Optimal Controller Design for Dynamic Systems with Perturbed Time-Varying Delay

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Abstract:

In this paper, a controller design is proposed to control systems subjected to uncertainties and perturbed time-varying delay. The proposed controller strategy is composed of three parts, the linear state feedback part is used for assigning the closed loop eigenvalues, and the nonlinear switching part of the sliding mode and the adaptive part are used to achieve the robustness of global stability. By using the stability theorem, the adaptive law is utilized for adapting the unknown bounds of the lumped perturbations so that the objective of asymptotical stability is achieved, and then to use the variable structure control method to enhance the robustness of stability of the controlled systems. Once the system goes inside of the sliding surface of the variable structure controller, the dynamics of the controlled systems are insensitive to effect of perturbations. The system and controller are simulated by using Matlab/Simulink. Finally, a real numerical example is given to demonstrate the feasibility of the proposed controller.

KEYWORDS

Time-varying delay Systems, uncertainty, Adaptive, Sliding Mode Controller

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1- Introduction:

The problem of stability for a class of uncertainties dynamic control systems with time-varying state delay is investigated in this work. An independent delay adaptive variable structure controller is designed to drive the states of the system to the equilibrium point (zero). The proposed controller strategy is composed of three parts, the linear state feedback part used to assign the closed loop eigenvalues, the nonlinear switching part of sliding mode and the adaptive part that are used to achieve the robustness of global stability. Variable structure controller (VSC)[1] with sliding mode (SM) has been traditionally recognized as a high gain control technique with outstanding robustness features for solving stabilization and tracking problems. The main feature of VSC is to employ a discontinuous control under the reaching law to drive the state from an arbitrary initial state in the state space toward a designed state along a pre-specified trajectory, i.e. switching hyperplane. The discontinuous high speed switching action maintains the state on this surface once the system enters the sliding hyperplane. Moreover, when the dynamics of the controlled system are in the switching hyperplane, it has been shown that VSC possesses several advantages, e.g., fast response, good transient performance, robustness of stability, insensitivity to the matching parameters variation and external disturbance [2].

1-1 Time-Varying Delay:

Due to the finite speed of information processing the transmission time-varying delay has been often encountered in various engineering systems, for example aircraft systems, microwave oscillator, rolling mill, chemical process, manual control and long transmission line in pneumatic, hydraulic systems. Since the existence of such delay is frequently a source of instability that cannot be ignored during the design of a control system, considerable attention has been paid to study systems with delay [1]. In this work, a robust control scheme is proposed for a class of uncertain dynamical systems with time-varying state delay. To increase the robust control efforts, one incorporates variable structure control with sliding mode to deal with the unknown parameters as well as the unknown gains in the bound of the delayed states. The uncertainties as well as the upper bound of the delayed states are dealt with by means of the well used robust control method.

1-2 System Description:

In this section, a class of systems that are subjected to time-varying state delays and perturbation are considered. The dynamic equation of these systems is governed by;

$$\dot{x}(t) = \left[A + \Delta A(t,x)\right]x(t) + \left[A_h + \Delta A_h(t,x)\right] x(t-h) + Bu(t) + D(t,x) \qquad (1)$$

Where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $D(t) \in R^m$ is an uncertain external disturbance and/or unknown nonlinearity of the system. The constant $A \in R^{m\times n}$, $A_h \in R^{m\times n}$, $B \in R^{n\times m}$ are known, and B is full rank. The term $\Delta A(t,x)$ and $\Delta A_h(t,x)$ are unknown real function representing time-varying parameter uncertainties of system matrices A and A_h respectively. The unknown scalar $h(t)$ denotes bounded and continuous delay function satisfying the following,

$$0 \le h(t) \le \bar{h} < \infty; \qquad \dot{h}(t) \le \eta < 1 \qquad (2)$$

Where $\bar{h}$ an unknown constant, but η is a known constant. $x(t)$ is an arbitrary known continuous state vector for specifying initial condition.

The following assumptions are assumed to be valid:

Assume that the system pair (A,B) is controllable, and all state variables are available for measurement. The Uncertain matrices and vector, ΔA, ΔA_h and D, that are continuously differentiable in *x*, and piecewise continuous in *t*. Then the objective is to design a variable structure controller for the system in Eq.(1) subjected to the previous assumptions. It will be shown that all the states of the controlled system will asymptotically approach zero in spite of the existence of perturbations, i.e.,

$$\lim_{t\to\infty} x(t) = 0 \tag{3}$$

Set all the uncertainties and perturbation to zero, i.e., $\Delta A = 0$, $\Delta A_h = 0$ and $D = 0$, the dynamic equation of Eq.(1) can be rewritten as a nominal system,

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + Bu(t) \tag{4}$$

Let a state feedback control $u = Kx$, where $K \in R^{m\times n}$ is a constant matrix. Then one can rewrite the dynamic equation of the nominal system Eq.(4) as,

$$\dot{x}(t) = (A + BK)x(t) + A_h x(t-h) \tag{5}$$

In order to examine the stability of the nominal system which is represented by Eq.(5), a Lyapunov function is defined as

$$V(t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(\tau)Rx(\tau)d\tau \tag{6}$$

Where P and R are symmetric positive definite matrices.

The derivative of Lyapunov function in Eq.(6), corresponding to the nominal system which is represented in Eq.(5), is then given by,

$$\begin{aligned}\dot{V} &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Rx(t) - (1-\dot{h})x^T(t-h)Rx(t-h) \\ &= -\begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix} Q \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}\end{aligned} \tag{7}$$

Where

$$Q \cong \begin{bmatrix} -[(A+BK)^T P + P(A+BK) + R] & -PA_h \\ -A_h^T P & (1-\dot{h})R \end{bmatrix} \tag{8}$$

According to the Lyapunov stability theorem, it is known that if $Q > 0$, the nominal system Eq. (4) will be uniformly asymptotically stable for all $x(t) \in R^n$.

It is well known that one can design the feedback gain matrix K using pole assignment method to obtain a set of pre-specified eigenvalues when (A,B) is controllable and nominal system, so that the desired system's performance can be achieved.

2- Proposed Controller:

When the design of a variable structure controller with sliding mode and the analysis of a nominal system are completed, one can do the design and can apply it to the original system and then take an example to show the simulation results for the designed controller.

2-1 Analysis of the Nominal System:

By setting all the uncertainties and perturbation to zero in Eq.(1), i.e. $\Delta A=0$, $\Delta A_h=0$ and D=0, from Eq.(1) one can obtain the dynamic equation of the nominal system as:

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + Bu(t) \tag{9}$$

Let a feedback control u_{eq} be as:

$$u_{eq} = Kx \tag{10}$$

Where $k \in \Re^{m\times n}$ is a constant matrix. Then, one can rewrite the dynamic equation of the nominal system in Eq.(9) as:

$$\dot{x}(t) = (A + BK)x(t) + A_h x(t-h) \qquad (11)$$

In order to examine the stability of the nominal systems Eq. (11), a Lyapunov function in Eq. (6) is used. The nominal system in Eq. (11) is globally, uniformly, asymptotically stable if there exist symmetric positive definite matrices P and R such as the following:

$$\left[(A+BK)^T P + P(A+BK) + R\right] > 0 \qquad (12)$$

And

$$(1-\dot{h})R + A_h{}^T P[(A+BK)^T P + P(A+BK) + R]^{-1} PA_h > 0 \qquad (13)$$

The derivatives of Lyapunov function in Eq.(6), corresponding to the nominal system Eq.(11), can be given by:

$$\dot{V} = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)R\,x(t) - (1-\dot{h})x^T(t-h)Rx(t-h)$$
$$= \left[x^T(t) \quad x^T(t-h)\right] \; Q \; \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \qquad (14)$$

Where

$$Q \cong \begin{bmatrix} -\left[(A+BK)^T P + P(A+BK) + R\right] & -PA_h \\ -A_h^T P & (1-\dot{h})R \end{bmatrix} \qquad (15)$$

According to the Lyapunov stability theorem, it is known that if Q>0, then the nominal system that is represented by Eq.(10) will be uniformly asymptotically stable for all $x(t) \in R^n$, and it is also known that Q is positively definite if and only if Eq.(12) and (13) are fulfilled.

2.2 Switching hyperplane:

In order to stabilize the perturbed time-varying delay system in Eq.(1), the VSC technique is utilized. In general, the design procedure of the VSC technique can be divided into two phases. The first phase is to design a switching hyperplane for the system so that once the controlled system enters the switching hyperplane, the desired dynamic performance can be generated. In this section the switching hyperplane of system in Eq.(1) is designed as:

$$\sigma = Cx - C\int_0^t (A+BK)x\,dt \qquad (16)$$

Where C $\in \Re^{m\times n}$ is a constant full rank matrix and is chosen so that the matrix CB is non-singular $k \in \Re^{m\times n}$ also as a constant matrix that satisfies [3]:

$$\mathrm{Re}[\lambda \max(A+BK)] \qquad (17)$$

After designing the switching hyperpalne, the second phase of the VSC design is to design an appropriate control law so that the sliding condition $\sigma\dot{\sigma}\langle 0$ is satisfied. The satisfaction of the sliding condition ensures that only the switching hyper plane will attract the trajectories of the controlled system and also the trajectories will stay thereafter. When the dynamics of the system in Eq.(1) are driven into the sliding phase, i.e.

$\dot{\sigma}(x) = 0$ And $\sigma(x) = 0$

One can know that,

$$\dot{\sigma} = CA_h x_{(t-h)} + CBu - CBKx = 0$$

Therefore it is known that there exists an equivalent control, $u = u_{eq}$ as:

$$u_{eq} = -(CB)^{-1} CA_h x(t-h) + Kx \qquad (18)$$

So that $\dot{\sigma}$=0. The closed-loop dynamic equation after system entering the switching hyperpalne can be obtained by substituting into Eq.(10) and the resulting equation is:

$$\dot{x} = (A + BK)x(t) + A_h x(t-h) - B(CB)^{-1} CA_h x(t-h) \quad (19)$$

$$= (A + BK)x(t) + A_h x(t-h) \quad (20)$$

3- Case Study:

Consider the dynamic model [4] with the following data:

$$A = \begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} 0.1\sin(2t) & -0.3\sin(t) \\ -0.1\sin(t) & -0.075\sin(3t) \end{bmatrix}$$

$$A_h = \begin{bmatrix} 0 & 0.1 \\ 0.5 & 1 \end{bmatrix}$$

$$\Delta A_h = \begin{bmatrix} -0.2\sin(2t) & 0.1\sin(3t) \\ 0.1\sin(t) & -0.175\sin(t) \end{bmatrix}$$

$$D = \begin{bmatrix} 0.2 + 0.6\sin(t) \\ 0.1 + 0.09\sin(3t) \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$\Delta B = \begin{bmatrix} 0.25\sin(t) & -0.15\sin(2t) \\ -0.1\sin(t) & 0.05\sin(3t) \end{bmatrix}$$

$$h(t) = 0.1|\cos(6t)|$$

It is clearly known that 0< h (t) <∞, $\dot{h}$(t) <1.

The main objective is to use the proposed control scheme of section two to design the controller to stabilize the system. According to Eqs. (2-8) the switching function is designed as:

$$\sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \{x - \int_0^t (\begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -20 & -8 \end{bmatrix})\} x dt \quad (21)$$

From Eq.(21), it is known that $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $CB = B^T C^T = 1$. Then the controller is designed as:

$$u = Kx$$

Where $K = \begin{bmatrix} 1 & 0 \\ -20 & -8 \end{bmatrix}$, and the sign function is used as a saturation for removing the chattering problem.

The closed-loop dynamic response of computer simulation using MATLAB/ Simulink software is done. For this simulation, one can use the initial condition as $X_{(0)} = [4 \quad -2]^T$.

From this information, it is clearly shown that 0< $h_{(t)}$ < ∞, $\dot{h}_{(t)}$ > 1, the uncertain matrices and vector ΔA, ΔA_h, D are continuously differentiable in x and piecewise continuous in t. Assume that there exists unknown continuous functions for the appropriate dimensions G,E and F such as:

$$\Delta A = BG$$
$$\Delta A_h = BE$$
$$D = BF$$

These conditions are so called matching conditions [4]. Then, one can compute G, E and F as:

$$G = \begin{bmatrix} 0.1\sin(2t) & -0.3\sin(t) \\ -0.2\sin(t) & 0.1\sin(3t) \end{bmatrix}$$

$$E = \begin{bmatrix} -0.1\sin(2t) & 0.1\sin(3t) \\ 0.2\sin(t) & -0.35\sin(t) \end{bmatrix}$$

$$F = \begin{bmatrix} 0.2 + 0.6\sin(t) \\ 0.2 + 0.18\sin(3t) \end{bmatrix}$$

Then the controller is designed as:

$$U = u_{eq} + u_n$$
$$= Kx + L\sigma$$

Where $K = \begin{bmatrix} 1 & 0 \\ -20 & -8 \end{bmatrix}$, L is positive scalar. Then the reachability condition can be given by:

$\sigma\dot{\sigma} \le u_n \le -L\sigma^2$

Then, since $\sigma^2 \geq 0$ always, the reachability condition is satisfied for any positive value assigned to *L*.

The computer simulation of a nominal closed-loop system without uncertainty is given as shown in figure (1); and applied under Matlab/Simulink as subsystem. Figure (2) illustrates the simulink implementation of the system delay. Uncertainty for both (A, A_h) and disturbances also make a subsystem for using MATLAB/Simulink.

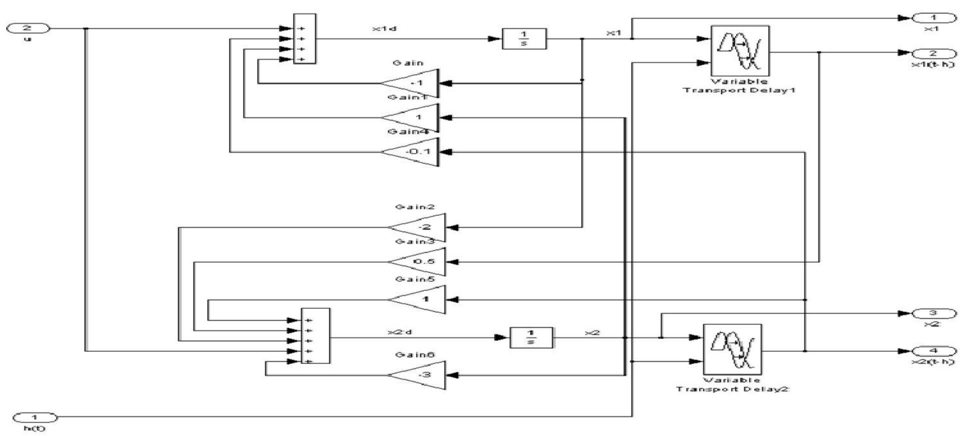

Fig.(1) The simulink implementation of the open loop system

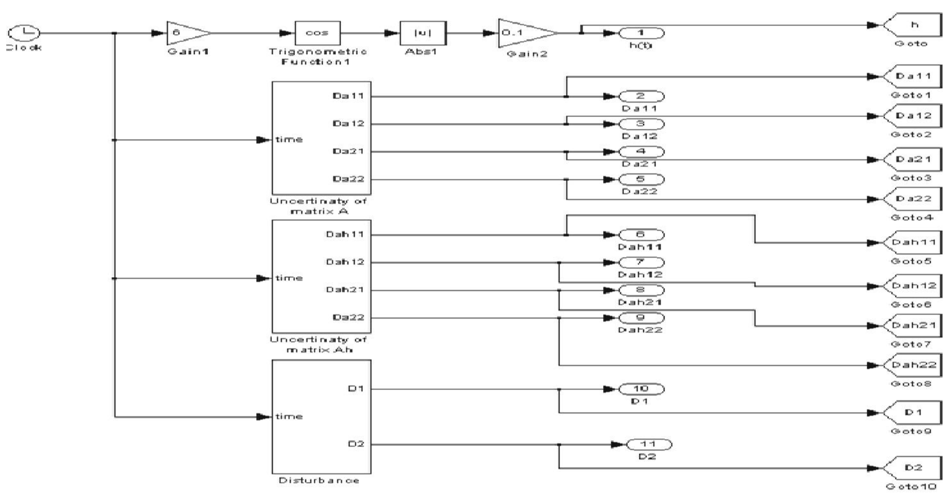

Fig.(2) The simulink implementation for system with uncertainty and disturbance.

The closed-loop dynamic trajectory of state variable X_1 (with and without delay) illustrated in Figure (3).

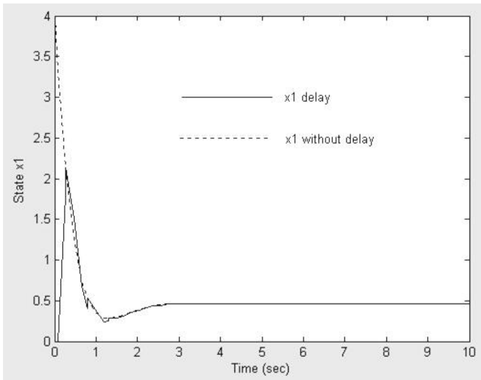

Fig.(3) State x1 response with initial condition =4.

The same procedures for state X_2 are used and the result is shown in Figure (4).

Figure (5) illustrates the states trajectory (X_1 against X_2) without using the controller.

Note that from Figure (3) to Figure (5) the initial condition used is $X_{(0)} = [4\ -2]^T$.

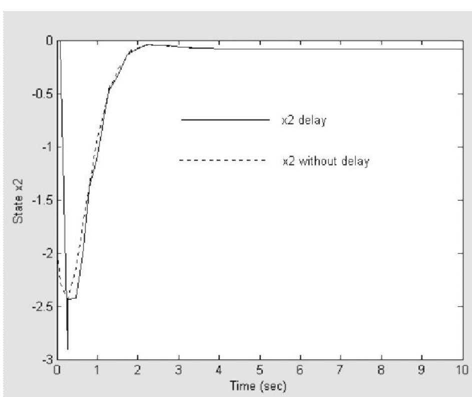

Fig.(4) State X_2 response with initial condition = -2.

Figure (6) illustrates the simulink implementation of the system with designed controller.

The state trajectory (X1and X2 against time) is illustrated in Fig. (7). It is clear that the equilibrium value of both states goes to zero.

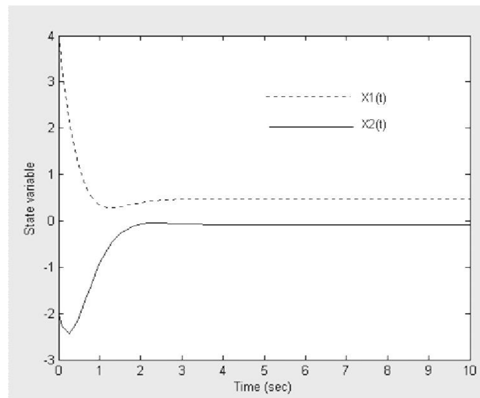

Fig.(5) State variable without controller.

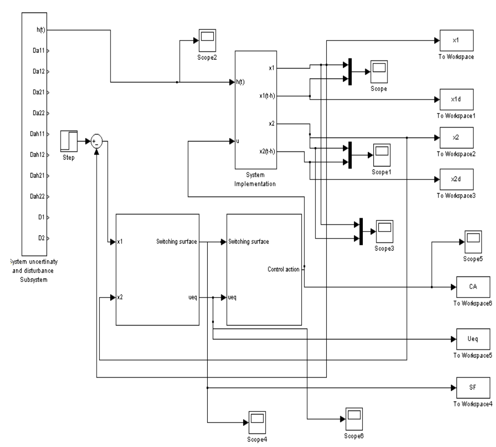

Fig.(6) The simulink implementation of the system with controller.

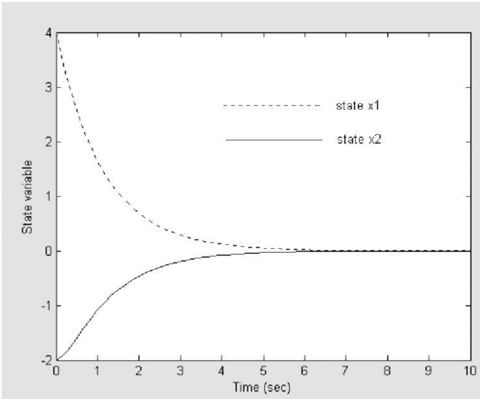

Fig.(7) State variable with controller.

Finally the phase plane plot of the system without controller and with controller is illustrated in figure (8). The sliding phase is clear when using the controller.

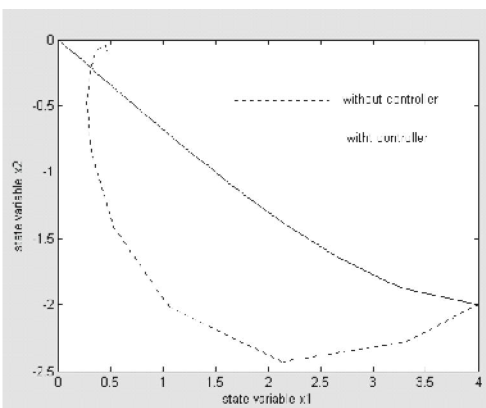

Fig.(8) Phase plane plot

4- Discussion and Conclusion:

The closed loop dynamic responses of the computer simulation are given from Fig.(3) to Fig.(5) with the initial condition $X_{(0)} = [4 \quad -2]^T$. It is clearly shown that each state variable approaches to a small bounded region in finite time as shown in Fig. (7). It is clear that the equilibrium value of both states goes to zero. Note that the saturation function is adopted, and both switching functions will enter a small bounded region. The case study clearly demonstrates a very good advantage of the proposed control scheme.

In this work, a variable structure with sliding mode controller is successfully designed for the system controlled with perturbed time-varying delays.

The VSC scheme is proposed for stabilizing a class of perturbed time-delay systems. Then, by using the proposed control scheme, the controlled system is guaranteed to have the global asymptotically stable property.

Furthermore, the knowledge of the upper perturbations is not required and the desired system performance is not required. Computer simulation results illustrated the design of a variable structure with sliding mode controller and states trajectory for system performance which are acceptable.

In general, the advantages of the proposed controller are summarized as follows:

- The knowledge of the perturbation is not required.
- The perturbation adaptation strategy is simple to implement.
- The tracking accuracy is adjustable.
- The exact function of time-delay is not required.

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