

# إستخدام بعض الطرائق التمييزية الحصينة

## لتشخيص أمراض القلب في البصرة

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### Abstract

In this research I've tired to find out to robust and efficient estimations mean vector (location parameter) and, variance covariance matrix (scull parameter) for linear and quadratic discriminant functions. And also attempting to provide the robust linear and quadratic discriminant functions in the case of existing outliers resents through using some of high quality robust methods estimation. of these methods are minimum covariance determinant estimator, reweighed minimum covariance determinant estimator and minimum volume ellipsoid estimator , moreover , comparing robust discriminant functions with classical discriminant functions to draw out the best discriminant function in order to diagnose two types of diagnosis diseases heart which are angina pectoris and clot hearty .

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(Outliers)

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. (C.H)

(A.P)

:

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:

.

(Box Sarhan)

(Robustness)

.

(MCD)

(RMCD)

$\underline{\mu}$

.(MVE)

(Hubert,Driessen,2004) .  $\Sigma$

: (MCD)

...

) $\Sigma$ (

( $\underline{\mu}$ )

---

(h)

(Rousseeuw, 1985) .(MCD)

(P+1) (h)

.(50%) (Breakdown Point)

:

$$H = C_{P+1}^n \quad \dots(2-1)$$

(MCD)

(Hubert,Driessen,2004):

(Affine Equivariant)

$$\bar{X}_{MCD} = \frac{1}{h} \sum_{i=1}^h x_i \quad \dots (2-2)$$

$$\hat{\Sigma}_{MCD} = dp \times \frac{1}{h} \sum_{i=1}^h (x_i - \bar{X}_{MCD})(x_i - \bar{X}_{MCD})' \quad \dots ( -$$

( Pison, Van Aelst,2002) :

: dp :

$$\left[1 + \frac{15}{(n-p)}\right]^2 \times \frac{1}{\chi_{p,\alpha}^2} \quad \dots(2-4)$$

:

. : P

. :  $\alpha$

:(RMCD)

. .

) $\underline{\mu}$ (

(MCD)

(h)

)  $\Sigma$ (

$(\bar{x}_{j,0})$

$(\hat{\Sigma}_{j,0})$

$\Sigma()$

<sup>8</sup>(Rousseeuw , Leroy, 1987)

:

....(2-5)

$$\bar{x}_{j,0} = \frac{1}{h} \sum_{i=1}^h x_{ij}$$

$$\hat{\Sigma}_{j,0} = dp \times \frac{1}{h} \sum_{i=1}^h (x_{ij} - \bar{x}_{j,0})(x_{ij} - \bar{x}_{j,0})'$$

....(2-6)

. (2-4)

: dp :

:

$$w_i = \begin{cases} 1 & \text{if } \sqrt{(x_{ij} - \bar{x}_{j,0})' \hat{\Sigma}_0^{-1} (x_{ij} - \bar{x}_{j,0})} \leq \chi_{P,1-\alpha}^2 \\ 0 & \text{if } o.w \end{cases}$$

....(2-7)

(RMCD)

:

(1)

$$\bar{X}_{RMCD} = \frac{\sum_{i=1}^h w_{ij} x_{ij}}{\sum_{i=1}^h w_{ij}}$$

....(2-8)

---


$$\hat{\Sigma}_{RMCD} = dp \times \frac{\sum_{i=1}^h w_{ij} (\underline{x}_{ij} - \bar{X}_{RMCD})(\underline{x}_{ij} - \bar{X}_{RMCD})'}{\sum_{i=1}^h w_{ij}} \quad \dots(2- )$$

$$\dots( - ) \quad \sum_{i=1}^h w_{ij} \neq 0$$

(RMCD)

. (Affine Equivariant)

<sup>9</sup> (Rousseeuw, Katrin, 1999) .

:(MVE)

3.1.2

(MVE)

(MVE)

:

(P) (Distribution Function)

F

$\mu_n(F)$

$\Sigma_n(F)$

:

$$\Sigma \in PSD_{(P)} \{ A \in R^{P \times P} \mid x' A x > 0 \text{ for all } x \neq 0 \in R^P \}$$

:

:  $x$  and  $\mu \in R^P$

$$D_{\mu, \Sigma}^2(x) = (x - \mu)' \Sigma^{-1} (x - \mu)$$

....(2-10)

.  $x$  ,  $\mu$

(MVE)

:

....(2-11)

$$p. \sum, \mu (x) = [\det(\Sigma)]^{-\frac{1}{2}} g([\frac{1}{2} (x-\mu)' \Sigma^{-1} (x-\mu)]^{\frac{1}{2}})$$

:

$$\mu_\alpha (F_{\mu, \Sigma}) = \mu \text{ and } C_\alpha \Sigma_\alpha (F_{\mu, \Sigma}) = \Sigma \quad \dots(2-12)$$

(Fisher Consistency)

) (MVE)

Rousseeuw , Leroy, .(  $C_\alpha = 1 + \frac{15}{n-p}$  :

(1987)

$$(\alpha = \frac{1}{2})$$

(EMVE)

(MVE)

(Exact Minimum Volume Ellipsoid Estimator)

) (P+1)

(MVE)

(Nonsingular)

.(

:

(P+1)

(J)

$J = \{i_1, i_2, \dots, i_{p+1}\}$  of  $\{1, 2, \dots, n\}$

( $\Sigma_j$ )

( $\mu_j$ )

( $\mu_j$ ) ( $x_i$ )

.  $\{x_j / j \in J\}$

(P+1)

:

( $\Sigma_j$ )

....(2-13)

$$D_i^2 (J) = (x_i - \mu_j)' \Sigma_j^{-1} (x_i - \mu_j)$$









**:Rules**

**:(MCD)**

**1.1.2.2**

(j = 1,2,3,...,L) j

(Σ<sub>j</sub>)

(μ<sub>j</sub>)

(h)

(MCD)

(p<sub>j</sub>)

(H)

:

$$\bar{x}_{j,MCD} = \frac{\sum_{i=1}^h x_{ij}}{h}, \quad j=1,2,\dots,L \quad \dots(2-24)$$

$$\hat{\Sigma}_{j,MCD} = dp \times \frac{\sum_{i=1}^h (x_{ij} - \bar{x}_{j,MCD})(x_{ij} - \bar{x}_{j,MCD})'}{h} \quad \dots(2-25)$$

<sup>4</sup>(Hubert, . (P<sub>j</sub><sup>R</sup>)

Driessen, 2004)

$$\hat{P}_j^R = \frac{\tilde{n}_j}{\tilde{n}} \quad \dots(2-26)$$

:

. j

: n<sub>j</sub>

: n

:

(Pooled Var-Cov Matrix)

$$\hat{\Sigma}_{PMCD} = \frac{\sum_{j=1}^L n_j \hat{\Sigma}_{j,MCD}}{\sum_{j=1}^L n_j} \quad \dots(2-27)$$

(2-24) (RLDR)

: (2-27) (2-26),

:  $\prod_k x$

$$\hat{d}_k^{RL}(\underline{x}) > \hat{d}_j^{RL}(\underline{x}), \forall j=1, \dots, L, j \neq k \quad \dots(2-28)$$

:

$$\hat{d}_j^{RL}(\underline{x}) = \hat{d}_j^{RL}(\underline{x}, \hat{\mu}_{j,MCD}, \hat{\Sigma}_{PMCD}) = \bar{\underline{x}}'_{j,MCD} \hat{\Sigma}_{PMCD}^{-1} \underline{x} - \frac{1}{2} \bar{\underline{x}}'_{j,MCD} \hat{\Sigma}_{PMCD}^{-1} \bar{\underline{x}}_{j,MCD} + Ln(\hat{p}_j^R) \quad \dots(2-29)$$

$$(2-29) \quad (L=2) \quad \pi_1 = \pi_2$$

:

$$\begin{cases} x \in \pi_1 & \text{if } (\bar{x}_1 - \bar{x}_2)' \hat{\Sigma}_{PMCD}^{-1} (x - (\bar{x}_1 + \bar{x}_2)/2) > 0 \\ x \in \pi_2 & \text{if } \text{other wise} \end{cases}$$

$$\dots(2-30) \quad ( - )$$

:(RMCD)

2.1.2.2

(RLDR)

(MCD)

( $\hat{\Sigma}_o$ )

( $\bar{x}_{j,o}$ )

(RMCD)

j

(1)

(RMCD)

:

$$\bar{x}_{j, \text{RMCD}} = \frac{\sum_{i=1}^{n_j} w_{ij} x_{ij}}{\sum_{i=1}^{n_j} w_{ij}} \quad \dots(2-31)$$

....(2-32)

$$\hat{\Sigma}_{\text{PRMCD}} = \frac{\sum_{j=1}^L \sum_{i=1}^{n_j} w_{ij} (x_{ij} - \bar{x}_{j, \text{RMCD}})(x_{ij} - \bar{x}_{j, \text{RMCD}})'}{\sum_{j=1}^L \sum_{i=1}^{n_j} w_{ij}}$$

$$x_{ij} \quad (\hat{P}_j^R)$$

:

. j

$$RD_{ij} = \sqrt{(x_{ij} - \bar{x}_{j, \text{MCD}}) \hat{\Sigma}_{j, \text{MCD}}^{-1} (x_{ij} - \bar{x}_{j, \text{MCD}})} \quad \dots(2-33)$$

:

$x_{ij}$

$$RD_{ij} > \sqrt{\chi_{P, 0.975}^2} \quad \dots(2-34)$$

: j

$\tilde{n}_j$

$$\tilde{n} = \sum_{j=1}^g \tilde{n}_j \quad \dots(2-35)$$

$$\hat{\Sigma}_o \quad (2- )$$

( -7)

$$\hat{\Sigma}_{j, \text{MCD}} \quad \hat{\mu}_{j, \text{MCD}}$$

(2-27)

(Pooled Var-Cov Matrix)

(2-31) (2-26)

(RLDR)

<sup>4</sup>(Hubert, Driessen, 2004) : (2-32)

$$\hat{d}_k^{RL}(\underline{x}) > \hat{d}_j^{RL}(\underline{x}), \forall j=1, \dots, L, j \neq k$$

:  $\pi_k$   $x$

:

....(2-36)

$$\hat{d}_j^{RL}(\underline{x}) = \hat{d}_j^{RL}(\underline{x}, \hat{\underline{\mu}}_{j, RMCD}, \hat{\Sigma}_{PRMCD}) =$$

$$\frac{1}{2} \hat{\Sigma}_{PRMCD}^{-1} (\underline{x} - \hat{\underline{\mu}}_{j, RMCD})' \hat{\Sigma}_{PRMCD}^{-1} (\underline{x} - \hat{\underline{\mu}}_{j, RMCD}) + Ln(\hat{p}_j^R)$$

:(MVE) 3.1.2.2

**(Minimum Volume Ellipsoid Estimator)**

. j

$$(\underline{\mu}_j) \quad (j=1, 2, 3, \dots, L)$$

(MVE)  $(p_j) \quad (\Sigma_j)$

(H)  $(h_p)$

(Median Distance)

$$\bar{x}_{j, MVE} = \frac{\sum_{i=1}^{h_p} x_{ij}}{h_p}, j=1, 2, \dots, L$$

....(2-37)

$$\hat{\Sigma}_{j, MVE} = \frac{1}{h_p} \sum_{i=1}^{h_p} \hat{\Sigma}_{(i).j}, j=1, 2, \dots, L$$

....(2-38)

$$(\hat{P}_j^R)$$

.(2- )

: (Pooled Var-Cov Matrix)

$$\hat{\Sigma}_{PMVE} = \frac{\sum_{j=1}^L n_j \hat{\Sigma}_{j,MVE}}{\sum_{j=1}^L n_j} \quad \dots(2-39)$$

(RLDR)

: (Hubert,Driessen,2004) (2-26) (2-39) (2-37)

$$\hat{d}_k^{RL}(\underline{x}) > \hat{d}_j^{RL}(\underline{x}), \quad \forall j=1, \dots, L, \quad j \neq k$$

:

$$\begin{aligned} \hat{d}_j^{RL}(\underline{x}) &= \hat{d}_j^{RL}(\underline{x}, \hat{\mu}_{j,MVE}, \hat{\Sigma}_{PMVE}) = \\ & \frac{1}{2} \hat{\Sigma}_{j,MVE}^{-1} (\underline{x} - \hat{\mu}_{j,MVE})' \hat{\Sigma}_{j,MVE}^{-1} (\underline{x} - \hat{\mu}_{j,MVE}) + Ln(\hat{p}_j^R) \end{aligned} \quad \dots(2-40)$$

: 2.2.2

: (MCD) 1.2.2.2

$$(\hat{\Sigma}_{j,MCD}) \quad (\bar{\underline{x}}_{j,MCD})$$

$$(2-25) (2- ) \quad j=1,2,3,\dots,L \quad j \quad (\hat{P}_j^R)$$

(RQDR) . (2-26)

: (Hubert,Driessen,2004)

$$\hat{d}_k^{RO}(\underline{x}) > \hat{d}_j^{RO}(\underline{x}), \quad \forall j=1,2,\dots,L, \quad j \neq k$$

:

....(2-41)

$$\hat{d}_j^{RQ}(\underline{x}) = \hat{d}_j^{RQ}(\underline{x}, \hat{\underline{\mu}}_{j,MCD}, \hat{\underline{\Sigma}}_{j,MCD})$$

$$= -\frac{1}{2}Ln|\hat{\underline{\Sigma}}_{j,MCD}| - \frac{1}{2}(\underline{x} - \bar{\underline{x}}_{j,MCD})' \hat{\underline{\Sigma}}_{j,MCD}^{-1} (\underline{x} - \bar{\underline{x}}_{j,MCD}) + Ln(\hat{p}_j^R)$$

:(RMCD)

2.2.2.2

$$(\hat{\underline{\Sigma}}_{j,RMCD}) \quad (\bar{\underline{x}}_{j,RMCD})$$

$$\cdot (2-26) (2-9) (2- ) \quad (\hat{p}_j^R)$$

(RQDR)

:(Hubert,Driessen,2004)

$$\hat{d}_k^{RQ}(\underline{x}) > \hat{d}_j^{RQ}(\underline{x}) \quad , \quad \forall j=1,2,\dots,L, j \neq k$$

:  
....( - )

$$\hat{d}_j^{RQ}(\underline{x}) = \hat{d}_j^{RQ}(\underline{x}, \hat{\underline{\mu}}_{j,RMCD}, \hat{\underline{\Sigma}}_{j,RMCD})$$

$$= -\frac{1}{2}Ln|\hat{\underline{\Sigma}}_{j,RMCD}| - \frac{1}{2}(\underline{x} - \bar{\underline{x}}_{j,RMCD})' \hat{\underline{\Sigma}}_{j,RMCD}^{-1} (\underline{x} - \bar{\underline{x}}_{j,RMCD}) + Ln(\hat{p}_j^R)$$

:(MVE)

3.2.2.2

$$(\hat{p}_j^R) \quad (\hat{\underline{\Sigma}}_{j,MVE}) \quad (\bar{\underline{x}}_{j,MVE})$$

$$\cdot (2-26) (2-38) (2- )$$

(RQDR)

:(Hubert,Driessen,2004)

$$\hat{d}_k^{RQ}(\underline{x}) > \hat{d}_j^{RQ}(\underline{x}) \quad , \quad \forall j=1,2,\dots,L, j \neq k$$

:

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$$\begin{aligned} \hat{d}_j^{RQ}(\underline{x}) &= \hat{d}_j^{RQ}(\underline{x}, \hat{\underline{\mu}}_{j,MVE}, \hat{\Sigma}_{j,PMVE}) \\ &= -\frac{1}{2} \text{Ln} \left| \hat{\Sigma}_{j,MVE} \right| - \frac{1}{2} (\underline{x} - \bar{\underline{x}}_{j,MVE})' \hat{\Sigma}_{j,MVE}^{-1} (\underline{x} - \bar{\underline{x}}_{j,MVE}) + \text{Ln}(\hat{p}_j^R) \end{aligned} \quad \dots(2-43)$$

$$\begin{aligned} \hat{d}_j^{RL}(\underline{x}) &= \hat{d}_j^{RL}(\underline{x}, \hat{\underline{\mu}}_{j,MCD}, \hat{\Sigma}_{j,PMCD}) = \\ & \bar{\underline{x}}_{j,MCD}' \hat{\Sigma}_{j,PMCD}^{-1} \underline{x} - \frac{1}{2} \bar{\underline{x}}_{j,MCD}' \hat{\Sigma}_{j,PMCD}^{-1} \bar{\underline{x}}_{j,MCD} + \text{Ln}(\hat{p}_j^R) \end{aligned} \quad \dots(2-44)$$

$$: \quad (2- \quad ) \quad (L=2) \quad \pi_1 = \pi_2$$

$$\begin{cases} x \in \pi_1 & \text{if } (\bar{x}_1 - \bar{x}_2)' \hat{\Sigma}_{j,PMCD}^{-1} (\underline{x} - (\bar{x}_1 + \bar{x}_2)/2) > 0 \\ x \in \pi_2 & \text{if } \text{other wise} \end{cases} \quad \dots (2-45)$$

<sup>3</sup>(Croux, Haesbroeck, 2001)

:  
:  
1.3.2

(Stepwise Selection)

Wilk's Lambda

Wilk's Lambda

<sup>5</sup>(Neil.H.Timm, 2002) :

$$\hat{P} = \frac{|W_{PP}|}{|T_{PP}|} \quad \dots(2-46)$$

:  
:  
:W  
:T



( ) /

$$F(\hat{P}) = \frac{[1 - \hat{P}] [n - P - 1]}{\hat{P}P} \quad \dots(2-46)$$

. F<sub>1,n-P-1</sub>

: 2.3.2

: ( ) . 2 . 3 . 2

:

.(Neil.H.Timm,2002)

$$\chi^2 = -[n-1-1/2(P+g)] \log \hat{e} \quad \dots(2-48)$$

:

. P :

. : g

. ( ) ( ) : ^

.P(g-1)

: . 2 . 3 . 2

.( Neil.H.Timm, 2002):

$$\mu = \left( \sum_{i=1}^k V_i \right) \text{Ln } |S| - \sum_{i=1}^k (V_i \text{Ln } |S|) \quad \dots(249)$$

: C<sup>-1</sup> μ (1949) Box

$$C^{-1} = 1 - \frac{2m^2 + 3m + 1}{6(m+1)(k-1)} \left[ \sum_{i=1}^k \frac{1}{V_i} - \frac{1}{\sum_{i=1}^k V_i} \right] \quad \dots(2-50)$$

χ<sup>2</sup> Box'M

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$$\frac{(m-1)(k-1)}{2}$$

:

:m

:k

:-

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.

:

( 1970 )

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	:		
	(	)	
(2010	2009	2008)	
	(	)	
)	(n <sub>1</sub> =70)	(A.P	)
		(n <sub>2</sub> =70)	(C.H
	(	)	
	(2=	)	(1=
( )			( )
	:		
.	( - )	:	(age) .1
.		:	(weight) .
		:	(Blood Pressure) .
.	( / )		
		:	(Sugar) .
.	( - )		
		:	(Urea) .5
		.	( %)
	:	(cholesterol)	.6
( - )			
.			
.	(2 = )	(1 = )	:
			(sex) .7

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:

Group	No. of Cases	Prior	Group Name
1	70	0.5	
2		0.	
Total	1		

:  
:

Wilk's (Stepwise Discriminant Analysis)  
Rao's V&Lambda

.(3) ( ) ( ) (Sex)

(1)

Variable in the Analysis							
( )							
	Rao's V		Wilk's Lambda	F to Remove	Tolerance	Variables	Step
		.000	.804	.565 2	1.000	Cholesterol	1
.000	.126 1	.000	.804	.429 2	.966	Cholesterol	2
.000	.555 2	.000	.878	02 21.	.966	Weight	
.000	.559 3	.000	.774	24.	.963	Cholesterol	3
.000	6 40.	.000	.771	.008 2	.950	Weight	
.000	.161 4	.000	.733	15.337	.980	Blood Pressure	
.000	.234 4	.000	.711	27.956	.920	Cholesterol	4
.000	.908 6	.000	.627	12.290	.888	Weight	
.000	.471 5	.000	.688	16.332	.977	Blood Pressure	
.000	.309	.000	.634	9.176	.876	Sugar	
.000	.318 5	.000	.777	31.930	.900	Cholesterol	5
.000	.513 7	.000	.654	13.588	.879	Weight	
.000	.21 6	.000	.666	18.383	.976	Blood Pressure	
.000	.837 2	.000	.654	7.555	.878	Sugar	
.000	.474 8	.000	.587	6.317	.955	Urea	

.000	85.	.000	.614	10.432	.687	Cholesterol Weight Blood Pressure Sugar Urea Age	6
.000	.006 7	.000	.644	15.578	.850		
.000	.581 8	.000	.678	13.739	.965		
.000	.473 8	.000	.665	9.325	.876		
.000	.327 9	.000	.566	7.576	.976		
.000	.976 9	.000	.509	5.991	.683		

( )

**Wilk's Lambda**

Sig.	df2	df1	Exact F		df3	df2	df1	Lamb da	Nu mbe r of Vari able s	Step
			Statistic	F						
.000	136.000	1	21.555		136	1	1	.758	1	1
.000	135.000	2	23.504		136	1	2	.638	2	2
.000	134.000	3	22.863		136	1	3	.562	3	3
.000	133.000	4	21.408		136	1	4	.420	4	4
.000	132.000	5	17.242		136	1	5	.491	5	5
.000	131.000	6	16.094		136	1	6	.370	6	6

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**Rao's V**

v in Change		Rao's V			Entered	Step
Sig.	Statistic	Approx. sig.	df	Statistic		
.000	25.655	.000	1	21.333	Cholesterol	1
.000	26.406	.000	2	48.222	Weight	2
.000	23.333	.000	3	68.321	Blood Pre	3
.000	15.322	.000	4	80.554	Sugar	4
.000	11.345	.000	5	90.564	Urea	5
.000	9.567	.000	6	105.666	Age	6

(Normal)

(SPSS.16)

(Weight)

(Blood Pressure )

(Urea)

(cholesterol)

(Sugar)

(Age)

(Box- and -Whisker Plot)

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( ) ( )

.( )

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(2.48)

0.01

$$\chi^2 = 66.841$$

(2.50) (2.49)

Box'sM=132.488

4.3 :

(Q.Basic)

(( )

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( )

: (5)



(4)

RQ MVE	RLM VE	RQR MCD	RLR MCD	RQ MCD	RLM CD	QDF	LDF	Variables
- 211.4 3	114.4 9	4.4319	- 137.7 2	-242. 35	4.210 8	- 160. 08	1.38 34	constant
1.12	1.11	.99	0.96	2.01	2.29	79.2 5	80.5 6	Age
1.01	1.14	3.07	0.69	2.12	51.45	4.73	5.81	Weight
0.4	0.96	1.1	1.06	1.97	2.61	18.7 5	21.9 3	Urea
0.79	0.68	-7.30	0.98	1.92	624.2 3	13.7 4	14.4 1	Blood Pressure
0.71	0.87	2.77	1.30	2.22	682.7 8	.01	.02	Sugar
0.77	1.12	1.39	1.30	1.93	419. 34	4.93	5.85	Cholesterol

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(5)

<b>RQ MVE</b>	<b>RL MVE</b>	<b>RQ RMC</b>	<b>RL RMC</b>	<b>RQM CD</b>	<b>RLM CD</b>	<b>QD F</b>	<b>LDF</b>	<b>Variable s</b>
- 301. 65	102. 65	6.55	- 403. 21	- 303.5 2	4.34	- 180. 09	4.21	<b>constant</b>
1.06	1.05	0.68	0.03	1.56	1.34	42.6 2	19.95	<b>Age</b>
0.77	1.18	2.29	2.03	2.16	1.94	0.44	0.29	<b>Weight</b>
0.76	0.99 0	1.88	1.88	1.94	1.93	4.61	3.81	<b>Urea</b>
- 1.03	0.78	4.14	2.26	2.56	1.77	3.01	2.56	<b>Blood. Pressure</b>
1.39	1.28	2.90	1.84	1.88	1.81	0.01	0.01	<b>Sugar</b>
0.73	1.17	1.54	2.26	1.85	2.09	1.08	1.29	<b>Choleste rol</b>

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(

.(7)(6)

( ) /

(6)

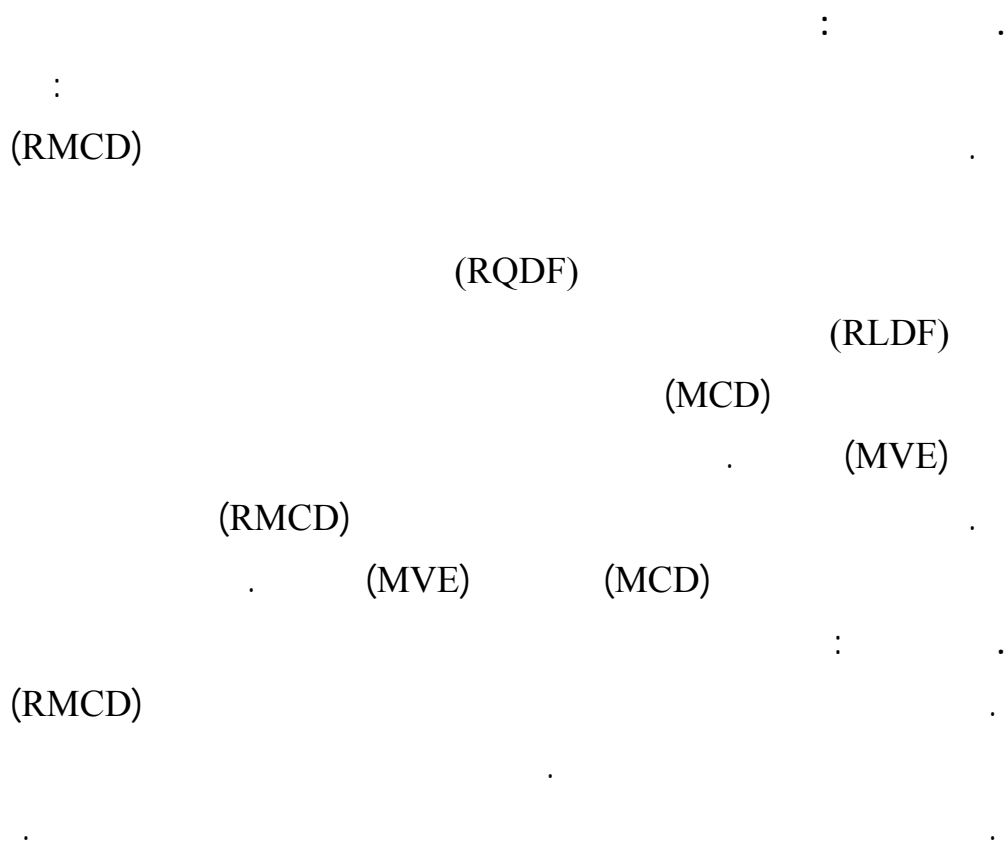
LDF	QDF	RLM CD	RQM CD	RLRMC D	RQRMC D	RLM VE	RQM VE
0.53	0.42	0.32	0.29	0.10	0.09	0.36	0.30

(7)

LDF	QDF	RLM CD	RQMC D	RLR MC D	RQR MCD	RL MV E	RQ MV E
0.42	0.31	0.22	0.19	0.08	0.04	0.26	0.23

) RMCD .( )

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- " (1990)
- " (2001) (26)
- "
- " ( )
3. Croux, C.& Hasbrouck, G.,(2001), "A Note on finite-sample efficiencies of Estimators for the Minimum Volume Ellipsoid ", Submitted.
  4. Hubert, M., Driessen, K.V., (2004),"Fast and Robust Discriminant Analysis", Computational statistics and Data Analysis, vol .45, 301-20.
  5. Neil.H.Timm. (2002),Applied Multivariate analysis,John Wiley & sons.
  6. Pison, G., Van Aelst, S., and Willems, G. (2002b), "Small Sample Corrections for LTS and MCD," *Metrika*, 55, 111-123.
  7. Rousseeuw, P.J., (1985) "Multivariate Estimation with High Breakdown Point", *Mathematical Statistics and Application*, B., pp. 283-397.
  8. Rousseeuw, P.J.&Leroy,A.M.,(1987)"Robust Regression and Outlier Detection", John Wiley & Sons, New York.
  9. Rousseeuw, P.J., &KatrinV.D.,(1999)"A fast Algorithm for the Minimum Covariance Determinant Estimator", *Technometrics*, 41, pp. 212-223.

