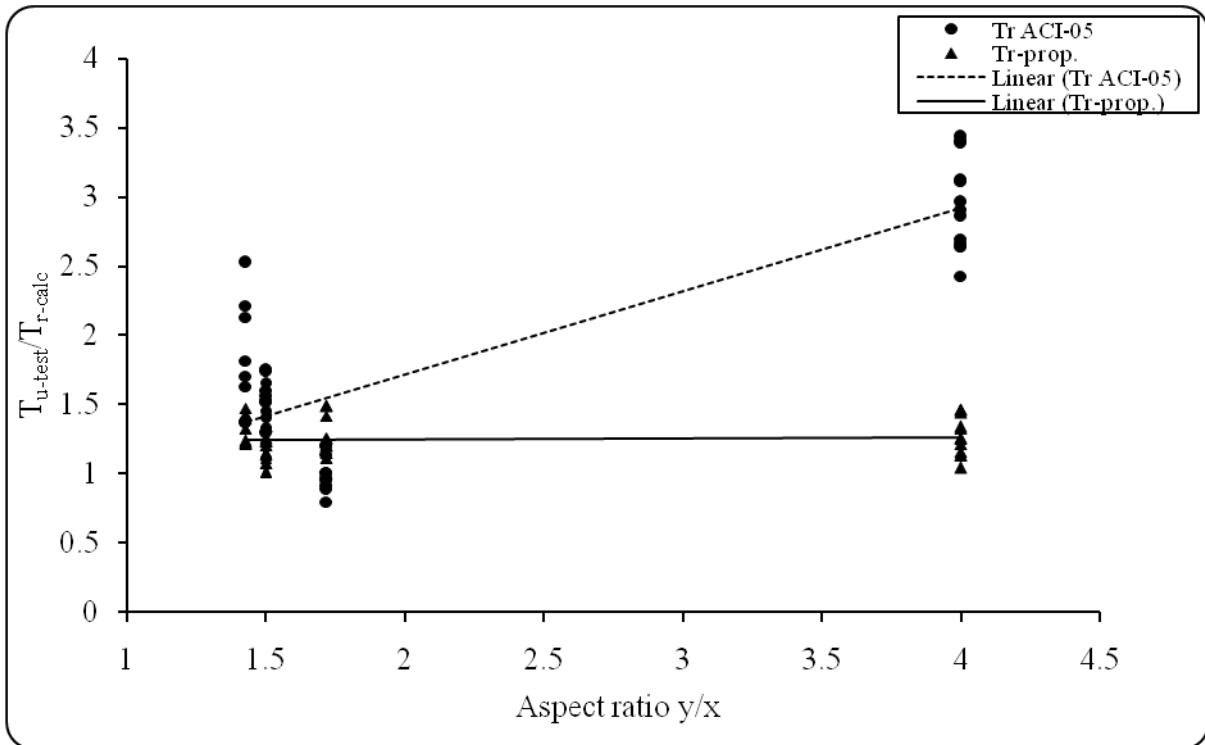


Fig.2 - Influence of aspect ratio  $y/x$  on test results

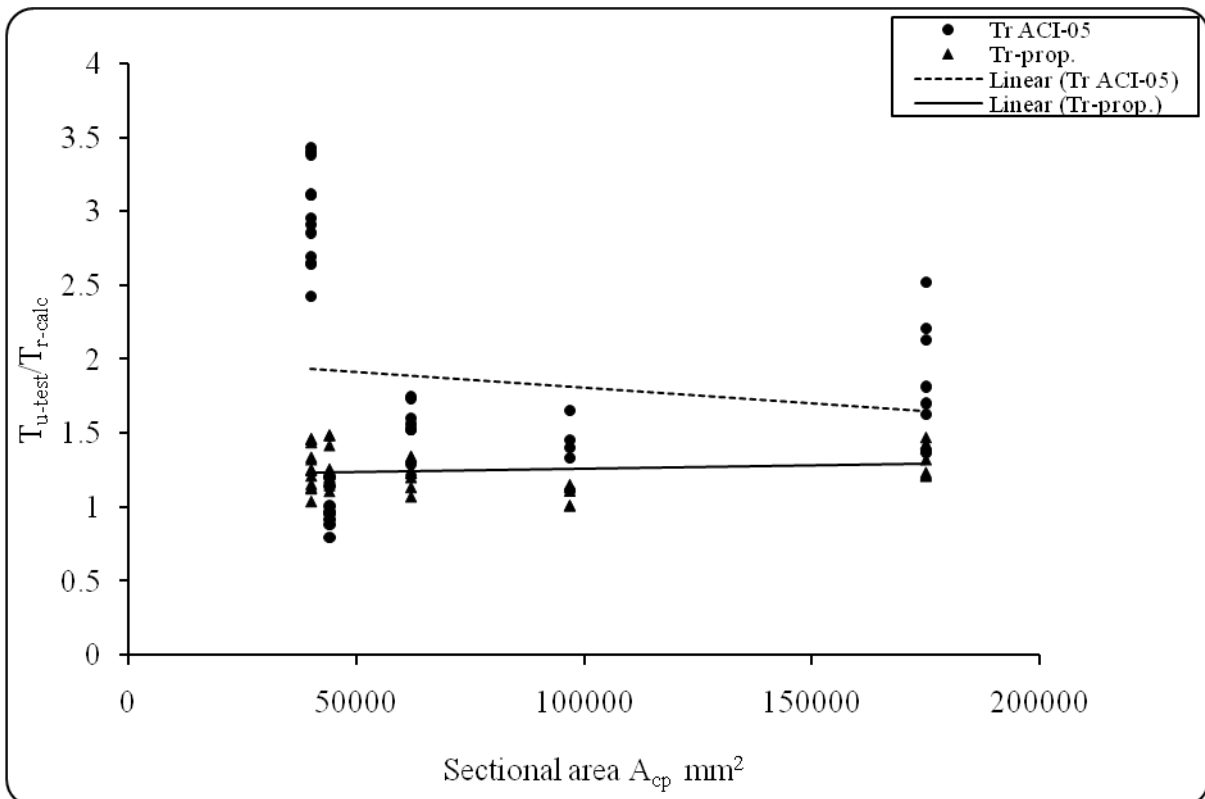


Fig.3 - Influence of sectional area  $A_{cp}$  on test results

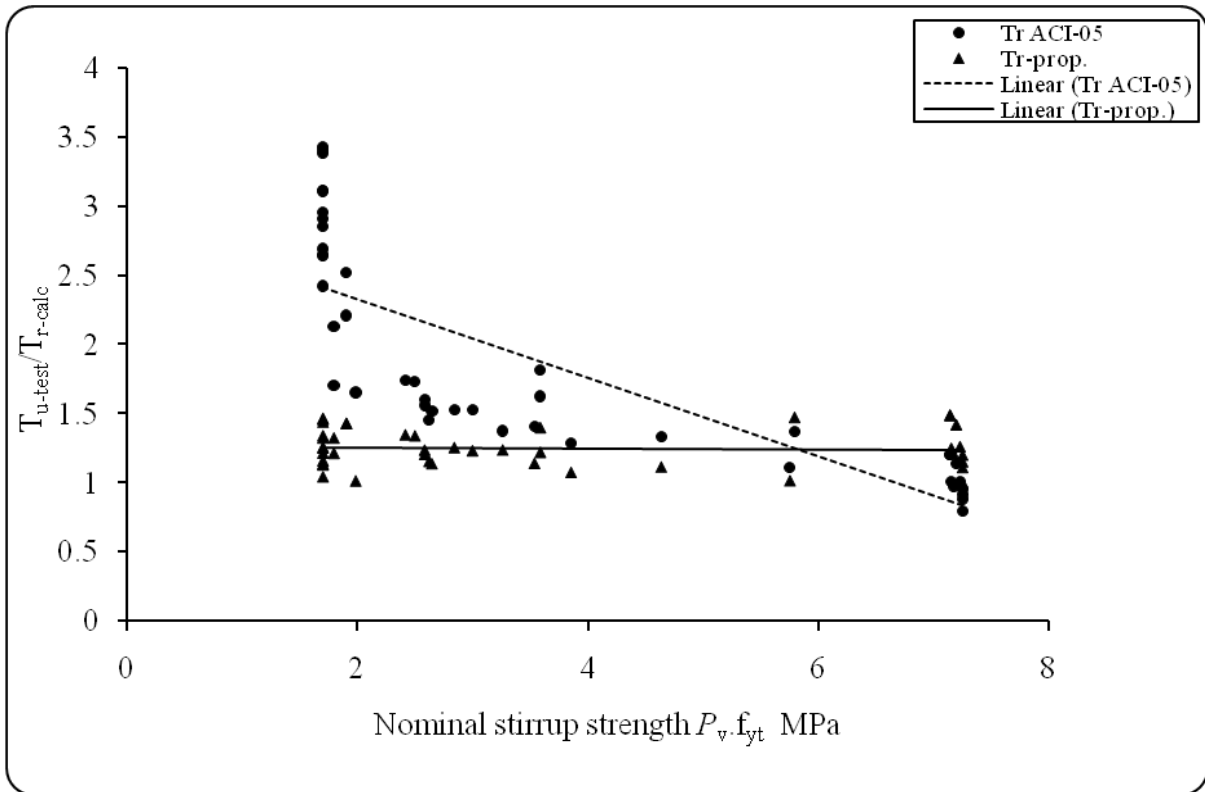


Fig.4 - Influence of nominal stirrup strength  $\rho_v.f_{yt}$  on test results

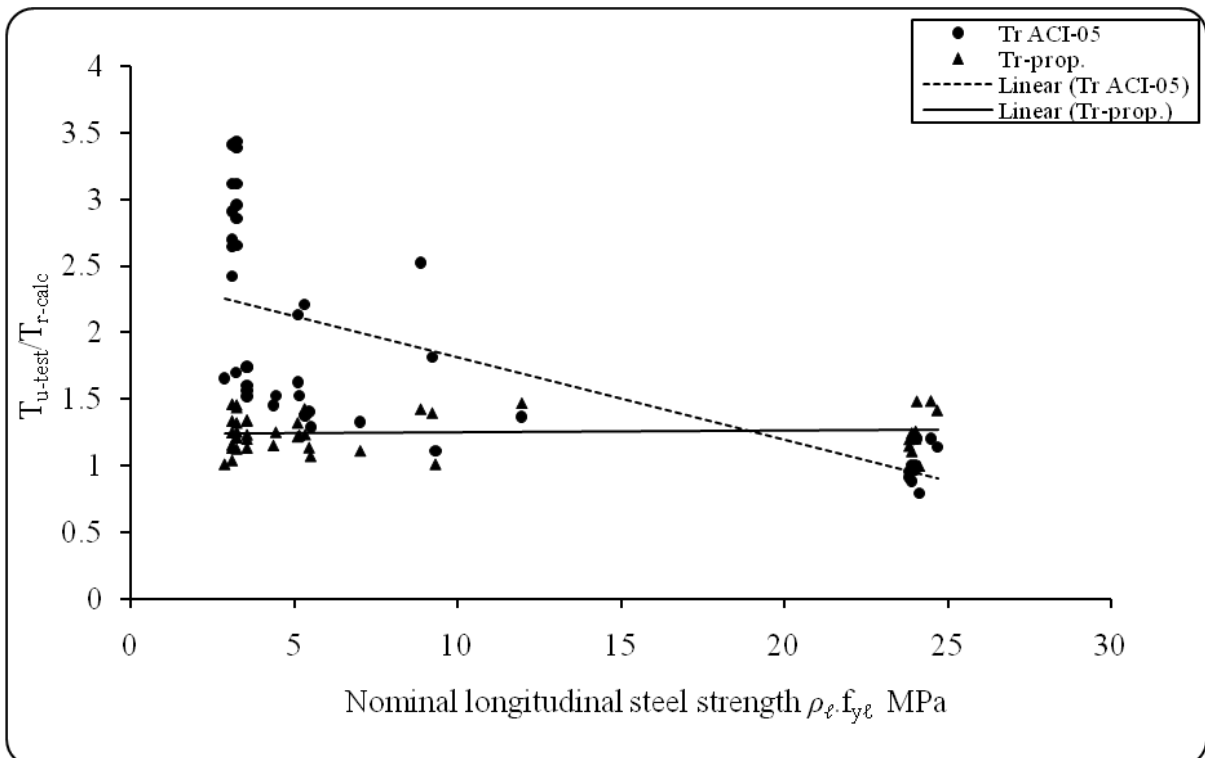


Fig.5 - Influence of nominal longitudinal steel strength  $\rho_e.f_{ye}$  on test results

**CONCLUSIONS:**

Based on this work, the following conclusions are made:

1. A simple equation (15) is presented to estimate cracking torsional moment  $T_{cr}$  in HSC rectangular section beams.
2. Another equation (16) is suggested for predicting torsional resistance moment  $T_r$  of such beams. This method agrees with the recent trend of space truss analogy of shear flow that bases strength only on the contribution of reinforcement- as in ACI 318M-90 and later ACI Code versions, Canadian, AASHTO-LRFD, and EURO methods.
3. The existing methods, give COV values between (9,38-9,48) percent for the ratio ( $T_{cr-test}/T_{cr-calc}$ ), while the proposed equation (15) leads to a COV value of (8,93) percent for this ratio.
4. The COV value of the existing code design methods ranges between (25,19-27,82) percent for the ratio ( $T_{u-test}/T_{r-calc}$ ). On the other hand, a significant reduction in COV value has been obtained when the proposed equation (16) was applied, which led to the best value of COV-11,17 percent for this ratio.
5. The proposed method – equation (16) is similar to the EURO one- equation (13), with one major difference. Proposed equation (16) uses powers of values less than 1,0 for  $\rho$  parameters:  $A_t$ ,  $A_\ell$ ,  $P_h$ ,  $f_{yt}$ , and  $f_{y\ell}$ . Therefore, for the ratio ( $T_{u-test}/T_{r-calc}$ ), equation (16) gives low value of essentially 1,999  $\cong$  1 (Table 3). In contrast, EURO method has respective values of 1,394 and 14.
6. The latest code design method (ACI 318M-08) has the highest mean value (at 1,86) of all the other methods and the highest ratio of  $T_{u-test}/T_{r-calc}$  (at 3,23). Despite this, ACI 318M-08 Code [11] method leads to  $\rho$  unsafe ratios, with a low value of (1,8).
7. For a range of  $f'_c$  between (21,7-10,8) MPa, the proposed equation (16) gives safe prediction (Fig.1), as well as a rising factor of safety with increasing  $f'_c$ . This is considered useful, since the number of available HSC tests in torsion is limited, compared to normal strength concrete ( $f'_c < 21,7$  MPa).
8. Figs. 2,3,4, and 5 show that the factor of safety of the proposed equation (16) is not influenced by rising values of  $\frac{y}{x}$ ,  $A_{cp}$ ,  $\rho_v \cdot f_{yt}$ , and  $\rho_\ell \cdot f_{y\ell}$ . On the other hand, the safety factor of the ACI 318M-08 Code method increases with rising value of  $\frac{y}{x}$  and decreases with rising values of  $A_{cp}$ ,  $\rho_v \cdot f_{yt}$ , and  $\rho_\ell \cdot f_{y\ell}$ . This may be that the proposed equation (16) relates to these 4 factors more closely than ACI 318M-08 method with practical tests.

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