New Proposed Method of Direction Finding Using ESPRIT

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الخلاصة

تم تكريس هذا البحث لاظهار مزايا الطريقة المقترحة (2D-F/B-ESPRIT) ومقارنتها مع الطريقة الاعتيادية المسماة (2D-ESPRIT) وتم الاستنتاج بان الطريقة المقترحة افضل من الطريقة الاعتيادية في عملية تقليل الاخطاء في ايجاد الاتجاه عند تقليل عدد الومضات (snapshots) وعندما تكون الاهداف متقاربة جدا ومترابطة جدا وقوة اشارة ضعيفة نسبة الى الضوضاء وكذلك عندما يكون عدد المتحسسات قليل .

ABSTRACT

This paper is devoted to introduce the high performance of a new proposed method called (2D-FB-ESPRIT). A study of the estimation of 2-D Angle of Arrival (2-D AOA) of the signals incoming from the sources by using the data, collected from equally spacing planar array based on the 2-D ESPRIT. This new proposed method is based on Forward/Backward spatial smoothing techniques. This method overcomes the error that occurs in the normal 2D-ESPRIT method. In this paper, the results show a substantial improvement occurs in performance of the proposed method as compared with the original method. The 2-D ESPRIT is two dimentional Estimation of Signal Parameters via Rotational Invariance Technique.

INTRODUCTION

Array processing deals with signals processing carried by propagating wave phenomena [10]. The received signal is obtained by means of an array of sensors located at different points in space in the field of interest. The most commonly used configuration is the linear array, in which the sensors (all of common types) are uniformly spaced along a straight line. Another common configuration is a planner array, in which the sensors form a rectangular grid or lie on concentric [8, 9, and 10]. Array signal processing is used in such diverse areas as radar, sonar, communication, and seismic exploration. Usually, the main parameter of interest is the Direction of Arrival (DoA) of observed signal. There are a set of methods which are used to estimate DoA. MUltiple SIgnal Classification (MUSIC) method was the first of the high resolution algorithms to correctly exploit the underlying data model of narrow band signals in additive noise. Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) is an approach to signal parameter estimation problem that exploits sensor array invariance. Its computational advantages are achieved by exploiting displacement invariance designed into the sensor array [11]. ESPRIT is similar to MUSIC in that it correctly exploits the underlying data model, while it has apparent significant advantages over MUSIC.

The generality of the fundamental concepts on which ESPRIT is based makes the extension to higher spatial dimensions and signal containing multiple frequencies possible .The significant computational advantage of ESPRIT becomes even more noticeable in multidimensional parameters estimation where computational load grows linearly with dimension in ESPRIT, while that of MUSIC and others grow exponentially [4].In this paper, the 2D- ESPRIT algorithm is modified to handle the problem of high relative error, which occurs when 2D-ESPRIT algorithm is used. The new proposed method exploits the (2D-F/B ESPRIT) processing scheme, which is named as (2D- F/B ESPRIT). The new proposed algorithm is tested and compared with the 2D-ESPRIT using different computer simulation programs built for this purpose.

The Modeling of Data

Figure (1) shows the essential modeling of data received by Equally Spacing Planar Array (ESPA) that consists of isotropic antennas, which radiate in all directions with equal amplitude and phase. The output of ESPA is considered as the input to the covariance matrix, which is used to compute simultaneously, both the azimuth and elevation angles for a far-field targets using Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT). The source signal $S_k(t)$ is assumed to be a random exponential waveform referred to use the kth signal, which defined by [1,2,3,10]:

$$S_k(t) = A_k e^{j\omega_k t} \qquad k = 1,2, \; ..... \; , K \qquad ...(1)$$

And as A_k is a vector of amplitude $|A_k|$ and phase of ϕ_k then $S_k(t)$ will be:

$$S_k(t) = | A_k | \, e^{j(\omega_k t + \phi_k)} \qquad ... \; (2)$$

The total incident signal $V_{m1m2}(t)$ received by the Uniform Rectangular Array (URA) are the sum of the K sources plus an Additive White Gaussian Noise (AWGN) $N_{m1m2}(t)$ and thus[12] :

$$V_{m1m2}(t) = \sum_{k=1}^{K} S_k(t).a(\theta_k, \gamma_k) + N_{m1m2}(t) \qquad ... \; (3)$$

Where

$a(\theta_k, \gamma_k)$=*exp*(j $\phi_{m1m2,k}$) Which is the 2-D steering vector. It is a function of both the azimuth and the elevation angles. The $N_{m1m2}(t)$ is two dimensional additive noise. The task here is to evaluate the 2-D steering vector $a(\theta_k, \gamma_k)$. The derivation of this evaluation is fully explained in references [3].

As the assumption that the space is far-field and the incoming rays are parallel and the measurement are done by taking samples in time domain, then

$$V_{m1m2n} = \sum_{k=1}^{K} | A_k | .e^{j(\omega_k(n-1)T_s+\phi_{kn})}.e^{j\phi} + N_{m1m2n} \quad ... \quad (4)$$

Where the noise samples N_{m1m2n} are assumed to be independent of each other and ϕ_{kn} is the sample source random phase, which is assumed to be uniformly distributed in $(-\pi,\pi)$. However, the equation (4) can be written in Matrix form

$$\overline{V} = \overline{A}.\overline{S} + \overline{\mathrm{N}} \quad ... \quad (5)$$

Where $\overline{\mathrm{V}}, \overline{\mathrm{A}}, \overline{\mathrm{S}}$ and $\overline{\mathrm{N}}$ are the received signals, steering vectors, source signals, and the noise matrices, respectively. And accordingly the covariance matrix R can be written as

$$R = E[\overline{V}.\overline{V}] = \overline{A}R_s\overline{A}^H + \mathrm{R_N} \quad ... \quad (6)$$

R_S is the source covariance matrix (i.e. the signal coherency matrix) and R_N is the noise covariance matrix. This matrix is defined as a Toeplitz matrix , since it is assumed that it is spatially independent (i.e. mathematically a Toeplitz structure is a matrix with equal entries along each diagonal [7].

The proposed two dimensions forward/Backward Spatial Smoothing Technique [3]:

Figure (2) is used t0 show the process of how the number of sensors used in calculation of the covariance matrix either in forward or backward spatial smoothing. This figure shows the process in one dimension. Figures (3a) and (3b) show the process of how the sensors are distributed in two dimensions. Also these two figures (3a) and (3b) show the sensors which are used in preparing the covariance matrices in forward/backward and vertical/horizontal spatial smoothing. The total number of sensors is assumed to be (M_1 x M_2) and the total number of sub arrays of sensors is L in both forward/backward and horizontal/vertical. The value of L is given as L=M_2-I+1 or L=M_1-I+1 , where I is the number of sensors in each sub array. L is the same value in vertical and horizontal grouping while I is different. The main difference between the proposed method and the classical one is the technique of sub grouping and this spatial smoothing method shown in figures (3a) and (3b). This method provides a high signal to noise ratio due to minimize the noise and maximize the signal. From these figures (3a) and (3b) one can estimate he horizontal forward covariance matrix

$$\tilde{R}_h^f = 1/L \sum_{l=1}^{L} R_{lh}^f$$

And the horizontal backward covariance matrix

$$\tilde{R}_h^b = 1/L\sum_{l=1}^{L} R_{lh}^b$$

Where R_{lh}^f is the l-th horizontal forward covariance matrix and $R_{lh}^{\ b}$ is the l-th vertical covariance matrix . The average estimated horizontal covariance matrix is

$$\tilde{R}_h^{fb} = (\tilde{R}_h^f + \tilde{R}_h^b)/2 \quad .$$

In a similar method, one can estimate the vertical covariance matrix $\tilde{R}_v^{fb}$ as below

$$\tilde{R}_v^f = 1/L\sum_{l=1}^{L} R_{lv}^{\ f} \text{ and } \qquad \tilde{R}_v^b = 1/L\sum_{l=1}^{L} R_{lv}^{\ b}$$

Then

$$\tilde{R}_v^{fb} = (\tilde{R}_v^f + \tilde{R}_v^b)/2 \quad .$$

And finally the resultant estimated average covariance matrix is

$$\tilde{R}^{fb} = (\tilde{R}_h^{fb} + \tilde{R}_v^{fb})/2 \quad . \qquad \ldots (7)$$

The calculations of above matrices are shown in the flowchart explained in figure (4).

The 2D- ESPRIT Algorithm summary :

The resultant covariance matrix can be decomposed into a number of matrices each of which is the outer product of an eigenvector of the covariance matrix viz. [7].

$$R = \sum_{i=1}^{N} e_i \, U_i^{*} \, U_i^{T} \qquad \ldots (8)$$

Where e_i 's are the eigenvalues . And U_i 's are the eigenvectors.

The eigenvectors space is divided into signal and noise subspaces respectively. The ESPRIT is based on the signal subspace E_s and it is widely explained in references [3, 4, and 11].

To summarize the proposed 2D-ESPRIT algorithm, as a step-by-step outline considering the (URA) as depicted in Figure (1), the following tips can be followed:

1- Obtain the signal subspace E_s and the subspace eigenvalues Λ_s from the Eigen-decomposition of the estimated covariance matrix R. The columns of matrix E_s are the signal eigenvectors .
2- A matrix E_w is obtained from the following equation
   $E_w = E_s^H . E_s$
3- This matrix E_w is again decomposed into eigenvectors and eigenvalues. The number of eigenvalues is equal to the number of sources and their values will be complex i.e. formed from real values and imaginary values .

4- The estimated directions are calculated from the imaginary part or the real part of eigenvalues calculated in step 3 using the following relations

$$\tilde{\theta}_k = \sin^{-1}\left[\frac{\lambda\angle\phi_k}{2\pi|\Delta_1|}\right]$$

And

$$\tilde{\gamma}_k = \sin^{-1}\left[\frac{\lambda\angle\phi_k}{2\pi|\Delta_2|\cos\tilde{\theta}_k}\right]$$

Where $\tilde{\theta}_k$ is the azimuth angle estimate and $\tilde{\gamma}_k$ is the elevation angle estimate. λ is the wavelength in meter unit , Δ_1 is distance separation between the sub arrays in azimuth plane , Δ_2 is distance separation between the sub arrays in elevation plane and $k = 1,2,3,\ldots. K$ where K is the total number of sources .

These steps are repeated for each assumed sources for different cases either changing number of snapshots or correlation factor or signal to noise ratio and other factors. Then the Root Mean Square error RMS in degree, obtained from both the Azimuth and the Elevation angles, are calculated as follows

$$RMS-error = \sqrt{((\theta_1-\tilde{\theta}_1)^2+(\theta_2-\tilde{\theta}_2)^2+\ldots\ldots+(\theta_K-\tilde{\theta}_K)^2)/(K*number\text{ of trials})}$$

RESULTS AND DISCUSSIONS

The same conditions are taken in the application of ESPRIT on the ordinary case and the proposed case 2D-F/B-ESPRIT. These results are used as a comparison between the behavior of the two methods (2D ESPRIT and 2D-F/B-ESPRIT) for different situations. Figures (6) and (7) show how the proposed 2D-F/B ESPRIT and the 2D-ESPRIT algorithms act, where the RMS error in degree is plotted versus the number of snapshots. It is clear that the 2D-F/B – ESPRIT gives a good performance as compared with the 2D- ESPRIT for all points. Also figures (8) and (9) depict the behavior of the suggested 2D-F/B – ESPRIT and the 2D-ESPRIT algorithms as a function of correlation coefficient (cc). Figures (10) and (11) show the behavior of the above methods as a function of the separation of targets in space (direction difference in degree). These results show how the 2D-F/B-ESPRIT is efficient even at the worse cases from number of snapshots and closely and correlated sources.

Figures (12) and (13) illustrate the RMS error in degree versus the SNR in dB. However, the performance of the 2D-F/B-ESPRIT and the 2D-ESPRIT is almost the same, since the two algorithms depend upon the ESPRIT approach to estimate the DoAs and of its advantages is the validity ever at low signal to noise ratio (SNR). Eventually, figures (14) and (15) show the number of sensors variation versus the RMS error in degree. From these results, it becomes clear that the conventional 2D-ESPRIT suffers from degradation in its performance when the number of sensors decreased, while the 2D-F/B- ESPRIT shows a superior performance evens at the above condition.

From the above results, one can conclude that the 2D-F/B-ESPRIT has the following benefits, as compared with the 2D-ESPRIT algorithm:

1. The proposed 2D-F/B- ESPRIT exhibits lower RMS error at coherent arrival.
2. This method exhibits lower RMS error at low SNR.
3. It exhibits lower RMS error at closely spaced sources.
4. Finally, the proposed method has a very good performance to find the (DOAs) using Equally Spacing Planner Array (ESPA) with low number of sensors, which is the main feature that gives an advantage to the proposed 2D-F/B-ESPRIT compared with the 2D- ESPRIT.

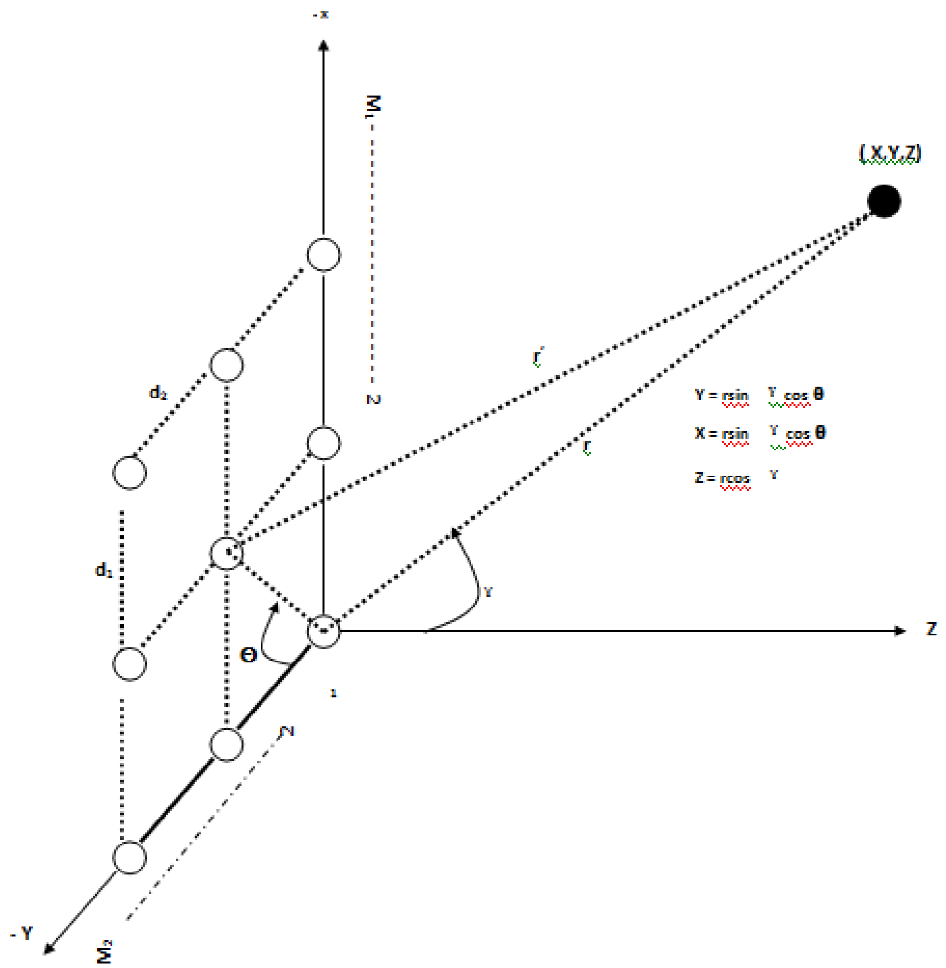

Figure -1 : The planner array geometry and the receiving signals

(Forward Subarrays)

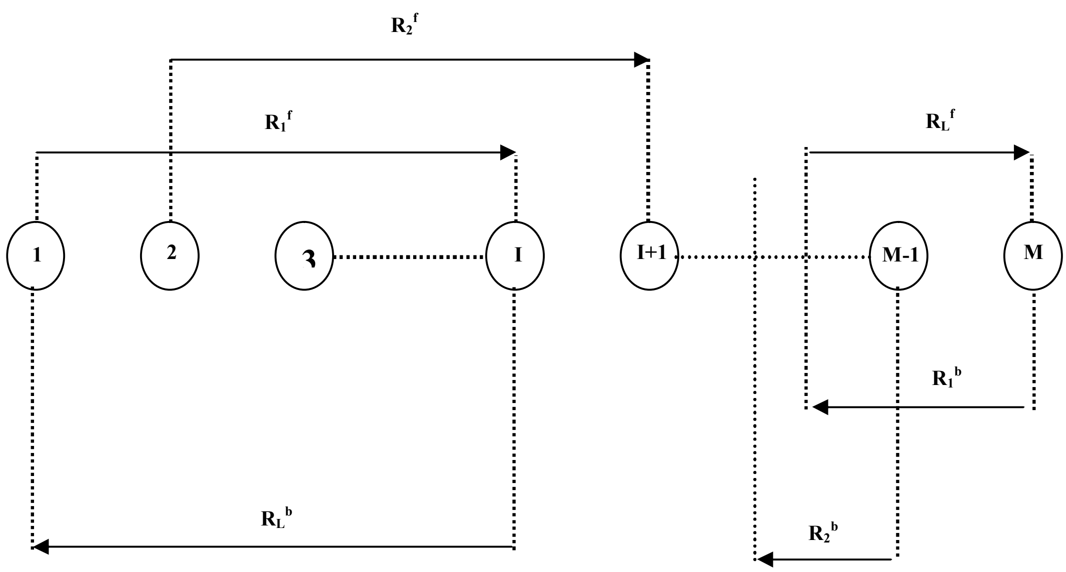

Figure -2: Forward / Backward spatial smoothing

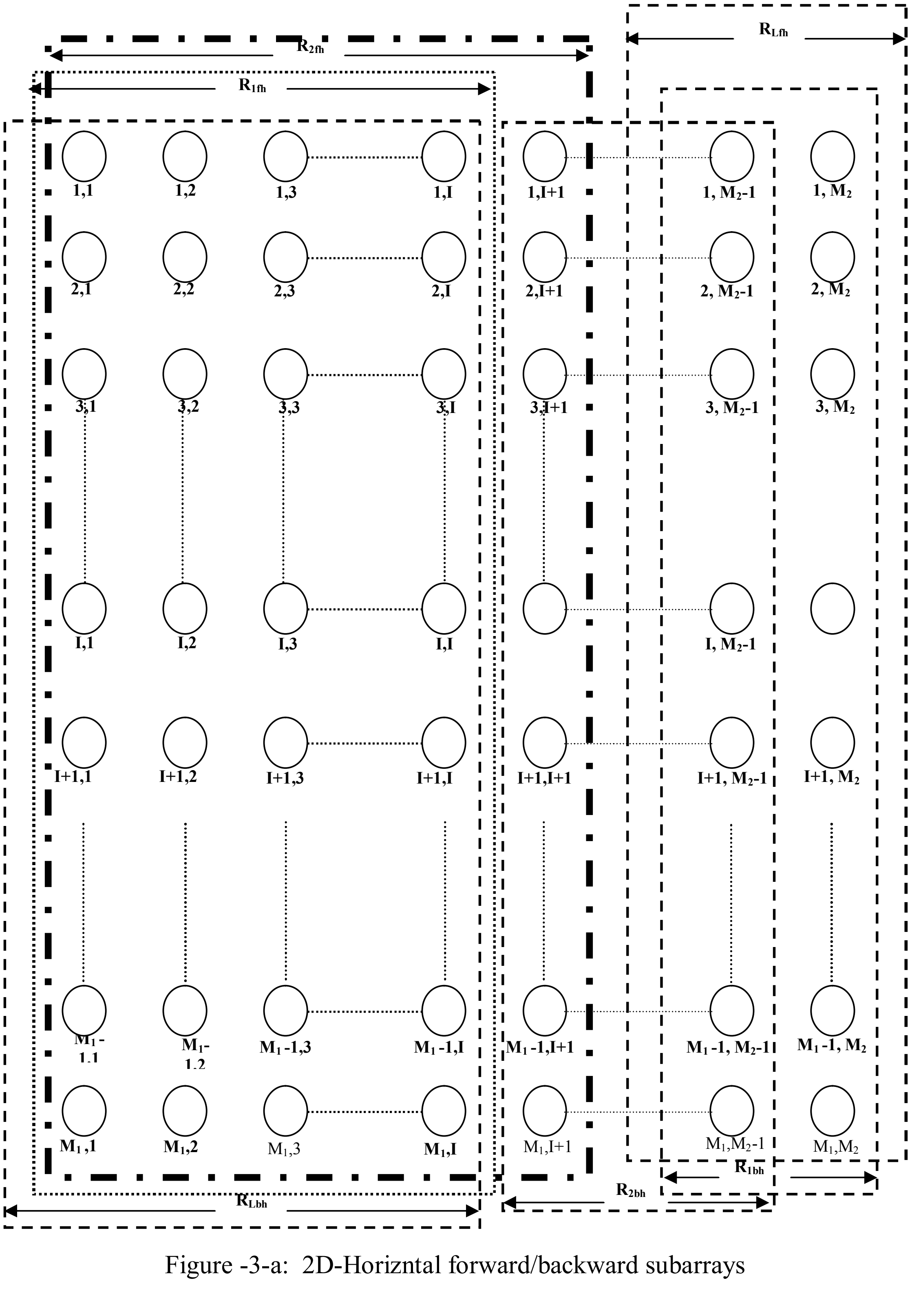

Figure -3-a: 2D-Horizntal forward/backward subarrays

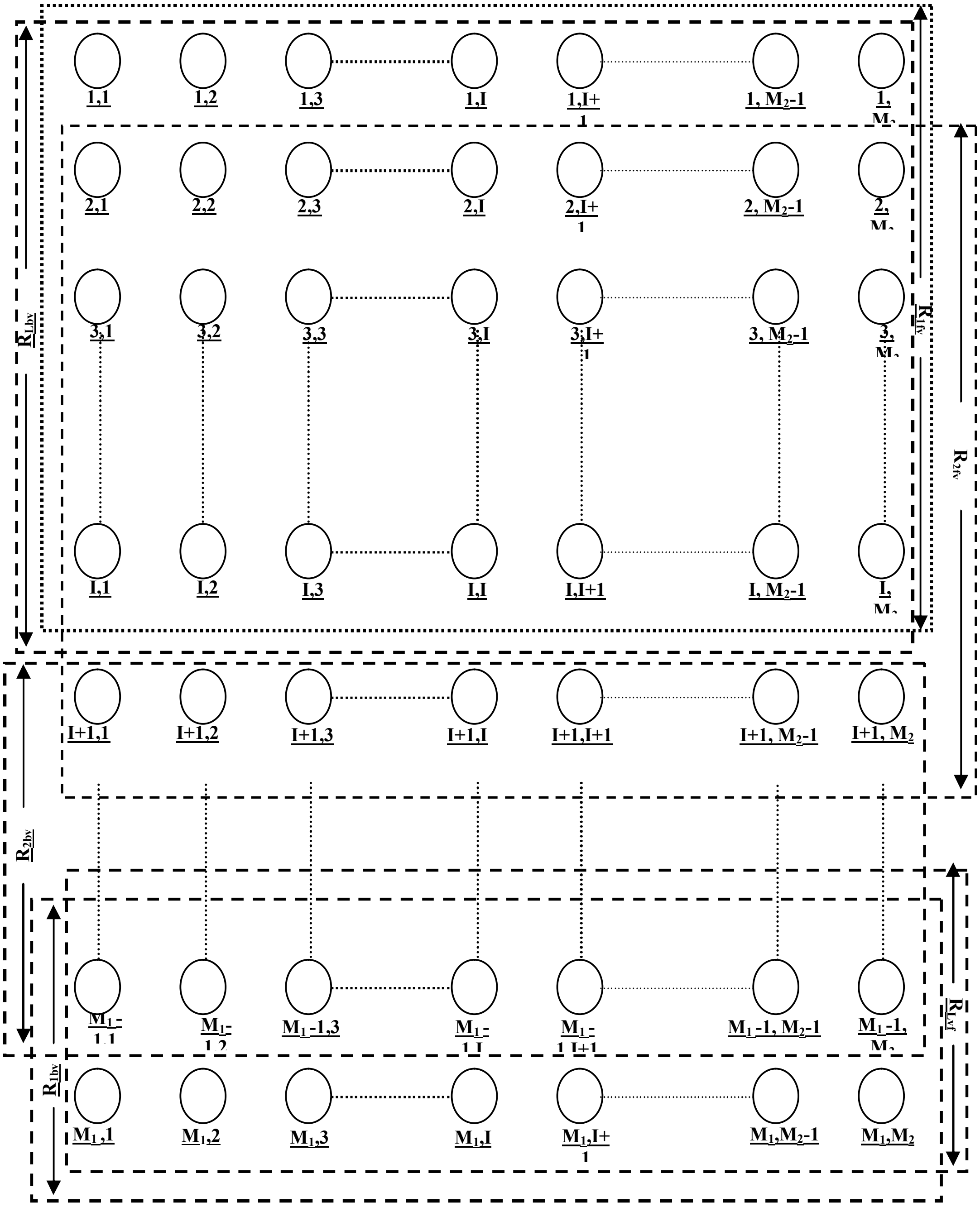

Figure -3-b: 2D-Vertical forward / backward sub arrays

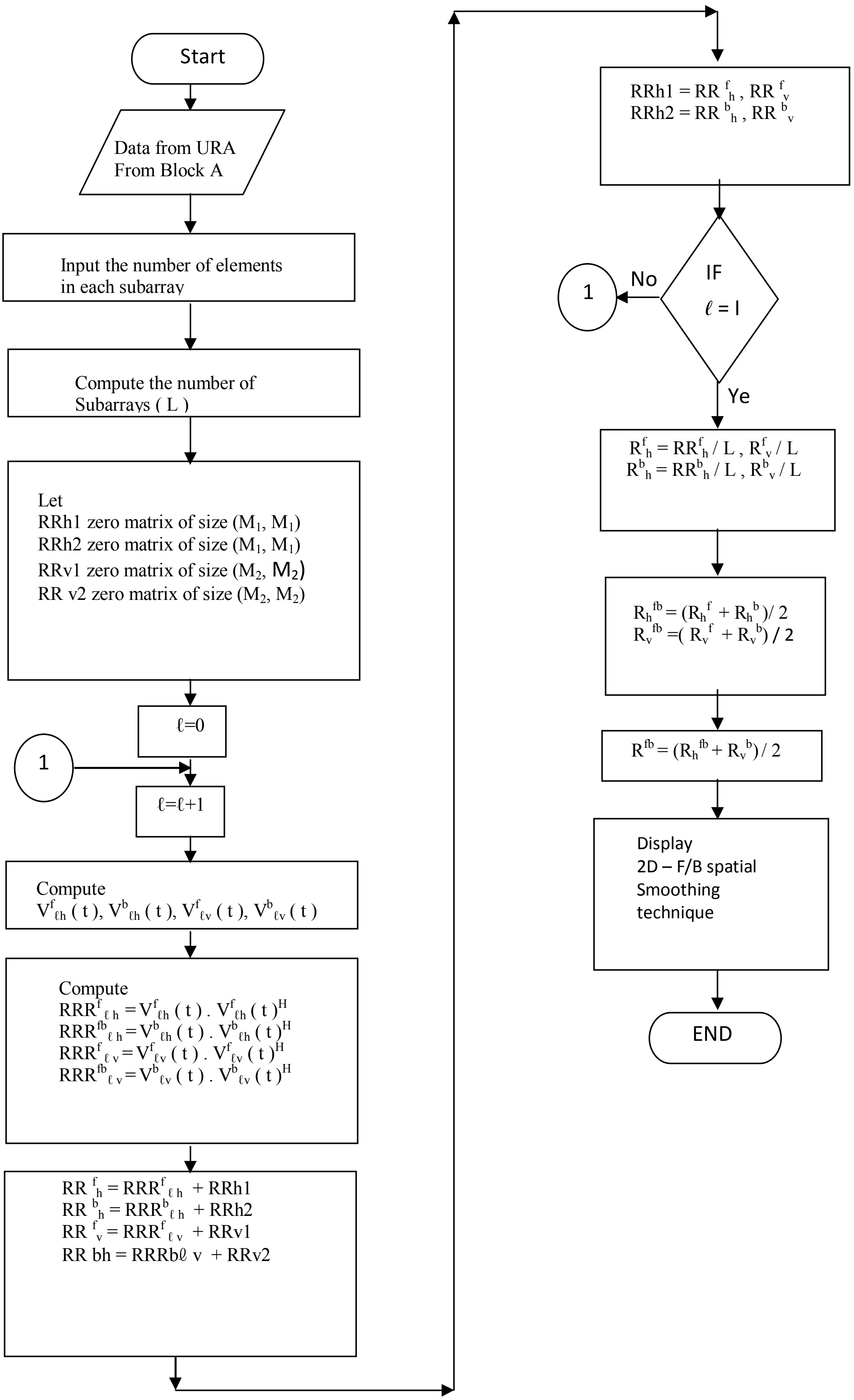

Figure -4: 2D-F/B flowchart

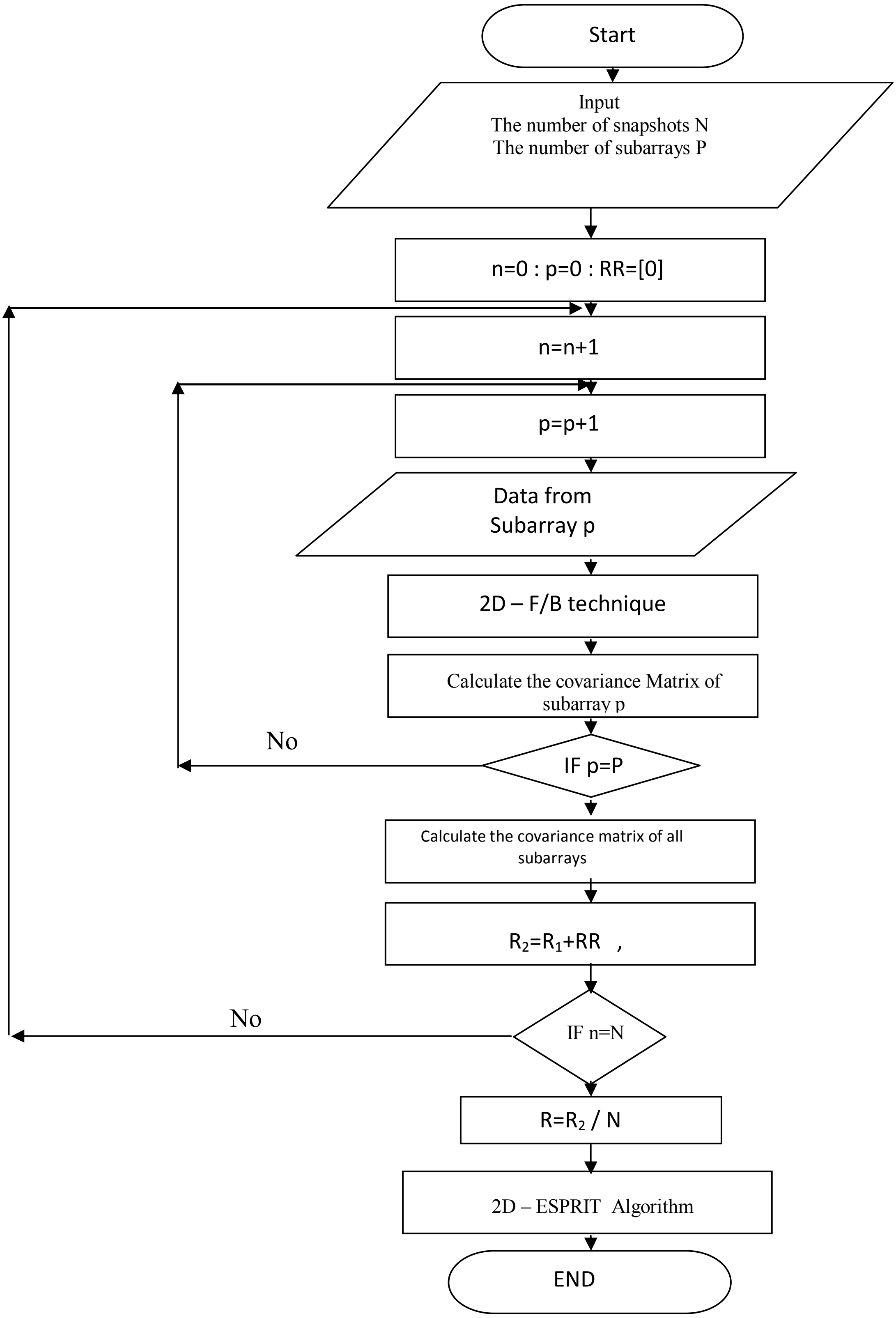

Figure -5: flowchart of 2D-F/B-ESPRIT

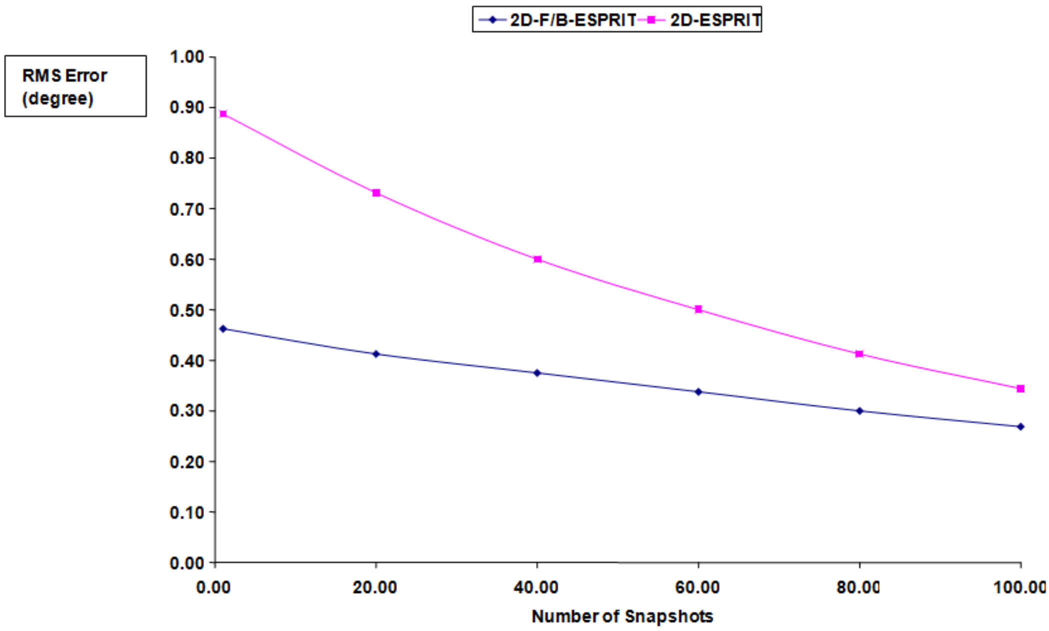

Figure -6: Root mean square error (RMS in degree) of the elevation angle versus the number of snapshots

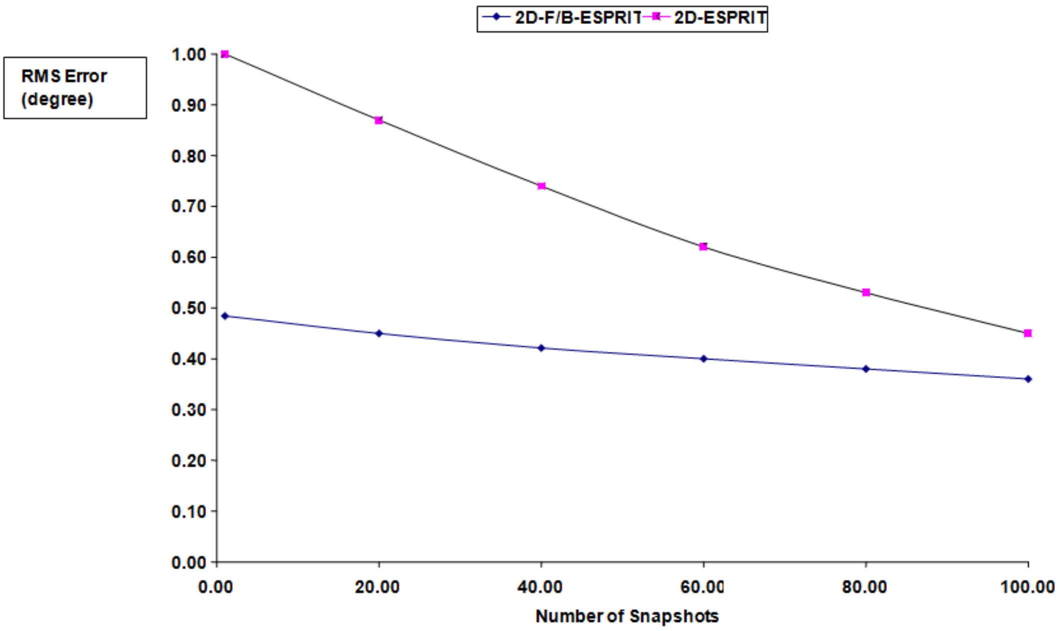

Figure -7:Root mean square error (RMS in degree) of the azimuth angle versus the number of snapshots

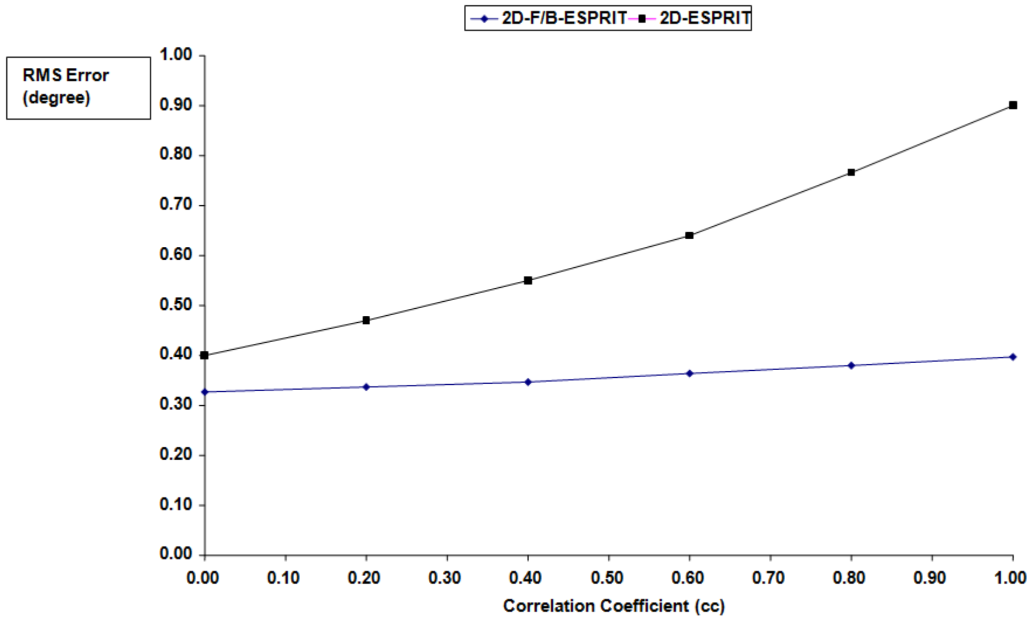

Figure -8:Root mean square error (RMS in degree) of the azimuth angle versus the correlation coefficient

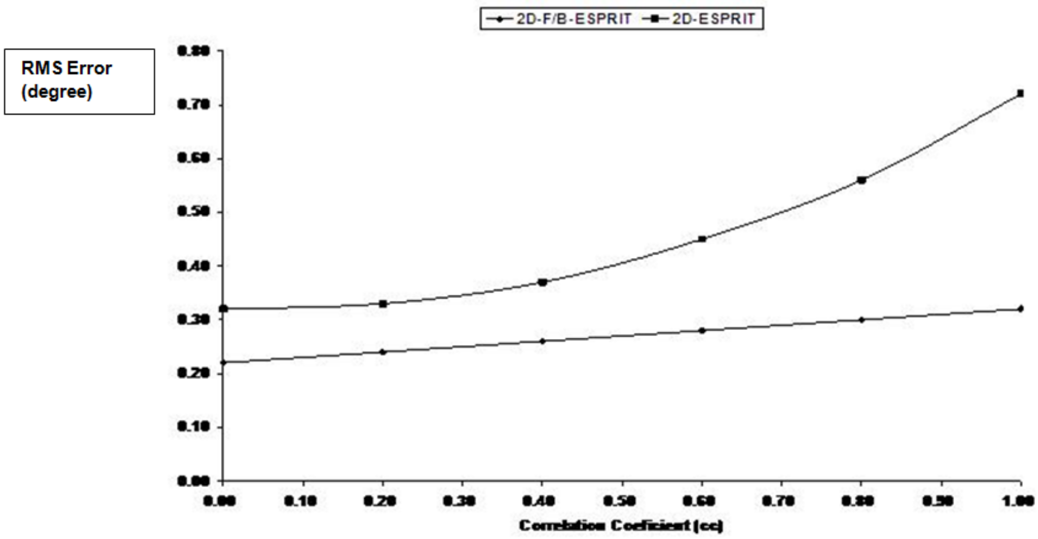

Figure -9: Root mean square error (RMS in degree) of the elevation angle versus the correlation coefficient

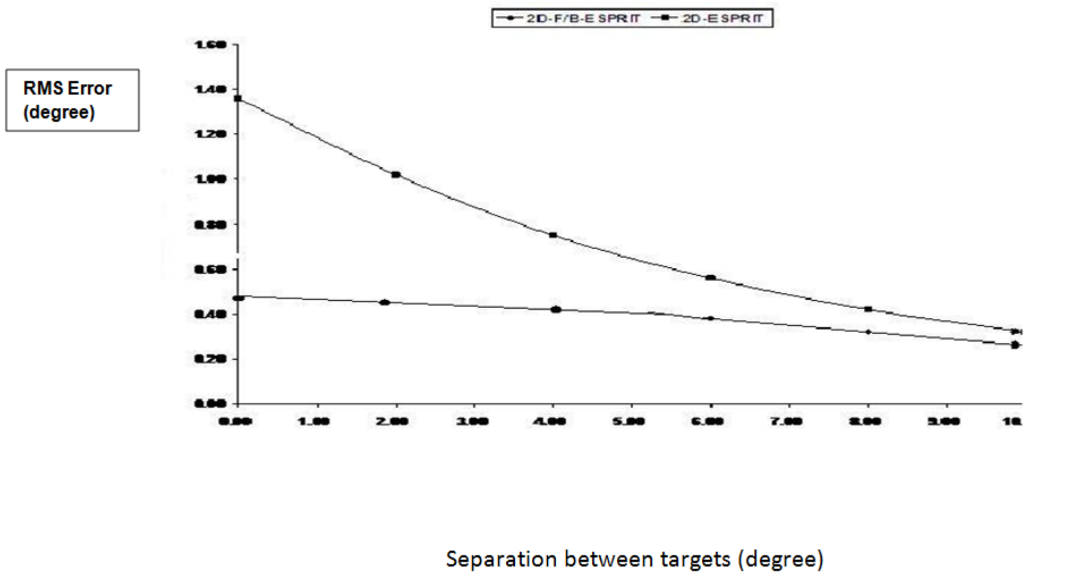

Figure -10: Root mean square error (RMS in degree) of the azimuth angle versus the separation between targets

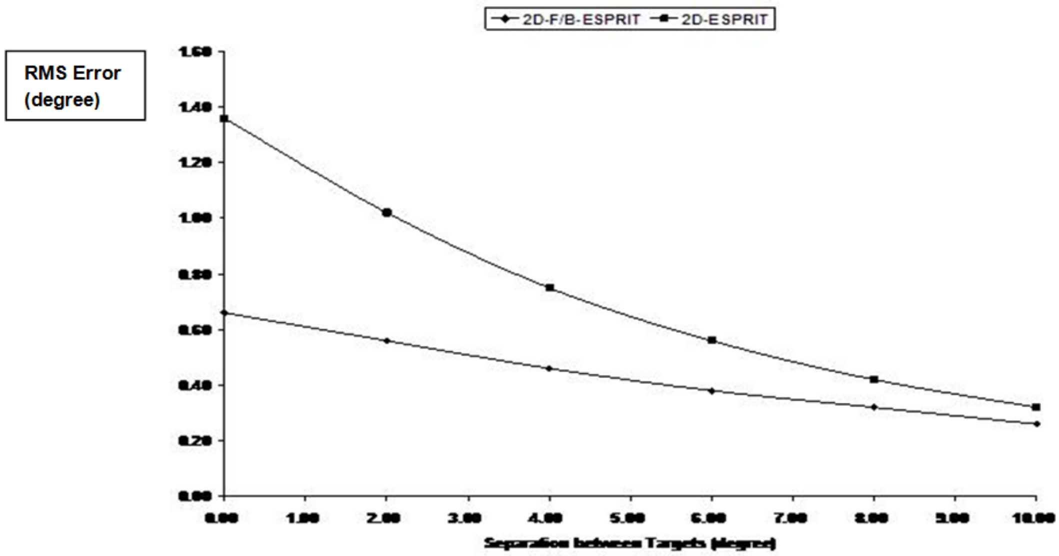

Figure -11: Root mean square error (RMS in degree) of the elevation angle versus the separation between targets

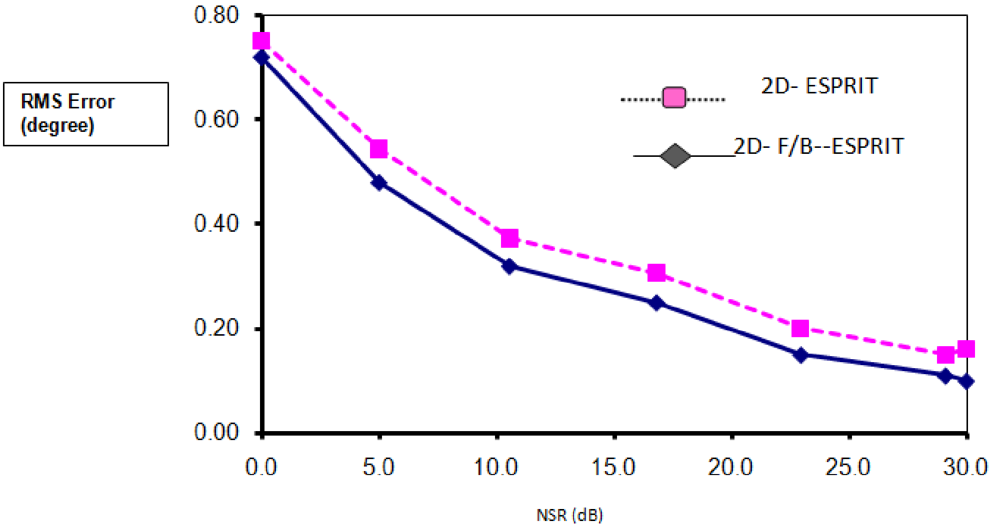

Figure – 12: Root mean square error (RMS in degree)of the azimuth angle versus the SNR(dB)

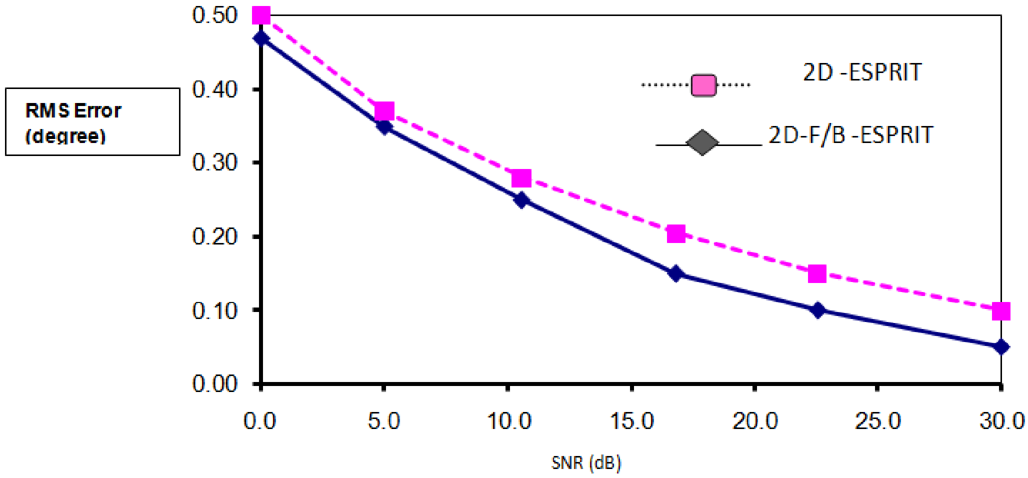

Figure -13: Root mean square error (RMS in degree)of the elevation angle versus the SNR(dB)

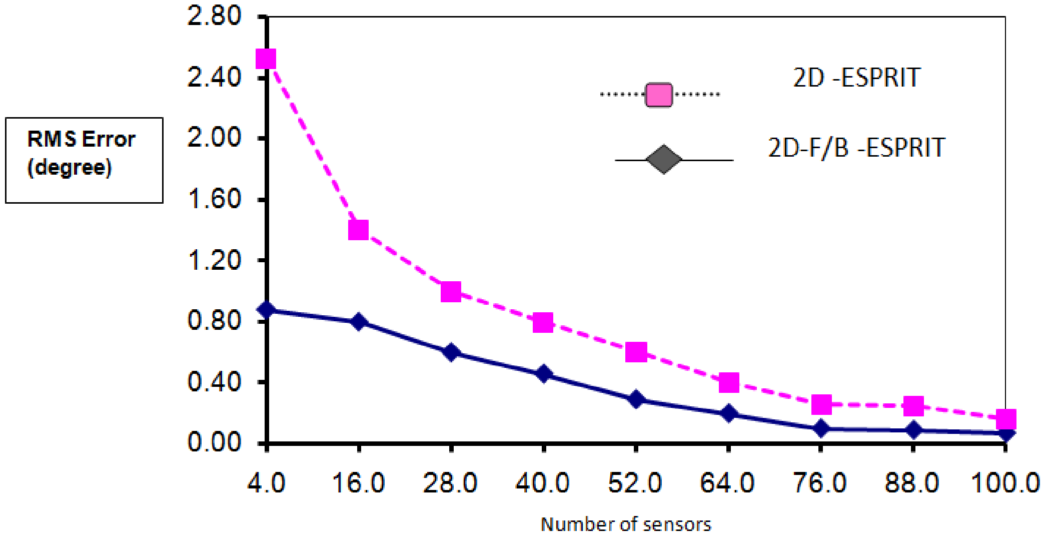

Figure-14: Root mean square error (RMS in degree)of the azimuth angle versus the Number of sensors

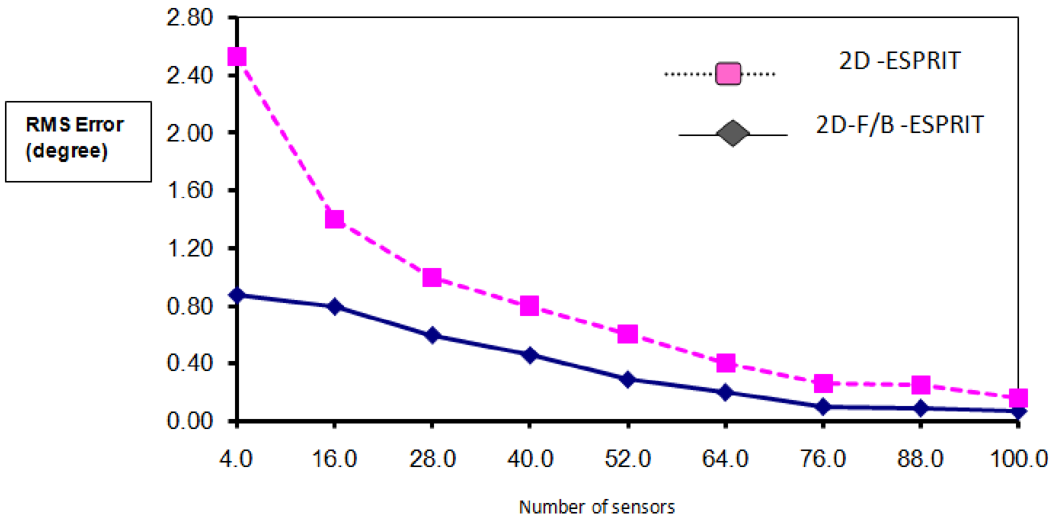

Figure -15: Root mean square error (RMS in degree)of the elevation angle versus the Number of sensors

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