THERMAL STABILITY ANALYSIS OF LAMINAR NANOFLUIDS FREE CONVECTION IN HORIZONTAL CYLINDER

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ABSTRACT :-
The effect of nanoparticles on the heat transfer of the fluids becomes one of the most interesting fields for researches. Metallic and nonmetallic materials are used to enhance the thermal performance of the fluids. In this work, an attempt was made to extend the work in this field to study the effect of addition of copper oxide nanoparticles on the transient behavior of free convection in horizontal cylinder. The dimensionless form of the energy and the momentum equations was solved using the commercial package COMSOL 3.5. A constant wall temperature CWT boundary condition was subjected to the surfaces of the cylinder. The lower half of the walls temperature was always higher than the upper one. The effect of volume fraction of nanoparticles on the stability of the transient behavior of the heat transfer and the fluid flow was examined. Also the effect of Rayleigh number on the velocities, temperature and Nusselt number are shown graphically. It is found that not only the onset of instability of the natural convection but also the type of the steady state profiles of temperature and velocities were affected by the existence of the nanoparticles. A phase plane plot was used to analysis the steady state types of dynamic behavior of the nanofluid natural convection under the CWT boundary condition.

KEYWORDS; stability, natural convection, nanofluid, CuO, COMSOL

الخلاصة :-

تأثير الجسيمات النانوية على انتقال الحرارة خلال السوائل يصبح واحدا من المجالات الأكثر إثارة لاهتمام الباحثين . وتستخدم المواد الفلزية واللافلزية لتعزيز الأداء الحراري للسوائل . في هذا العمل ، كانت هناك محاولة لتوسيع العمل في هذا المجال لدراسة تأثير إضافة أكسيد النحاس على السلوك الغيرمستقر للحمل الحراري الحر في اسطوانة أفقية . تم حل نموذج غيربعدي لمعادلات الطاقة و الزخم باستخدام برنامج COMSOL 3.5 . وقد تم اختيار حدود المعادلات في حالة تعرض الاسطوانة الى درجة حرارة الجدار ثابت CWT . وكان النصف السفلي من جدران الاسطوانة دائما بدرجة حرارة أعلى من العلوي . تم فحص تأثير النسب الحجمية من الجسيمات النانوية على ثبات سلوك استقرار لنقل

الحرارة و تدفق السوائل . أيضا وتظهر تأثير عدد رايلي على السرعات ، ودرجة الحرارة وعلى عدد نسلت بيانيا . وقد وجدت أنه ليس فقط بداية حدوث عدم الاستقرار للحمل الحراري الطبيعي ولكن أيضا نوع منجني الحالة المستقرة لدرجة الحرارة و السرعات تأثرت بوجود الجسيمات النانوية . تم استخدام مخططات مستوي المرحلة لايجاد تحليل أنواع حالة الاستقرار للسلوك الديناميكي للموائع النانوية للحمل الحراري الطبيعي تحت شرط الحدود CWT .

NOMENCLATURE

Cp	Specific heat (kJ /kg K)	t	Time (s)
g	Gravity acceleration (m/s^2)	T	Temperature (K)
Gr	Grashof number	u	Velocity in x direction (m/s)
k	Thermal conductivity (W /m K)	U	Dimensionless x component of velocity
Nu	Nusselt number	v	Velocity in y direction (m/s)
p	Pressure (Pa)	V	Dimensionless y-component of velocity
P	Dimensionless pressure	x	Distance in x direction (m)
Pr	Prandtl number	X	Dimensionless x coordinate
Ra	Rayleigh number (Gr Pr)	y	Distance in y direction (m)
s	Arc length	Y	Dimensionless y coordinate

Greek symbols		Subscripts	
α	Thermal diffusivity (m^2/s)	c	Cold
β	Volumetric thermal expansion coefficient (1/K)	f	fluid
θ	Dimensionless temperature	h	Hot
μ	Dynamic viscosity (kg/m s)	n	normal direction
ν	Kinematic viscosity (m^2/s)	p	particle
ρ	Density (kg/m3)	∞	environment
τ	Dimensionless time	1,2	Upper / Lower surface
Γ	Boundary		
λ	Dummy-variable		

1. INTRODUCTION

Natural convection heat transfer is an important phenomenon in engineering systems due to its wide applications in electronic cooling, heat exchangers, phase change materials, etc. Due to the low thermal conductivity of convectional heat transfer fluids such as water and oils, different techniques were used to enhance the thermal performance of such systems. One of the promising techniques is the addition of nano-scale particles with high thermal conductivity to the base fluid which enhance the heat transfer process in these systems. Such fluids are called as nanofluid which is firstly used by Choi Recently, some numerical and experimental works with nanofluids include enhancement of thermal conductivity (Kang et al.), forced convection heat transfer (Maiga et al,), heat transfer with boiling and free convection (Xuan and Li,). Some reviewed works are published by Putra et al. Wang et al. Trisaksri and Wongwises, Abu-Nada &Wang and Mujumdar

For natural convections transfer using nanofluids, relatively few studies have been carried out. Hwang et al. investigated the buoyancy-driven heat transfer of water-based Al_2O_3 nanofluids in a rectangular cavity. Khanafer et al. investigated the effect of nanoparticles on enhancement of the heat transfer in a two-dimensional enclosure. Jou and Tzeng used nanofluids to enhance natural convection heat transfer in a rectangular enclosure. Jang and Choi investigated the Benard regime in nanofluid filled rectangular enclosures. Wang et al. conducted a study on natural convection in nanofluid filled vertical and horizontal enclosures. Also, a recent study by Polidori et al., analyzed the heat transfer enhancement in natural convection using nanofluids. Oztop and Abu-Nada studied the heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using nanofluids using different types of nanoparticles.

A little effort has been apply to investigate the natural convective instability of nanofluid in enclosures. A few studies have also been concerned with the problem of the onset of Rayleighe Bénard convection in a horizontal layer of nanofluids heated from below has been investigated by Kim et al. on the basis of a factor which measures the ratio of the Rayleigh number of a nanofluid to that of a base fluid.

Tzou investigates thermal instability of nanofluids taking into account the combined behaviors of Brownian motion and thermophoresis of nanoparticles.

In the present paper, we consider natural convection in a horizontal cylinder of copper oxide water nanofluid (Cuo-water) subjected to constant different temperature at the lower and the upper halves. The effect of addition of nanoparticles on the stability of thermal behavior of the enclosure will be investigated.

2. MATHEMATICAL MODEL

A schematic of the two-dimensional system with geometrical and boundary conditions is shown in Fig.(1). In the present analysis, Cartesian coordinate system will be applied to the horizontal cylinder. The nanofluid in the enclosure is Newtonian, incompressible, and laminar and is assumed to have uniform shape and size. The nanofluid used, which is composed of copper oxide nanoparticles in suspension of water, has been used at various particle concentrations ranging from 0 to 20% in volume. Moreover, it is assumed that both fluid phase and nanoparticles are in thermal equilibrium state The cylinder is heated from below with constant wall temperature, where the lower half of the cylinder is heated and maintained at a constant temperature (T_h) higher than the upper cold half temperature (T_c) which equals to the environment temperature ($T_c = T_\infty$). The Boussinesq approximation will relate the variable density to the local temperature.

The unsteady state equations governing the transport of mass, momentum and thermal energy in the two dimensional systems are as follows:

The thermo-physical properties of the nanofluid atare listed in Table (1).

The viscosity and the thermal conductivity of the nanofluid are given by the following models:

$$\frac{k_n}{k_f} = \frac{k_p + 2k_f - 2(k_f - k_p)\varphi}{k_p + 2k_f + (k_f - k_p)\varphi} \qquad \text{Maxwell–Garnetts model} \qquad (1)$$

$$\mu_n = \frac{\mu_f}{(1-\varphi)^{2.5}} \qquad \text{Brinkman-Model} \qquad (2)$$

The unsteady state equations governing the transport of mass, momentum and thermal energy in the two dimensional systems are as follows:

Continuity equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

Momentum equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_n}\frac{\partial p}{\partial x} + \frac{\mu_n}{\rho_n}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (4)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_n}\frac{\partial p}{\partial y} + \frac{\mu_n}{\rho_n}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta_n(T - T_\infty) \qquad (5)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_n\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (6)$$

The following dimensionless variables are used:

$$X=\frac{x}{L};\quad Y=\frac{y}{L};\quad \tau=\frac{t\alpha_f}{L^2}$$

$$U=\frac{uL}{\alpha_f};\quad V=\frac{vL}{\alpha_f};\quad P=\frac{pL^2}{\rho_f{\alpha_f}^2};\quad \theta=\frac{T-T_c}{T_h-T_c}$$

$$k=\frac{k_n}{k_f};\quad \alpha=\frac{\alpha_n}{\alpha_f};\quad \mu=\frac{\mu_n}{\mu_f};$$

The governing equations are re-written in above dimensionless form as follows:

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\Phi(\rho)}\frac{\partial P}{\partial X}+Pr\frac{\mu}{\Phi(\rho)}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (7)$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{\Phi(\rho)}\frac{\partial P}{\partial Y}+Pr\frac{\mu}{\Phi(\rho)}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+RaPr\Phi(\beta)\theta \quad (8)$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial x}+V\frac{\partial \theta}{\partial y}=\frac{k}{\Phi(\rho C_p)}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (9)$$

where

$$\Phi(\lambda)=1-\varphi+\varphi\frac{\lambda_p}{\lambda_f}$$

is the property function; e.g;

$$\Phi(\beta)=1-\varphi+\varphi\frac{\beta_p}{\beta_f}$$

The following dimensionless initial boundary conditions are imposed:

And the following dimensionless initial and boundary conditions are imposed:

Initial boundary

$$\theta=0;\ U=0;\ V=0 \quad @\,\tau=0 \quad (10)$$

Boundary conditions

$$\theta=0;\ U=0;\ V=0 \quad @\Gamma_2$$

$$\theta=1;\ U=0;\ V=0 \quad @\Gamma_1 \quad (11)$$

The local Nusselt number along the hot wall is defined as

$$Nu=k\frac{-L}{(T_h-T_c)}\frac{\partial T}{\partial n}\bigg|_{\Gamma_1}=-k\frac{\partial \theta}{\partial n}\bigg|_{\Gamma_1} \quad (12)$$

and the average Nusselt number is expressed as

$$Nu_{av}=\int Nu\,ds. \quad (13)$$

3. NUMERICAL SOLUTION

The set of PDEs (7-9) and the associated dimensionless initial and boundary conditions (10-11) have been solved using COMSOL 3.5. The package used the finite element method to solve the dimensional forms of governing equations represent the multiphysics problems. However, in this work an attempt was made to write a code with the COMSOL Script and solve the dimensionless form of the governing equations.

In order to have an independent grid size numerical solutions, the Nusselt number was determined for different number of elements as shown in Fig.(2). It is found that the discretization of the problem with about of ($9x10^3$) elements is suitable to have convenient solutions.

4. RESULTS AND DISCUSSION

First of all, the effect of increasing the nanoparticle volume fraction on the transient behavior of the laminar natural convection inside the enclosure was examined. A typical example of this problem for Rayleigh number of *5.2*$x10^4$, Fig(3), shows the temperature profile of the nanofluid enclosed in the cavity during different time steps. Besides the field velocity profiles are shown for the same time steps. It is shown that the heat-momentum dynamic behavior of the system for a pure water (ϕ=0) at Ra= $5.2x10^4$ are unstable with sustainable periodic variation in temperature-velocities profiles. As (ϕ) increases to (0.1) the transient behavior of the system becomes stable. The temperature-velocities profiles reach the steady state after some time intervals and all variables become time independent.

To investigate the onset of instability in the dynamic behavior of this problem, some experiments were performed near the value of Ra=$5x10^4$. Fig.(4) shows the bifurcation line which separates the two; stable and unstable regions. The upper region of (Ra-ϕ) indicates where the behavior is dynamically unstable whereas the lower region is for stable behavior.

Fig.(5) shows the effect of increasing the volume fraction of nanoparticles on the average Nusselt number. For Ra=$6x10^4$, as is less than the critical value defined by the bifurcation line, the transient behavior is always unstable. As (ϕ) increases the system becomes stable and the Nu increase also. The average value of the average Nu is about (3) for pure water whereas the enhancement due to addition of copper oxide nanoparticles with (20%) volume fraction reaches to three times than the pure water at the same Ra.

The same behavior is shown in Fig.(6) where the increase of Ra affect the velocity field indicated as average velocity through the whole enclosure.

The most interested representation of the numerical analysis of the problem is the plot of the so called the (phase plane) graph that demonstrates the average temperature gradient in the cavity versus the average velocity field. Fig.(7) shows that the addition of nanoparticles to the base fluid increase the stability of the thermal behavior. The steady state type of the system changes from sustainable steady state *limit cycle* at Ra=6x10^4 for pure water to *focus* at ϕ =0.2) .This manipulation of steady state behavior is called as Hopf bifurcation.

5. CONCLUSIONS

The transient behavior of copper oxide-water nanofluids natural convection in a horizontal cylinder has been studied numerically in this work. The natural convection flow is caused by the temperature difference between the cold top and hot bottom part of the cavity. It is found that the effect of addition of nanoparticles to the base water changes the thermal behavior of the fluids. Both the velocities and the temperature were changed from the sustainable periodic profiles to the stable profiles at constant Rayleigh number. It was found that the critical Rayleigh number is higher than that for pure fluid. It was found that the solution was more stable than the base fluid. The stability of the system is enhanced as the volume fraction of the nanoparticles increases and their size decreases or the average temperature of nanofluids increases. The phase plane of average temperature difference - velocity field analysis indicates that the type of the steady state developed from stable limit cycle to stable focus as the nanoparticles volume fraction exceeds the bifurcation values for each Rayleigh number.

Table (1) :Thermophysical properties of Cuo-water At room Temp

Property	CuO	Water
Cp (J/kg K)	540	4179
ρ (kg/m^3)	6500	997
k (W/m K)	18	0.61
β (1/K)	0.85x10^{-5}	2.1x10^{-4}
μ (kg/m s)		0.85x10^{-3}

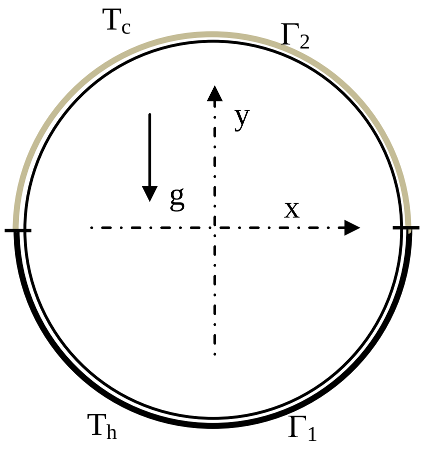

Fig.(1) : Physical model.

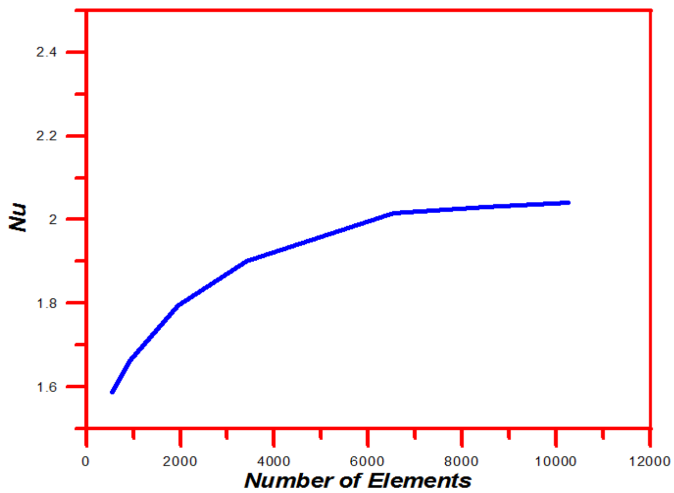

Fig.(2): Steady State Averge Nusselt number vs. Number of Elements (Ra=10^4)

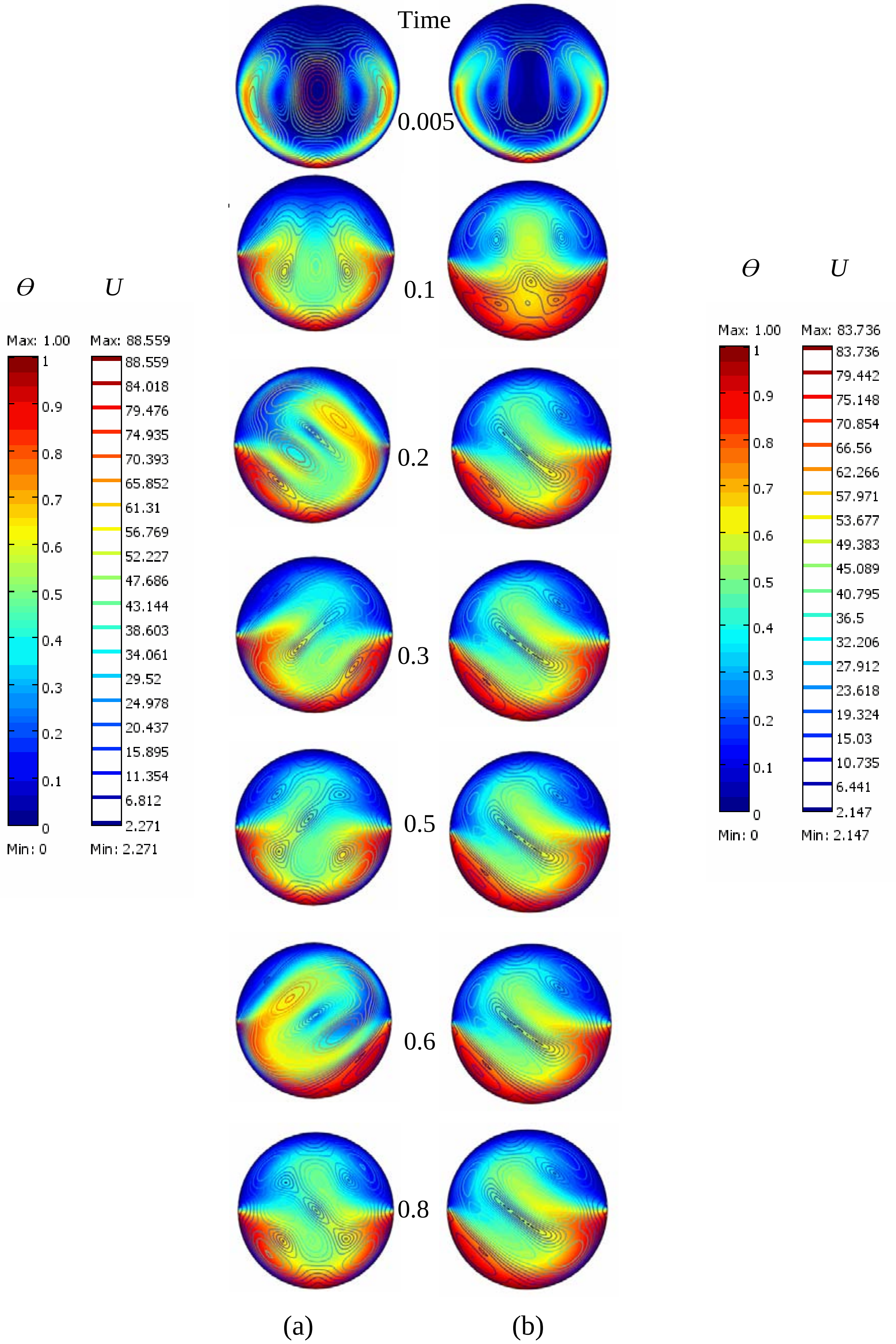

Fig.(3): Isotherms (Θ) and contours(U) at different time steps for Rayleigh Number=5.2x10^4
a- *φ=0.0* b- *φ=0.1*

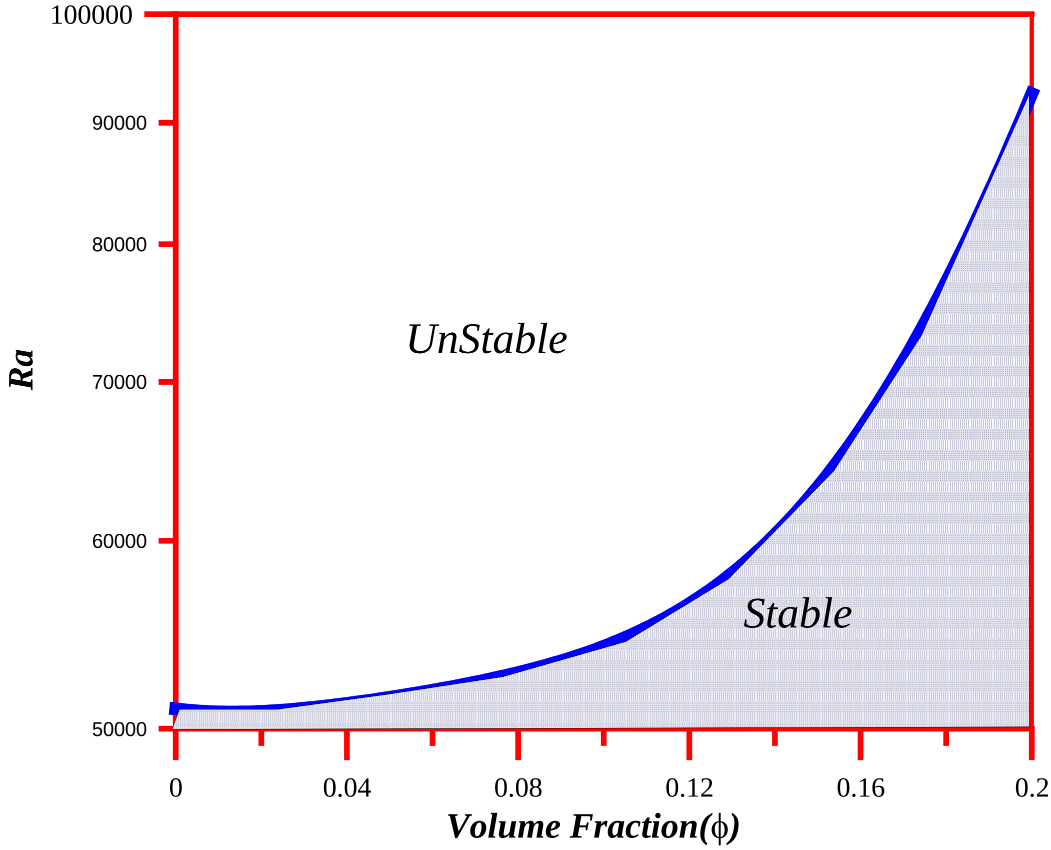

Fig.(4): Bifurcation line for Water-Copper Oxide Nanofluids.

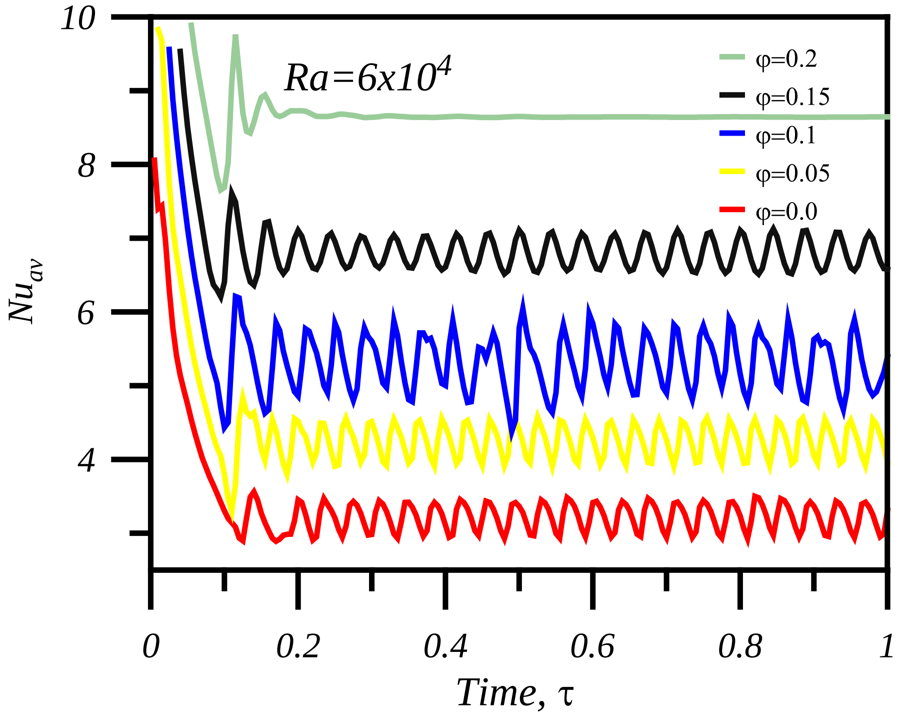

Fig.(5): Variation of average Nusselt Number with time for different values of volume fraction (ϕ) at Rayleigh number ($6x10^4$).

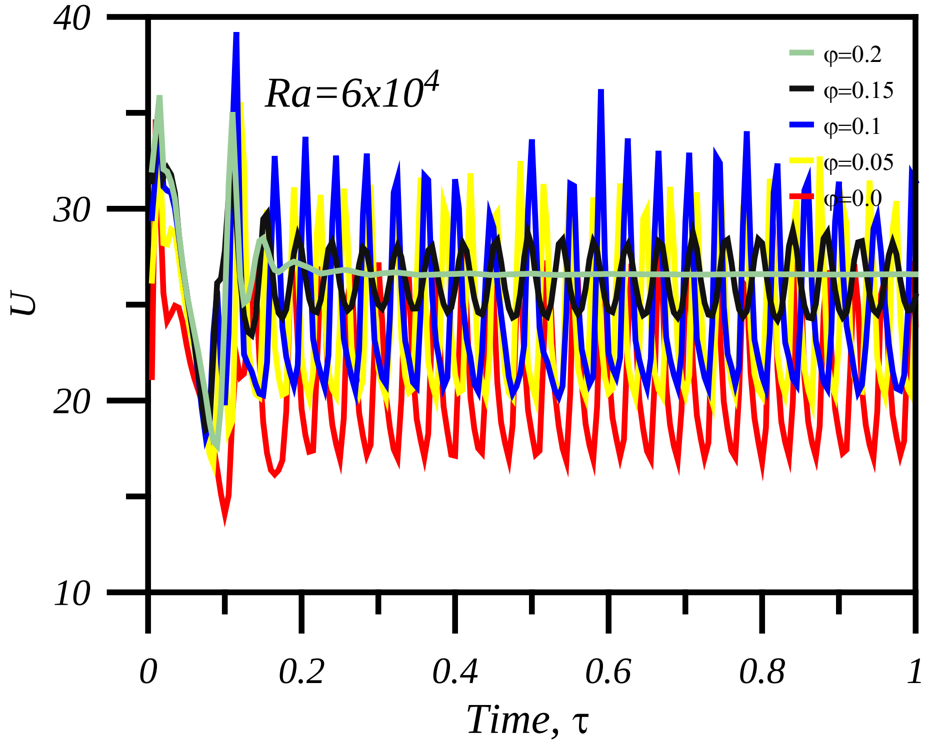

Fig.(6): Variation of average Velocity Field with time for different values of volume fraction (ϕ) at Rayleigh number ($6x10^4$).

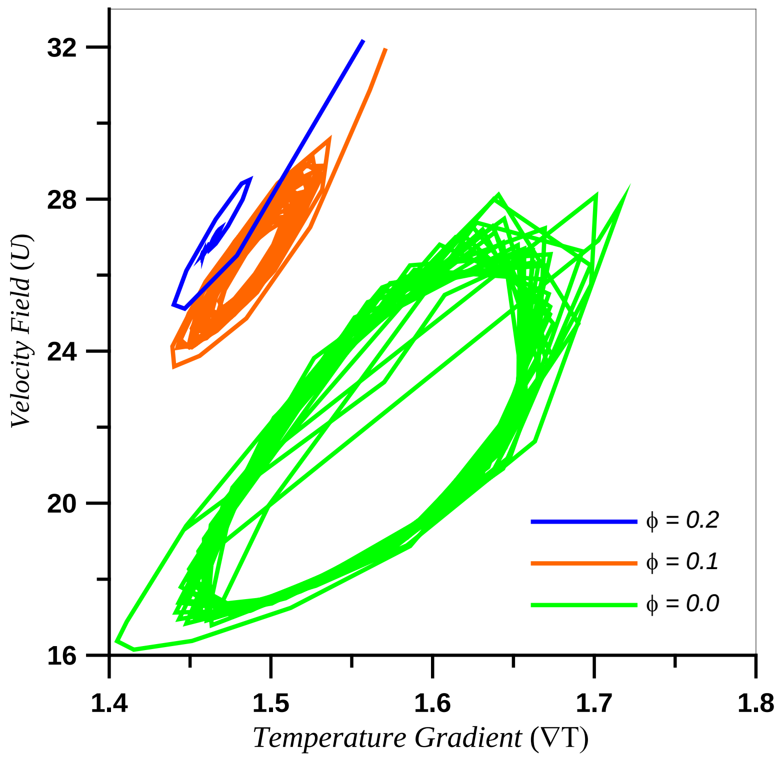

Fig.(7): Phase Plane of Nusselt Number- Velocity Field for different values of volume fraction (ϕ) at Rayleigh number ($6x10^4$).

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