

## Two-Stage and Walsh Interleavers for Interleave Division Multiple Access Systems

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### ABSTRACT

Interleavers are essential system components for interleave division multiple access (IDMA). In this paper, two-stage interleavers are proposed based on two optimization criteria combines minimum spreading distance with regular permutation and randomization properties to have almost the same advantages of the random distribution, and yet guaranties an enlarged minimum spread distance. Simulation results show sufficiently good performance with much less resource consumption compared to multiple random interleavers. The proposed method may also find other application areas such as multi-dimensional concatenated codes. In addition, an efficient and simple procedure is proposed to generate orthogonal interleavers by using the well-known Walsh-Hadamard transform.

**Keywords:** IDMA, Two-Stage Interleavers, Walsh Interleavers

### موزعات المرحلتين والولش لانظمة تقسيم موزعات متعددة الوصول

#### الخلاصة

الموزعات هي مكونات النظام الأساسية لتقسيم موزعات متعددة الوصول (IDMA). يقترح هذا البحث، الموزعات المرحلتين على أساس معيارين للامثلية يجمع بين الحد الأدنى لمسافة الانتشار مع اساليب التقليل العادية وخصائص التوزيع العشوائي للحصول على نفس المزايا تقريبا التي تتمتع بها التوزيع العشوائي، واعطاء ضمانات لتكبير الحد الأدنى لمسافة الانتشار. تظهر نتائج المحاكاة الأداء الجيد بما فيه الكفاية مع استهلاك أقل بكثير من الموارد مقارنة بالموزعات العشوائية المتعددة. قد تجد الطريقة المقترحة مجالات تطبيقية أخرى مثل الرموز المتسلسلة ذات الابعاد المتعددة. وبالإضافة إلى ذلك، طريقة فعالة وبسيطة لتوليد الرموز بشكل متعامد تم اقتراحها باستخدام طريقة Walsh-Hadamard المعروفة.

### INTRODUCTION

Efficient methods to spread the users in the mobile systems called interleave division multiple access (IDMA) has recently been introduced [1]-[3]. In IDMA systems, interleavers play a key role, since interleaving is the only mean to separate the users. In order to immunize noise and multiple access interference (MAI) for the receivers in IDMA systems, it is vital to choose good interleavers that are

weakly correlated between different users. Therefore, a careful design of the interleavers set is crucial for these systems [4]. The difference between the Turbo code and IDMA is the use of interleaver sets to spread the users rather than to split the information bits in Turbo code [5]. The correlation between interleavers should measure how strongly signals from other users affect the decoding process of a specific user [6].

Generally, the user specific interleavers are generated in independent and randomly methods. If this is the case, the base station (BS) is forced to consider a limited amount of memory to store these sets of interleavers, which may cause serious problem when the number of users is huge [7]. Moreover, there is a minor concern that piece of interleaver streams might be generated with high correlation. Thus, it is important to build a model of interleavers with low correlation by analyzing the characters of them [8].

The contributions of this paper are: (i) construct two-stage mother interleavers for IDMA based on two optimization criteria, maximize the spread spectrum distance by increasing the minimum distance of the crossing interleaving and design the interleavers in a way that reduces the cross correlation properties, and (ii) propose simple and efficient method for designing orthogonal interleavers.

In Section II, an introduction to IDMA system is presented. Section III, correlation upper bound of interleavers and minimum spreading distance of spreading code are discussed. Two-stage mother interleavers and multiple interleavers are given in Section IV. Section V presents a method for generating orthogonal interleavers using Walsh-Hadamard transform. Numerical results are presented in Section VI, and in Section VII conclusions are drawn.

**IDMA TRANSCEIVER PRINCIPLE**

The transmitter structure of an IDMA downlink is illustrated in Fig. 1. The input data sequence  $\mathbf{b}_k$  of user  $k$  is encoded using a convolutional encoder. The next step is a repeating encoder with scrambling, that generates a coded sequence  $\mathbf{c}_k = [c_k(0), c_k(1), \dots, c_k(J)]$ , where  $J$  is the frame length. The scrambling sequence  $\mathbf{S}$  includes alternated +1 and -1 as adopted in [1] and [3], which is required for receivers use the parallel interference cancellation (PIC) approach [9]. The elements in  $c_k$  are then fed to an interleaver producing  $\mathbf{x}_k = [x_k(0), x_k(1), \dots, x_k(J)]$ . The elements in  $c_k$  and  $x_k$  are called “chips” [10]. Users are slightly spread by their unique interleavers. These interleavers analyze the coded sequences so that the adjacent chips are approximately uncorrelated [11].

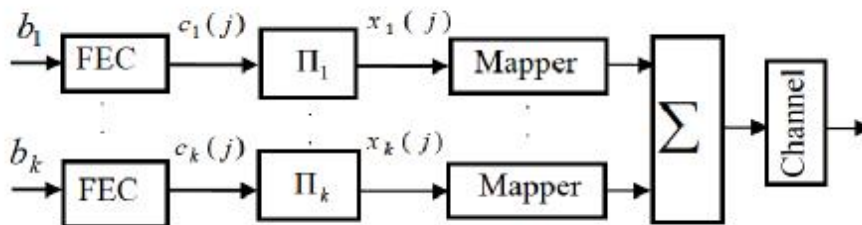


Figure (1) An IDMA downlink transmitter.

In this paper, binary phase shift keying (BPSK) modulation over a transmission channel that include  $L$  tap is adopted. Let  $h_k = h_{k,0}, \dots, h_{k,L-1}$  be the fading values related to the user  $k$ . The received signal at time instant  $j$  is expressed as

$$r_k(j) = \sum_{l=0}^{L-1} h_{k,l} \sum_{k=1}^K x_k + n_k(j), \quad j = 1, \dots, J + L - 1 \quad \dots (1)$$

where  $n_k(j)$  are samples of an Additive White Gaussian Noise (AWGN) process with zero-mean and variance  $\sigma^2$ . For simplicity, only real  $\mathbf{h}_k$  is considered, but the principle can be easily extended to complex signaling [3]. Assume that  $\mathbf{h}_k$  are known a priori at the receiver. Figure (2) shows a downlink IDMA receiver, which consists of an elementary signal estimator (ESE) and decoder (DEC), operating in an iterative manner. The received signal  $r_k(j)$  in (1) can be rewritten as

$$r_k(j + 1) = h_{k,l}(j) + \zeta_{k,l}(j) \quad \dots (2)$$

where

$$\zeta_{k,l} = h_{k,l} \sum_{k' \neq k} x_{k'}(j) + n(j), \quad \dots(3)$$

represents the influences of users interference and noise related to a user. Therefore, the calculation of the overall log likelihood ratio (LLR) for each chip is representation by the summation of the LLRs for all the related multipath coefficients. The user spreading algorithms for uplink is adopted in this paper [3]. The detailed algorithm for LLRs over a real multipath channel is described as [11]:

(i) *Estimation of Interference Mean and Variance*

$$E(r(j)) = \sum_{k,l} h_{k,l} E(x_k(j - 1)) \quad \dots(4)$$

$$Var(r(j)) = \sum_{k,l} |h_{k,l}|^2 Var(x_k(j - 1)) + \sigma^2 \quad \dots(5)$$

$$E(\zeta_{k,l}(j)) = E(r(j + 1)) - h_{k,l} E(x_k(j)) \quad \dots(6)$$

$$Var(\zeta_{k,l}(j)) = Var(r(j + 1)) - |h_{k,l}|^2 Var(x_k(j)) \dots(7)$$

(ii) *LLR Generation and Combining*

$$e_{ESE}(x_k(j))_l = 2h_{k,l} \frac{r^{(j+1)} - E(\zeta_{k,l}(j))}{Var(\zeta_{k,l}(j))} \quad \dots(8)$$

$$e_{ESE}(x_k(j)) = \sum_{l=0}^{L-1} e_{ESE}(x_k(j))_l \quad \dots(9)$$

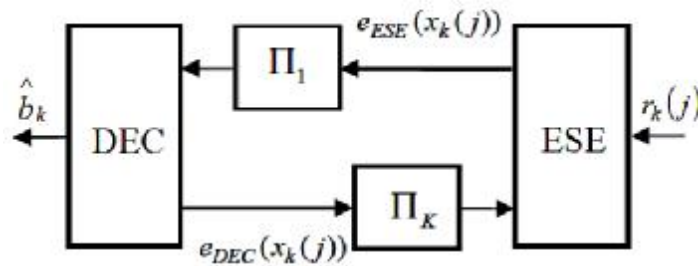


Figure (2) IDMA downlink receiver for user  $k$ .

**CORRELATION UPPER BOUND AND SPREADING DISTANCE**

An interleaver rearranges the ordering of a data sequence by means of a deterministic bi-jjective mapping. Let  $c_k = [c_k(0), c_k(1), \dots, c_k(J)]$ , be a sequence of length  $J$  with elements defined over  $GF(2)$ . An interleaver maps  $c_k$  onto a sequence  $x_k = [x_k(0), x_k(1), \dots, x_k(J)]$ . Such that  $x_k$  is a permutation of the elements of  $c_k$  [11]. The mapping function can be expressed as an ordered set called interleaving vector

$$\Pi = [\Pi(1), \dots, \Pi(j), \dots, \Pi(J)], \quad \dots (10)$$

with the proper deinterleaver, the permuted elements can be shifted back to their original positions [12]:

$$\Pi^{-1}[\Pi(d)] = \Pi[\Pi^{-1}[d]] = d \quad \dots (11)$$

The orthogonality implies no collision among interleavers, the two interleavers  $\Pi_i$  and  $\Pi_j$  are called orthogonal if

$$C(c_i(\Pi_i), c_j(\Pi_j)) = 0, \quad \dots (12)$$

Where the cross correlation  $C$  is the product value of two vectors  $c_i$  and  $c_j$ . The successful correlation is achieved when the term  $C$  close to  $0$  for  $i \neq j$ . Since correlation is a function of data length, evaluating the correlation  $C$  of two sets of interleavers is very computationally expensive. Therefore, to analyze the correlation coefficients, the peak basis correlation function in [6] is used with some simplification. Let the canonical basis  $e_i$  and the generating set  $w_j$  are defined as [6]

$$e_i(j) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \end{bmatrix}, i, j \in (1, 2, \dots, J), \dots (13)$$

$$w_i(j) = \begin{bmatrix} 1 & -1 & \dots & -1 \\ 1 & 1 & \dots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \dots & 1 & 1 \end{bmatrix}, i, j \in (1, 2, \dots, J), \dots (14)$$

The peak basis correlation of  $\Pi_i$  and  $\Pi_j$  is represented by  $\Pi(\Pi_i \text{ and } \Pi_j)$  and can be written as

$$P(\Pi_i, \Pi_j) = \max_{w_j} \sum_{i=1}^J \langle \Pi_j(c(w_j)) \rangle \dots (15)$$

for all vectors of generating set  $w_j$ , there exists a maximum value  $\Pi(\Pi_i \text{ and } \Pi_j)$  represent the correlation between  $\Pi_i$  and  $\Pi_j$ . The minimum distance  $MD$  similar to the concept of spread was proposed in [13] and redefined by Crozier [14], which is more effective in the context of turbo codes. The  $MD$  during all the interleavers is defined as [9]

$$MD = \text{Min}(DIST(n)), n \in (1, \dots, N), \dots (16)$$

Where  $N$  is the total number of interleavers and  $DIST(n)$  represents the minimum distance of the cross interleaving for the interleaver  $n$  and defined as

$$DIST(n) = \text{Min}|\Pi_n(i) - \Pi_n(i - 1)|, i \in (2, \dots, J) \dots (17)$$

**TWO-STAGE INTERLEAVER**

The main point in design of such algorithms is to generate interleavers in a simple and effective way. The desired master interleaver is chosen after two stages.

**FIRST-STAGE**

As it is known that the interleaver is a single input single output (SISO) function. Let the interleaver output at time  $i$  is  $\Pi_i$ , then for each coded frame with  $J$  bits, the following relation holds

$$\Pi(i) = J \times (s - 1) + j, s \in (1, \dots, S), j \in (1, \dots, J), \dots (18)$$

The position of every data bit is deterministic according to the above relation, while for random interleaver the position of each data bit is randomly positioned. A simple illustration for this stage is shown in Figure (3), where  $J = 4$  bits and spreading sequence  $S = 2$ . For an input sequence  $c_k = \{c_k\}, j = 1, 2, \dots, J$  any of its

$j - th$  data will be positioned to the output  $\Pi(i)$ . Consequently,  $\Pi(i)$  and its adjacent positioned data satisfy the following expression:

$$|\Pi(i) - \Pi(i \pm j)| \geq S, j = 1, i \in (1, \dots, J \times S), \dots (19)$$

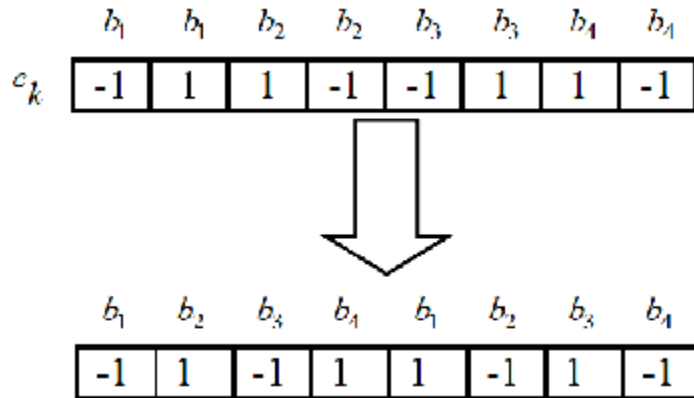


Figure (3) Example of stage 1 mapping,  $\Pi(i) = [1, 5, 2, 6, 3, 7, 4, 8]$ .

SECOND-STAGE

The stage commences from the interleaver stored at the end of the first stage. It will enable a random search in order to have almost the same advantages of the random distribution with better minimum cross interleaving. Figure (4) shows the concrete design steps of the second stage, which are:

- Write the pre-defined interleaver indices column-wise into a matrix  $A$  with  $M$  rows and  $N$  cols as depicted in Figure (4), where  $M \times N \geq J \times S$ .
- Updating matrix  $A$  by scrambling columns and rows indices.
- Read out row-wise and choose all the values  $\leq J \times S$ .

A mathematical representation of the stage can be written as follows:

$$\Pi(i) = A(\Psi(i), \Psi(i)), i \in (1, \dots, M), j \in (1, \dots, N), \dots (20)$$

Where  $\Psi$  represents the columns and rows scrambling rule, with number of rows and columns equal to  $2^{sf}$  and  $sf$  refers to spreading factor.

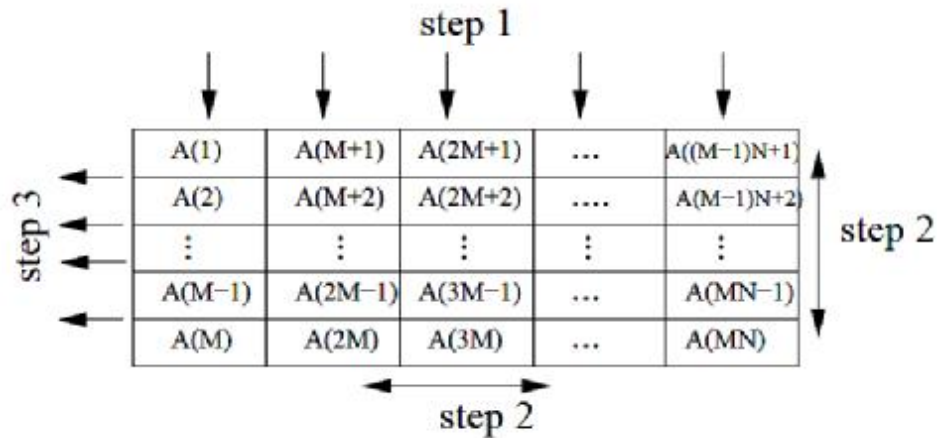


Figure (4) Sequential steps of stage 2.

**MULTIPLE INTERLEAVERS**

The remaining interleavers are generated by cyclically shifting the mother interleaver indices depending on a channel, block length and the index of user number.

$$shift_k = k \times \lfloor J/K \times L \rfloor, k \in (1, \dots, K), \dots (21)$$

Where  $\lfloor . \rfloor$  returns the nearest smaller integer,  $L$  is the number of channel taps and  $shift_k$  is the step of shifting for user  $k$ . Furthermore, each interleaver can be characterized by only a single distinct parameter of a random seed, which are sufficient for good user separation [15]. Suppose that the master interleavers is known to the receiver, the whole interleavers set can be generated efficiently if the receiver knows the step shifting for each user. Compared to pseudo-random interleavers [6], the proposed approach is independent on the number of users and can reduce the information exchange between transmitter and receiver.

**WALSH INTERLEAVERS**

The proposed method generates a set of  $S - 1$  orthogonal interleavers based on Walsh-Hadamard transform. The interleavers can be obtained in few steps [11]:

- Generate a Walsh sequence using Hadamard transform [17]:
- 

$$H(2^{sf}) = \begin{bmatrix} H(2^{sf-1}) & H(2^{sf-1}) \\ H(2^{sf-1}) & H(2^{sf-1}) \end{bmatrix}, 2 \leq sf \leq N \dots (22)$$

- Generate length  $S$  spreading sequence with altered  $+1$  and  $-1$  (i.e.,  $+1, -1, 1, -1, \dots$ ).
- For each Walsh sequence  $PN_i, i \in (2, \dots, S)$ , map elements of spreading sequence to the next place with  $PN_i$  sequence  $\{+1 \rightarrow (1), -1 \rightarrow (0)\}$  and keep the

indices of new places, which represent  $\Pi_i$ . The chips in the sub blocks are obtained by applying

$$\Pi((n - 1) \times S + d) = (n - 1) \times S + \Pi_i(d), \quad \dots (23)$$

where  $n \in (1, \dots, J)$  and  $d \in (1, \dots, S)$ .

**COMPUTER SIMULATION  
PERFORMANCE RESULTS**

The numerical results using of IDMA system model in Figure (1 and 2) with distinct interleavers are introduced. For the uncoded IDMA, 512 data bits are coded with the rate of  $R = 1/16$  repetition code and therefore, the size of the interleaver is 8192, with uniform transmission angle allocation. The chips of the interleaved are linearly superimposed and transmitted. For comparison, each figure contains the single user bound. The BER plot of the proposed interleavers is drawn in Fig. 5 after 15 iterations and compared with the case of using completely randomly generated multiple interleavers. It is noted that the performance of the interleavers generated by proposed simple strategy is slightly better than that of the completely randomly generated multiple interleavers. It is worth mentioned that the Matlab package is used in all simulations as well as C language in coded cases.

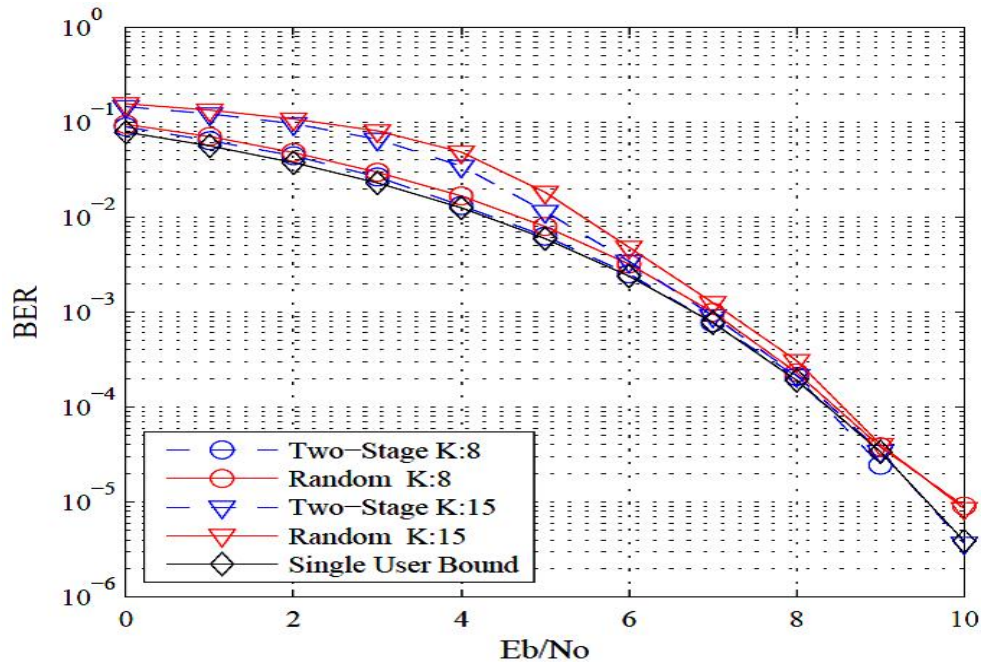


Figure (5) BER performance for uncoded IDMA with random and two-stage interleavers on an AWGN channel.



For the coded IDMA, simulation parameters are described as follows: 256 information bits are encoded by the rate  $R = 1/2$  memory 4 standard convolutional code with the generator polynomial  $[23, 35]_8$  where the octal notation has the least significant bit on the right, resulting in 522 coded bits. The coded bits are further encoded by the rate  $S = 1/8$  repetition code that gives 4167 coded bits. Thus, the resulting interleaver size is 4167. Fig. 6 shows the BER performance for the proposed interleavers on the AWGN channel. The generation matrix of master interleaver has 8 rows  $M$  and 522 columns  $N$ , with  $K \leq 8$  and 8 iterations. Since the two-stage interleaver still have the randomization distribution with better minimum crossing distance, the performance gain with the two-stage interleavers is about 0.5 dB at  $10^{-3}$ .

In Figure (7), the BER performance with  $S = 1/8$  for  $K = 4$  and  $S = 1/16$  for  $K = 8$  of two-stage interleavers is plotted on the 3 paths real-valued invariant channel  $h = [0.407, 0.815, 0.407]$  defined in [16]. Due to the weak cross correlated properties, the deinterleaving operation of proposed interleavers recover the initial sequence orders of other users. Thus, other user signals are effectively spread better than the interleavers introduced by [6]. The memory requirement of the random interleavers is  $(4 \times 8 \times 522 \times 14)$  bits for 4 users and  $(8 \times 8 \times 522 \times 13)$  bits for 8 users, whereas the proposed interleavers need  $(1 \times 8 \times 522 \times 13)$  for the users 4, 8 with shifting rule.

Thus, two-stage interleavers are much convenient than the conventional one for the IDMA system implementation.

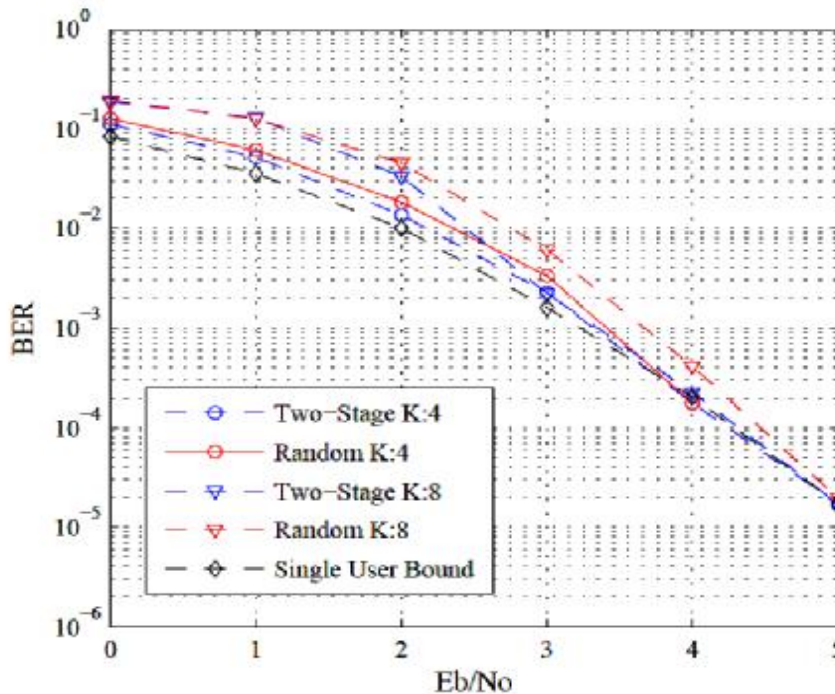


Figure (6) BER performance for coded IDMA with random and two-stage interleavers on an AWGN channel.

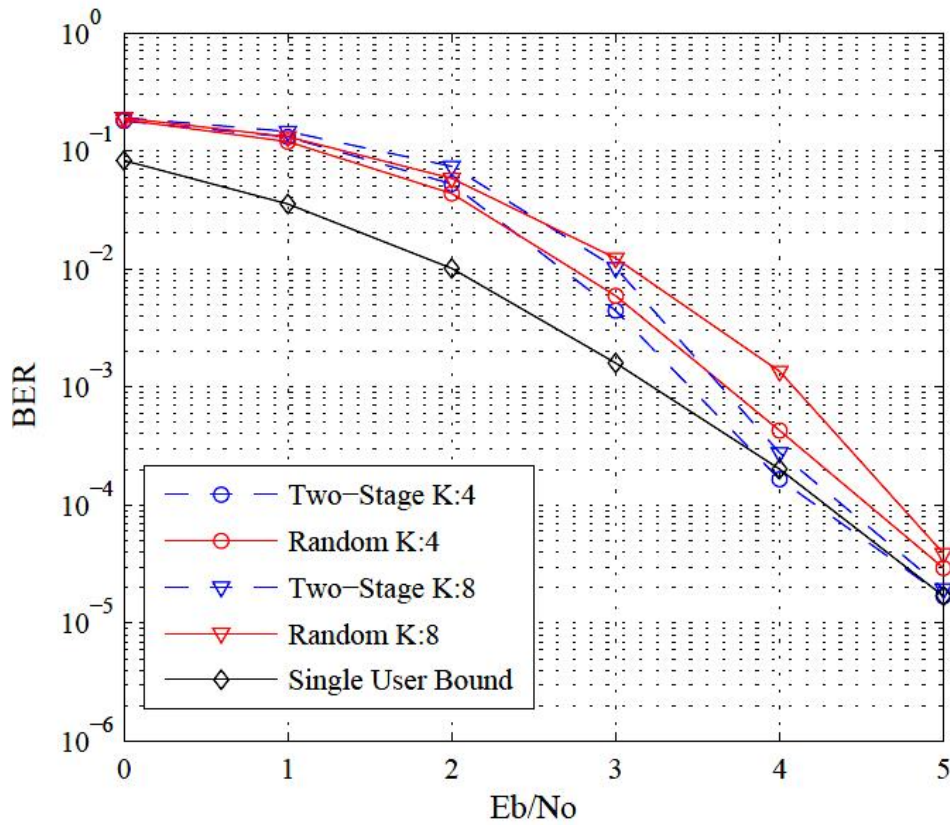


Figure (7) BER performance for coded IDMA system with random and two-stage interleavers on a multipath channel.

Figure (8) shows the performance results of the proposed Walsh interleavers on an AWGN channel. The simulation model was designed as the uncoded case. Since the orthogonal interleavers have a large portion of small interfering weights that degrade the error performance, the performance of proposed interleavers is better with low  $E_b/N_0$  and close to the random interleavers with high  $E_b/N_0$ .

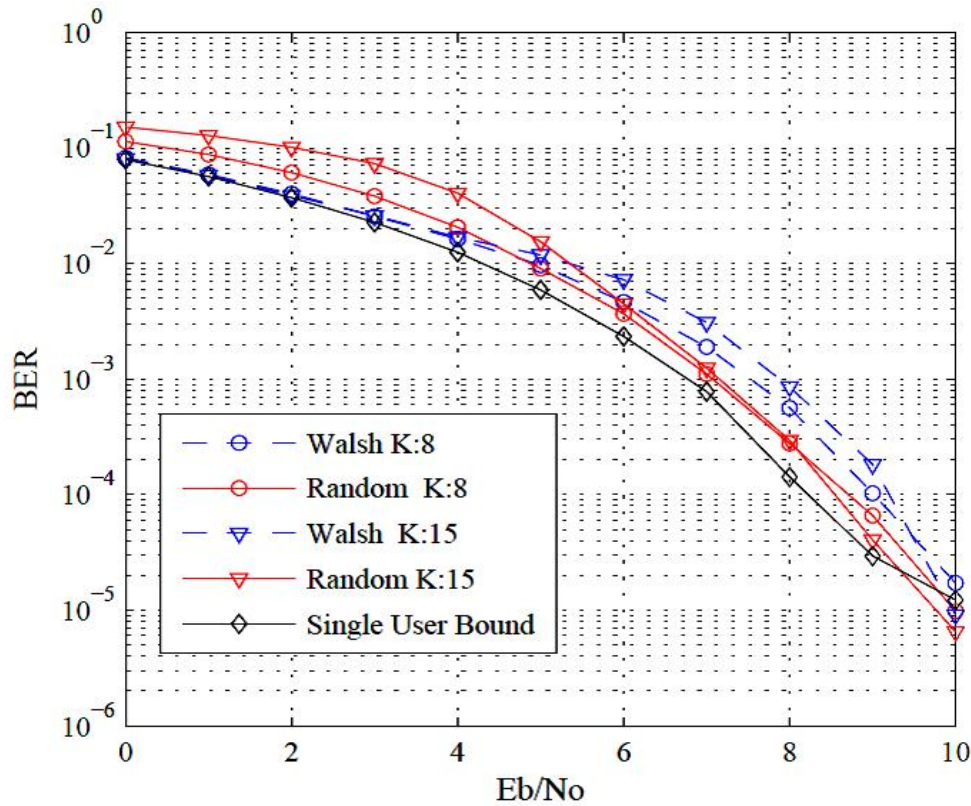


Figure (8) BER performance for uncoded IDMA system with random and orthogonal interleavers on an AWGN channel.

**CORRELATION AND MINIMUM DDISTANCE RESULTS**

For the comparison purposes, the peak basis correlation  $P(\Pi_i, \Pi_j)$  in (15) of the proposed interleavers in Section 4.B, section 6 and multiple random interleavers are calculated and presented in Tables (1-3). The 16 bits spreading code, 256 frame lengths and 5 users are used in the numerical analysis as in [4, 6]. It is shown that the cross correlation properties of the proposed two stage slightly better than random interleavers with small  $K$ . Whereas, the cross correlation properties of the Walsh interleavers are similar to the orthogonal interleavers in [4, 6]. Table (4) shows the minimum distance of the proposed, which is better than others using (17).

**Table (1)  $P(\Pi_i; \Pi_j)$  for Random Interleavers.**

	1	2	3	4	5
1	4096	844	892	792	904
2	780	4096	852	768	792
3	788	756	4096	828	796
4	744	768	780	4096	772
5	764	808	804	816	4096

**Table (2)  $P(\Pi_i; \Pi_j)$  for Two-Stage Interleavers.**

	1	2	3	4	5
1	4096	596	700	816	828
2	808	4096	496	700	716
3	802	632	4096	596	700
4	604	702	790	4096	596
5	640	704	892	672	4096

**Table (3)  $P(\Pi_i; \Pi_j)$  for Walsh-Interleavers.**

	1	2	3	4	5
1	4096	0	0	0	0
2	0	4096	0	0	0
3	0	0	4096	0	0
4	0	0	0	4096	0
5	0	0	0	0	4096

**Table (4): Minimum Distance**

Interleaver Type	MD
Random Interleaver	1
Walsh Interleaver	1
Two stage Interleaver	65

**CONCLUSIONS**

In this paper, a two-stage design criterion has been suggested to enhance the performance of random interleavers set for user separation of IDMA system. Other multiple interleavers are generated by cyclic shifts from a single mother interleaver. The simulations in Section 6 show that the proposed interleavers have significant performance gain and are very similar to the performance of random interleavers with less memory resources. Also, Walsh interleavers method is proposed as simple and efficient method to generate orthogonal interleavers.

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